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Bigger Slice or Larger Pie?
Optimal Marketing Strategies for New Firms

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Abstract

Just as their established counterparts do, new firms that pioneer nascent industries strive to increase profit by acquiring the sales of rival firms. But these new firms can also increase sales by attracting new customers to their nascent industry. This article investigates this tradeoff in marketing expenditure. By deriving conditions under which an increase in the sales of its rival can augment a firm's profit in a single-period model, we highlight how new firms can distinguish themselves. Extensive numerical analysis on a multi-period framework suggests that an increase in a firm's unit profit margin, effectiveness in gaining new sales, or initial sales level, but a decrease in sales decay, may cause a positive spillover for its rival. This research thus provides guidance to new firms attempting to find a balance between growing their industry (i.e. the pie) and competing on market share (i.e. the slice).

Keywords: Marketing, Nascent Industry, Expenditure Strategies, Optimal Control, Differential Games

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1. Introduction

Investing in advertising based on product characteristics and product comparisons can allow a new firm to acquire existing industry sales units belonging to other new firms. The targeting of another firm's sales to gain market share – i.e. increase the slice of the pie – can be quite direct. An example of such targeting is Microsoft's campaign against AOL in the Internet service provider (ISP) market. AOL had become the market leader among ISPs by offering users an easy-to-use product. MSN then offered, among other benefits that targeted AOL users, an application for AOL users who chose to switch to MSN that transferred the e-mail addresses and calendar information from the AOL program to the MSN program (Hu 2002).

But a new firm can also benefit from investing to "create" customers – i.e. increase the whole pie. Such a customer has not yet been attracted to any firm and is considered an "unrealized" customer for the industry until he becomes a customer to one of the firms. Unlike in established industries, entrepreneurs must inform unrealized customers about their new ventures and also convince prospective customers that they are legitimate. Aldrich and Fiol (1994) refer to this as cognitive legitimation, whereby the new firm needs the spread of knowledge among unrealized customers to become wider so that more unrealized customers will trust that firm and become "realized" customers. To do so, entrepreneurs often use awareness or informational advertising (Dorf and Byers 2005). Sometimes the investment in attracting unrealized customers may be indirect and aimed at changing industry conditions or government regulations. For instance, AOL, MSN, and other ISP firms made investments to remove threats to industry sales growth, such as choosing to file joint lawsuits in an attempt to slow down Internet spam communication.

Brandenburger and Nalebuff (1996) refer to this simultaneous need to compete and cooperate as co-opetition. What makes nascent competition interesting is the need for competing new firms to build primary demand (i.e. increase the overall customer demand for the industry) by investing to attract unrealized customers. Once a firm makes an unrealized customer aware of its product, all nascent industry members can compete for the business of that now realized customer. Thus, firms may potentially benefit from the spillover of other firms' increased primary demand as well as their own. In fact, "most entrepreneurs will never be able to independently create a new market simply by building their companies" (Goldstein 2004: B2). The radio-frequency identification tag (RFID) industry, for instance, had been around since the

1980's with relatively low sales. Industry sales grew rapidly when several new firms agreed to common standards, which made it possible for Wal-Mart and the Department of Defense to mandate RFID tag usage throughout their supply chain. Orders for hundreds of millions of tags for Wal-Mart's suppliers will now be fulfilled by many different manufacturers within the RFID industry.

New firms certainly are aware that market share activities alone (increasing their slice of the pie via sales acquisition) without any market growth activities (increasing the size of the pie via sales attraction) is likely to lead to a decrease in the size of the market. The decrease occurs because realized customers depart the industry due to their own forgetfulness or dissatisfaction with some aspect of the industry, and no new customers are replacing them. In the action pursuit games industry, Bernstein (2004) argues that "because communication has been focused almost exclusively on product sales, this ignores the fact that improving the environment for making sales IMPROVES [emphasis by the author] sales […] [U]ltimately, there won't be a market unless all of us learn a little more about good PR" (p 140). Consequently, optimal promotional strategies should not only evaluate the proper expenditure amount, but also how much to spend on unrealized as compared to realized customers. We intentionally use the term *expenditure* rather than advertising because we want to consider the full range of investment activities that affect purchases. As outlined in Aldrich and Fiol (1994), legitimizing activities that help grow sales include common standard-setting, influencing third-party actors, or governmental lobbying—not just traditional advertising. We therefore ask *how a new firm attempting to maximize profit should choose between investing to attract unrealized customers, which also increases total industry sales, and investing to acquire realized customers, which only increases the new firm's sales.*

The selection of the time frame between product launch and significant sales penetration allows us to complement the literature on new product marketing, as well as on market share games in oligopolies. The gap in the entrepreneurial marketing literature is articulated by Morris et al. (2002), who comment that an unanswered question is whether optimal levels of entrepreneurial marketing exist and, if so, what parameters influence those optimal levels. Soberman and Gatignon (2005) also draw attention to the lack of marketing research directed towards understanding the relationship between competitive dynamics and market evolution. Studies on market share games in oligopolies model only one advertising expenditure variable, which thus assumes the effect of a new firm's advertising to be the same on all customers. Fruchter (1999) points out that her oligopoly advertising model would be more accurate if it would specify the difference in efficiency between acquiring a customer from a rival firm and attracting unrealized customers. Thus, we address a gap in the literature by modeling customers according to whether they are realized or as yet unrealized.

The remainder of the article is organized as follows. We review relevant literature and then formulate a model where the expenditure for attracting unrealized sales units is modeled as a separate decision than that of the expenditure for acquiring sales units from the firm's rival. We derive optimal expenditure strategies for two competing firms in a single-period and a multi-period (differential game) model. For the latter, we use numerical and graphical methods to further our understanding of industry growth. Sensitivity analysis of the optimal expenditures is the emphasis of this numerical analysis. We conclude with a discussion on extensions and opportunities for future work.

2. Literature Review

We focus on the literature on market share games in oligopolies. First, we review studies that deal with spillover benefits occurring simultaneously with competition among firms. Then, we highlight the work on continuous-time monopoly settings and oligopoly settings (differential games) where customer segmentation occurs.

Wrather and Yu (1979) directly model spillover benefits to other firms in an oligopoly setting. In their framework, a firm can invest in product expenditures (which have spillover benefits for other firms) or brand expenditures (which are predatory in nature and acquire the sales of other firms). The objective of both firms is to maximize sales. They derive equilibria and comment on incentives for cooperation; however, there is no dynamic component to their model. In a multi-period, discrete-time oligopoly setting, Friedman (1983) sets up a game where multiple firms simultaneously decide on their quantity sold and advertising level. A parameter is used to model an industry level of cooperative or predatory behavior. However, that parameter is the same for the entire industry and unaffected by advertising expenditures or time. Also, Friedman (1983) does not distinguish between unrealized and realized sales, as we do. In Krishnamurthy's (2000) single-period game, instead, firms choose generic advertising (which expands the market) and brand advertising (which increases firm market share). He assumes that generic advertising is cooperative and focuses on when cheap-riding occurs. Krishnamurthy et al. (2003) argue that cooperative generic advertising is limited to industries subject to government regulation or experiencing sales decline. Neither phenomenon is likely for a nascent industry.

The Lanchester model of combat – first suggested for advertising applications in Kimball (1957) – assumes that a firm acquires competitors' sales units via its advertising expenditures and loses its own sales units via competitors' advertising expenditures. Most Lanchester models consider industry sales to be of fixed size. On the other hand, Vidale and Wolfe (1957) examine a situation where there are unrealized sales and, as a result, industry sales are not fixed. They model the growth and decay of the sales of a monopolist who can attract new sales from unrealized sales by increasing advertising expenditures. The waning effect of advertising expenditures is modeled by a decay factor that acts on realized sales of the firm. Scholars have combined the concepts of the Lanchester and Vidale-Wolfe advertising models to relax their limitations. The Vidale-Wolfe model is augmented by Sethi (1973), who expands the modeling of a monopoly and Deal (1979), who includes competition in a duopoly. Some of the more recent combinations of the two models are Fruchter (1999), who adds growth to a Lanchester model, and Wang and Wu (2001), who include decay and growth in a Lanchester formulation. However, this literature still represents advertising activity by one variable that equally impacts all sales not belonging to the firm. (Extensive reviews on differential games that use advertising expenditure as decision variable and sales or market share as state variable appear in Erickson 1995, 2003 and Feichtinger et al. 1994.)

There has recently been increased interest in customer segmentation in differential games, using different variables to represent investments in retaining the firm's own customers, acquiring customers from another firm, and attracting new customers to the industry. For multiple firms and fixed industry sales, Fruchter and Zhang (2004) model expenditures for retaining and acquiring customers. Instead, Hartl and Kort (2005) model expenditures for retaining and attracting customers in a single firm context. Because of our focus on nascent industries, we treat retention only in passing. The multi-period duopoly model that most closely resembles our work is that of Bass et al. (2005a, 2005b), which models brand and generic advertising for two firms in an infinite and finite horizon context, respectively. The generic advertising of one firm directly benefits the other just the same as if the other firm would have done the generic advertising. Their model assumes that the market size expands limitlessly with generic advertising and thus that industry sales are non-decreasing over time. For the infinite horizon model, they present as an extension a model that assumes that the market has a ceiling. We model a market with a ceiling and sales decay in a finite horizon context. We also differ from this work in that our expenditure toward unrealized customers is not a "public good" type of generic advertising. Instead, the other firm only benefits from a firm's expenditures that impact unrealized customers by acquiring the newly realized customers of that firm.

Our modeling contribution to the oligopoly literature is twofold. First, we are able to represent the competitive components of the Lanchester model and the market growth and decay components of the Vidale-Wolfe model. To our knowledge, no other work has yet included the decay component as well; we show that sales decay has a strong effect on expenditure policies and profits. Second, we separate the sales of the rival firm from unrealized sales in a multi-period context. As stated above, most previous research models expenditure as one variable. However, for a new firm, the one-variable modeling approach assumes that firm expenditures primarily aimed at acquiring another firm's sales have the same impact on unrealized sales, and firm expenditures meant to attract unrealized sales have the same impact on another firm's sales. Such an assumption may hold true in mature industries, where even customers who have yet to buy are familiar with the product and an established firm gains sales only at the expense of other firms. However, this assumption is likely to be violated in a nascent industry. Realized customers are typically early adopters and innovators (Moore 2000), while most unrealized customers need to be educated about the product (Dorf and Byers 2005) and overcome legitimacy concerns about the industry (Aldrich and Fiol 1994) before becoming realized customers. Also, a new firm can gain sales from attracting unrealized customers, which does not lower the sales of any other firm. The separation in sales necessitates using two expenditure variables for distinct investments in realized and unrealized sales (as done in Wrather and Yu 1979 and in Bass et al. 2005a, 2005b).

3. Optimal Control Model

3.1. Control Variables

Let t represent continuous time and T be the length of the planning horizon. Firm i chooses to invest in acquiring the sales of another firm at a cost $a_i(t)$; this investment is in *share expenditures.* Symmetrically, firm i chooses to invest in attracting new customers via the unrealized group of customers at a cost $w_i(t)$; this investment is in *growth expenditures.* Growth expenditures can also be thought of as a firm's investment in speeding up product diffusion to potential customers. As the nascent industry grows to maturity, the total sales of the firms approach the market ceiling. As a result, there are fewer and fewer unrealized customers to be acquired over time. Yet, to increase industry sales, firm *i* must attract unrealized customers to itself via $w_i(t)$, whereas to turn realized industry sales into sales, firm *i* must invest to acquire customers of firm *j* via $a_i(t)$.

3.2. Law of Motion for Sales

We focus on a duopoly of firms *i* and *j*. Let $S_i(t)$ be the number of units sold by firm *i* at *t*, and $S_j(t)$ be that of firm *j*. Hence,

$$I(t) = S_i(t) + S_j(t) \tag{1}$$

is the number of units sold by the entire industry at *t*. We were inspired by the Lanchester model (Kimball 1957) and Vidale and Wolfe (1957) for our formulation of the law of motion of $S_i(t)$. The share of realized industry sales held by each firm varies over time as a result of share expenditures $a_i(t)$. The effectiveness of share expenditures depends on the firm as well as product and market conditions. We denote the effectiveness of share expenditures as a constant $\rho_i \in (0,1]$. Firm *i*'s share expenditures have an impact of value $\rho_i f_i(a_i(t))$ on acquiring the sales of firm *j*, whereas it loses its own sales at a rate determined by the share expenditures of firm *j*. Thus, each dollar of share expenditures that impacts the other firm's sales via $f_i(a_i(t))$ yields a ρ_i fraction of the other firm's realized sales. The function $f_i(a_i(t))$ is positive and increasing in its argument. Firm *j* benefits from firm *i*'s growth via $\rho_j f_j(a_j(t))$. These combined effects on firm *i*'s sales are

$$\rho_i f_i(a_i(t))S_j(t) - \rho_j f_j(a_j(t))S_i(t). \tag{2}$$

Industry sales reach market saturation as firms build primary demand. Let *m* be the fixed number of units at which the market saturates, which corresponds to the maximum possible sales for the industry (the assumption that the market ceiling can be known is also used by Vidale and Wolfe 1957, Deal 1979, Fruchter 1999, and Wang and Wu 2001). Industry growth can only occur by attracting unrealized sales units that no firm has yet acquired, specifically $m - I(t)$ such units. We allow firm attraction of unrealized customers to depend on the impact of its growth expenditures, as do Vidale and Wolfe. Effectiveness of growth expenditures for firm *i* is denoted as a constant $\beta_i \in (0,1]$. Firm *i*'s expenditures have an impact of value $\beta_i g_i(w_i(t))$ on attracting unrealized sales units to firm *i*. Thus, each dollar of growth expenditures that impacts unrealized sales via $g_i(w_i(t))$ yields a β_i fraction of unrealized sales. The function $g_i(w_i(t))$ is positive and increasing in its argument. There are also non-expenditure related factors that affect sales. Let $[1-\delta_i] \in [0,1]$ be the sales decay parameter for firm *i*; it represents the proportion of sales units that depart the firm (i.e. become unrealized) due to factors such as forgetfulness and product and market characteristics. These combined effects on firm *i*'s sales are

$$\beta_i g_i(w_i(t))[m - S_i(t) - S_j(t)] - [1-\delta_i]S_i(t). \tag{3}$$

From combining (2) and (3), the change in firm's *i* sales over time can be expressed as

$$\dot{S}_i(t) = \rho_i f_i(a_i(t))S_j(t) - \rho_j f_j(a_j(t))S_i(t) + \beta_i g_i(w_i(t))[m - S_i(t) - S_j(t)] - [1-\delta_i]S_i(t). \tag{4}$$

The four terms in (4), respectively, are the gains in sales units acquired from competition, losses in sales units due to competition, gains in sales units due to the entry of unrealized sales units into the industry, and losses in sales units due to the exit of previously realized sales units from the industry. We note that $f(\cdot)$ and $g(\cdot)$ can be chosen so that equation (2) is the Lanchester equation that describes firm competition for existing sales, and equation (3) is the Vidale-Wolfe model describing growth and decay of sales.[1]

3.3. Objective Function

Let π_i be firm *i*'s unit profit margin, which is assumed fixed over time. Following the lead of Deal (1979), firm *i*'s objective is to select on the finite horizon [0,*T*] a stream over time of share expenditures and of growth expenditures that maximize its profit (net of advertising costs) plus a valuation of the firm's end-of-horizon sales. Formally,

$$P_i = \underset{\substack{a_i(t), w_i(t) \\ t \in [0,T]}}{\text{Max}} \int_0^T [\pi_i S_i(t) - a_i(t) - w_i(t)]dt + r_i S_i(T), \tag{5}$$

[1] The original 1957 model uses [$m - I(t)$]/m as opposed to $m - I(t)$. We can add 1/m by adjusting β_i accordingly.

where r_i is the salvage value firm *i* places on end-of-horizon sales. For a firm solely concerned with profit, r_i equals zero. However, a firm that cares about maximizing its end-of-horizon sales level more than profit would set r_i at a relatively high positive level. Similar to Wang and Wu (2001), we do not include a discount factor in the objective function. This allows for the model recommendations for investment over time to be more interpretable. Also, because we are interested in the nascent period of the industry, *T* would be relatively small, making discounting unessential.

Firm *i*'s complete optimal control formulation includes (4), (5) and the following conditions. Initial sales are set to a constant, $S_i(0) = S_i^0$, whereas end-of-horizon sales $S_i(T)$ are free and not required to meet a pre-set level (similarly for firm *j* by symmetry). The control variables are assumed to be nonnegative with $a_i(t) \in [0, \infty)$ and $w_i(t) \in [0, \infty)$. We discuss how $S_i(t)$, $S_j(t)$, and $m - S_i(t) - S_j(t)$ are kept nonnegative as we solve the single-period model.

4. Single-Period Analysis

In this section we characterize the optimal levels of share and growth expenditures for a scenario with two firms where each makes a one-time decision. We investigate the single-period model for two reasons. First, it helps build insight for the multi-period setting, as comparative static results cannot be obtained in that setting. Second, the solutions for the single-period model require little information relative to that of the multi-period one and could thus conceivably be used as a decision support tool in practice. The solutions of most differential games (and ours) require knowing the effectiveness and decay parameters for the other firm(s) and, as a result, empirical verifications (e.g. as in Erickson 1992 and Wang and Wu 2001) have tended to focus on mature commodity oligopolies (e.g. Anheiser Busch vs. Miller, Coke vs. Pepsi) with a well-known advertising history. Further, sensitivity analysis from a single-period setting is likely to provide insights as to what adjustments should be taken in a multi-period setting when a parameter change only affects the results of a single period – e.g. a firm's future sales may be unaffected by a decrease in sales decay due to current weather.

As a special case of the previous problem formulation we therefore solve for optimal expenditures in a single-period context with no salvage value. Because there is no sales overlap from period to period, there is no Nash equilibrium; the optimal expenditures of the firm do not change the optimal expenditures of its rival. As suggested earlier, initial values for sales of firm *i* and *j* are denoted as S_i^0 and S_j^0. Because there are no upper bound constraints on share and growth expenditures and this is a one-shot decision, end-of-period sales could result in infeasible total industry sales or sales acquisition from a rival firm (i.e. $S_i + S_j > m$, $S_i < 0$ or $S_j < 0$). Infeasible solutions are prevented by scaling the effectiveness parameters, unit profit margin and sales decay appropriately (empirical literature cited above offers applications in practice).[2]

Firm *i* must thus choose expenditure levels that solve

$$P_i = \max_{a_i, w_i} \pi_i S_i - a_i - w_i$$

$$\text{with } S_i = S_i^0 + \rho_i f_i(a_i) S_j^0 - \rho_j f_j(a_j) S_i^0 + \beta_i g_i(w_i)[m - S_i^0 - S_j^0] - [1 - \delta_i] S_i^0 . \tag{6}$$

The second order conditions for optimality is met (and there exists a unique optimal solution) if f_i and g_i are strictly concave in their respective parameter.[3] These conditions result in diminishing returns from the expenditure variables.

We select exponent functions (as e.g. Erickson 1985) to further our analysis. These functions are desirable because by varying the value of the exponent it is possible to "swipe" an infinite set of possible curves, making it easier to transform our theoretical model into an empirically testable framework. Thus,

$$f_i(a_i) = a_i^{1-\gamma_i} \text{ and } g_i(w_i) = w_i^{1-\theta_i}, \tag{7}$$

where γ_i, $\theta_i \in (0,1)$ represents the elasticity of share and growth expenditures, respectively, for firm *i*. The relationship between elasticity and effectiveness for expenditures is that elasticity measures the firm's operational ability, and effectiveness measures the firm's marketing ability. In other words, elasticity measures how much of the firm's investment is directly applied to realized and unrealized sales, and effectiveness measures the fraction of realized and unrealized sales that are acquired and attracted by the firm's investment. This yields the optimal solution

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[2] The formulation in (6) can be constrained by requiring that $\rho_i f_i(a_i) \leq \delta_i$ and $\beta_i g_i(w_i) + \beta_j g_j(w_j) \leq 1$, followed by using the Karesh-Kuhn-Tucker conditions to solve for optimal expenditures. If the constraints are not binding, optimal expenditures are obtained by using the first-order conditions on the objective function with respect to expenditures. If a constraint is binding, because of the linear costs in the objective function optimal expenditures are

$$f_i(a_i^*) = \delta_i / \rho_i \text{ or } g_i(w_i^*) = [1 - \beta_j g_j(w_j)] / \beta_i, \tag{*}$$

depending on which constraint is binding. However, note that implementations of (*) mean that firm competition is over, as either $S_j = 0$ or $S_i + S_j = m$, respectively. Thus, we focus on the unconstrained solutions of the expenditure variables for subsequent analysis.

[3] Negative second order derivatives and $\partial^2 f_i(a_i)/\partial w_i \partial a_i = \partial^2 g_i(w_i)/\partial a_i \partial w_i = 0$ yields a negative definite Hessian.

$$a_i^* = \left[\pi_i \rho_i S_j^0 [1-\gamma_i]\right]^{\frac{1}{\gamma_i}} \text{ and } w_i^* = \left[\pi_i \beta_i [m - S_i^0 - S_j^0][1-\theta_i]\right]^{\frac{1}{\theta_i}}. \quad (8)$$

Therefore, share expenditures depend on how many industry sales do not currently belong to firm *i*, and growth expenditures depend on how many unrealized sales still remain. Because of this dependence on industry behavior in aggregate, the strategy of firm *i* as it pertains to the other firm only depends on its initial sales level. We analyze how the solution in equation (8) is affected by changes in key model parameters by performing sensitivity analysis, which is summarized in Table 1. Results are obtained by taking the partial derivatives of the expenditure variables in (8) with respect to each parameter, holding all other parameters constant and assuming, as needed, that $0 < S_i^0 + S_j^0 < m$. We note that the conditions for concavity are sufficient to derive via comparative static analysis the results in Table 1 on optimal share and optimal growth expenditures[4].

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Insert Table 1 about here

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In discussing our findings, we refer to firm *i* as *the firm* and to firm *j* as *its rival*. We first consider firm parameters that have no effect on the rival's performance – the firm's growth expenditure effectiveness and sales decay. Then, we explore the impact of the firm's parameters that do affect the rival's performance, namely unit profit margin, share expenditure effectiveness, and initial sales levels.[5]

Effects of the firm's growth expenditure effectiveness (β_i). An increase in β_i, which also means an increase in the firm's number of sales that can be realized, also causes an increase in total expenditures, end-of-period sales, and profit but only for the firm. In contrast to Krishnamurthy (2000), the rival does not increase profit because growth expenditures are not cooperatively spent.

[4] Comparative static analysis for the general functions f_i and g_i is done using the Implicit Function Theorem (e.g. Currier 2000). Since all cross-derivatives equal 0, given parameter μ for firm *i*, $\frac{\partial a_i}{\partial \mu} = \frac{\partial^2 P_i / \partial a_i \partial \mu}{-\partial^2 P_i / \partial a_i^2}$ and $\frac{\partial w_i}{\partial \mu} = \frac{\partial^2 P_i / \partial w_i \partial \mu}{-\partial^2 P_i / \partial w_i^2}$. The results hold when f_i and g_i are increasing concave functions for both firms.

[5] For both firms, an increase in *m*, the industry ceiling, means an increase in the number of sales that can be realized, and thus causes an increase in their growth expenditures, end-of-period sales, and profit.

Effects of the firm's sales decay ($1-\delta_i$). An increase in ($1-\delta_i$), however, does not affect the firm's total expenditures or those of its rival, but it does mean that the firm can retain fewer realized sales and loses both end-of-period sales and profit (although the rival's end-of-period sales and profit are unchanged). The expenditure finding is striking: firms should either spend more to offset the effects of sales decay or spend less because they cannot retain the sales they gain. We investigate this further in the multi-period setting.

Effects of the firm's unit profit margin (π_i). An increase in π_i leads to the firm spending more on both attracting unrealized sales and acquiring sales of its rival. This finding is supported in the single-period model of Monahan (1987). Since an increase in the firm's total expenditures (not due to any sales losses) leads to increases in its end-of-period sales, one would also expect an increase in its profit. However, this is not necessarily the case as exhibited in Table 1 with a non-monotonic relationship between π_i and profit. Further, its rival's profit and end-of-period sales decrease if the firm obtains higher profits for each unit sold. For example, if the firm announces a major breakthrough in production that results in cost savings, then unit profit margin for the firm increases, and its rival should be prepared for the firm to spend more to acquire its sales.

Effects of the firm's share expenditure effectiveness (ρ_i). If some element of the firm's product or advertising ability that affects existing sales units improves, causing an increase in ρ_i, then the firm should always spend more to attract sales units from its rival. This unconditional finding is in contrast to Monahan (1987). In Monahan's market share attraction game, a firm's market share depends on the ratio of its advertising impact to the sum of advertising impact for both firms. Because there were zero unrealized sales, he found that the firm may decrease expenditures if it already is superior to its rival in the ratio of unit profit margin or share expenditure effectiveness. An increase in ρ_i increases end-of-period sales and profit for the firm because of the extra sales and revenue from share expenditures. An increase in ρ_i, however, does not affect the rival's expenditures, but its rival's profit and end-of-period sales decrease if the firm improves the effectiveness of its share expenditures. The profit results complement the findings of Krishnamurthy (2000).

Effects of the firm's initial sales level (S_i^0). We consider at length the relationships between initial sales levels and both expenditures and firm performance because these relationships are counter-intuitive and several are non-monotonic. If S_i^0 increases, then the firm should keep constant spending on acquiring the rival's sales units and decrease spending to attract unrealized ones. The decrease occurs because the pool of unrealized sales has grown smaller and, as a result, less is spent on pursuing those sales. If sales decay is low, then the firm may increase end-of-period sales and profit from a high S_i^0. Because this is a single-period model, if sales decay is high, the firm may have an incentive to have low initial sales levels. But what happens to the expenditures and profit of the firm if its rival experiences a change in its initial sales? We next identify the condition that determines whether total expenditure increases or decreases.

Specifically, an increase in initial sales for the rival results in the firm increasing its share expenditures but decreasing its growth expenditures. The rationales for these findings are intuitive. The rival is found to have more initial sales units than the firm expected. Because all other variables, including initial sales of the firm, are considered constant for the sensitivity analysis, this rival sales growth must be due to rival attraction of unrealized sales. Therefore, the firm transfers some of its expenditures from unrealized to realized sales units. If the elasticity for growth and share expenditures are the same (i.e. $\gamma_i = \theta_i$) – e.g. due to departmental overlap in overhead and other costs – we can state a relatively simple condition ($\frac{S_j^0}{[m - S_i^0 - S_j^0]} > \left[\frac{\beta_i}{\rho_i}\right]^{\frac{1}{1-\gamma_i}}$) under which an increase in initial sales for its rival leads to an increase in the firm's total expenditures. A high elasticity of expenditures makes it more difficult for the condition to be met if the firm is better at attracting unrealized sales than acquiring sales from its rival (i.e. its effectiveness ratio β_i/ρ_i exceeds 1) and easier to meet the condition otherwise ($\beta_i/\rho_i < 1$). If the effectiveness ratio of growth to share expenditures is less than one, an increase in the rival's initial sales level yields larger total expenditures, end-of-period sales, and profit for the firm if the rival's initial sales (S_j^0) exceed unrealized sales ($m - S_i^0 - S_j^0$).

In a Vidale-Wolfe single-period model with indirect firm competition, an increase in its rival's initial sales always lowers the firm's profit. In a Lanchester single-period model where unrealized sales are always zero, an increase in its rival's initial sales always lowers the firm's end-of-period sales. We balance these results by showing that an increase in its rival's sales may actually increase the firm's sales (and profit). This may seem counter-intuitive at first sight, but consider a firm that is inefficient in attracting unrealized sales, yet efficient at convincing customers of its rival to switch to its product. Due to its small efficiency ratio, such a firm can benefit from increases in the sales of its rival from being able to acquire them. We verified that the left-hand side of the above inequality is largest, and therefore that inequality is most likely to hold, when $S_j^0 = I$, the industry level of sales. In other words, a firm is more likely to profit from increased sales of its rival if that rival dominates the industry in terms of number of sales. This observation, along with our findings, shows that the firm is less (more) likely to profit from the growth of its rival when either that rival's sales or total industry sales are small (large, respectively). We next explore the robustness of some of these findings in a multi-period setting.

5. Multi-Period Analysis

5.1. Optimal Expenditure Strategy

The appropriate solution method for an open-loop Nash equilibrium for the optimal control problem formulated in (4) and (5) is that of differential games. Using Pontryagin's maximum principle, we maximize the Hamiltonian for firm *i* as given by

$$H_i(t) = \pi_i S_i(t) - a_i(t) - w_i(t) + \lambda_i(t)\dot{S}_i(t) + \varphi_i(t)\dot{S}_j(t), \tag{9}$$

with boundary equations $\lambda_i(T) = r_i$ and $\varphi_i(T) = 0$. The immediate profit of the firm is $\pi_i S_i(t) - a_i(t) - w_i(t)$, as we saw in the analogous single-period case. In a multi-period setting, however, we must also consider the time remaining until the end of the planning horizon and the fact that a firm's current sales can be acquired by its rival in the future. The future value of a sales unit to firm *i* is $\lambda_i(t)\dot{S}_i(t) + \varphi_i(t)\dot{S}_j(t)$, as co-state variable $\lambda_i(t)$ measures the future value of one additional sales unit for firm *i* to firm *i*'s profits and co-state variable $\varphi_i(t)$ measures the future value of one additional sales unit for firm *j* to firm *i*'s profits. Solving the necessary conditions for the Hamiltonian with respect to the control variables yields optimal expenditures

$$a_i^*(t) = \left[\left[\lambda_i(t) - \varphi_i(t)\right]\rho_i S_j(t)[1-\gamma_i]\right]^{\frac{1}{\gamma_i}} \text{ and } w_i^*(t) = \left[\lambda_i(t)\beta_i[m - S_i(t) - S_j(t)][1-\theta_i]\right]^{\frac{1}{\theta_i}}.^6 \quad (10)$$

A closed form solution for the control variables, which are affected by future sales values, requires a closed form solution for the co-state variables. However, the co-state variables cannot be solved in closed form, and thus no direct sensitivity analysis can be conducted in order to make comparisons with the single-period model. Instead, we gain insights through numerical analysis on the behavior of the control variables, state variables, and profit over time.

5.2. Numerical Analysis

We use a solution algorithm that discretizes time using difference approximations, and solves for the state and co-state variables using forward and backward passes as described in Erickson 2003 (p 61-62). We did extensive testing over a wide range of parameter values. Care was taken in choosing the parameters; large values of growth expenditure effectiveness or unit profit margin may result in solution instability. To simplify the solution process, we confined the numerical analysis to the case where $\gamma_i = \gamma_j = \theta_i = \theta_j = 1/2$. For the sensitivity analysis, the market ceiling *m* is standardized by setting it equal to 1; consequently unit profit margin for one firm refers to the value of all sales to that firm. The effects of salvage value were found to be similar to the results of Deal (1979) in that increasing the salvage value that a firm places on its end-of-horizon sales increases its total expenditures and end-of-horizon sales, but decreases its profit. As assumed in Bass et al. (2005b) and for ease of comparison, we use $r_i = r_j = 0$.The length of the planning horizon affects the end-of-horizon sales values. A very short finite horizon is dominated by the values chosen for initial sales level and salvage value at the end of the horizon. However, an infinite horizon is unrealistic as we investigate herein the nascent period of the industry; initial sales levels and salvage value should impact end-of-horizon sales and profit. We chose a common planning horizon of $T = 6$ periods to perform sensitivity analysis.[7] Because financing is important for new firms, we also record first period earnings (profit from $t = 0$ to $t =$

[6] We note that the optimal decision variables in (8) and (10) are nearly identical except for the unit profit margin. Profit is now a function of sales' future value over time rather than the fixed unit profit margin, π_i. In equation (10), the unit profit margin is the difference in future profit from firm *i* possessing a sale over and above that of the rival possessing that sale ($\lambda_i(t)$ - $\varphi_i(t)$). In contrast, only the future profit for firm *i* is included in equation (10) for growth expenditure. This is because an unrealized customer has zero value to the firm before being realized.
[7] A different time horizon may yield, for instance, different end-of-horizon sales for both firms, but the direction of the sensitivity relationships appears robust to the length of the time horizon.

1). Simpson's 1/3 Rule, a calculus technique for approximating a definite integral, was used to sum first period earnings, profit, and expenditures. A sampling of the run results are offered in Table 2. The first half of each run is when the firm has an advantage on the rival in growth effectiveness, whereas the second half of each run is when the firm has an advantage on the rival in share effectiveness.

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(Insert Table 2 about here)

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The discussion that follows focuses on how the dynamic dimension changes some of the results from our single-period model and previous models in the literature. We provide in Table 3 a summary of the conjectures derived from our numerical analysis. To showcase the impact of firms having different levels of growth and share effectiveness, we choose parameters that allow us to demonstrate the full effects of parameter changes on share expenditure, growth expenditure, end-of-horizon sales, and profit. In discussing our findings, we still refer to firm *i* as *the firm* and to firm *j* as *its rival*.

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(Insert Table 3 about here)

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Effects of growth expenditure effectiveness (β_i). As per Run (1) in Table 2, an increase in β_i yields increased end-of-horizon sales and profit for the firm, but unlike Wang and Wu (2001) it does not yield diminished total expenditures. From a comparative static analysis, Bass et al. (2005a) show in an infinite-horizon game that an increase in growth expenditure effectiveness for one firm increases only that firm's growth expenditures but increases profit for both firms. Instead, Run (1) shows that regardless of each firm's effectiveness in expenditures, higher growth expenditure effectiveness for the firm decreases the growth expenditures, but increases the share expenditures, of its rival as there are fewer unrealized sales than before and more realized sales for the firm.

There are also several non-monotonic results from an increase in growth expenditure effectiveness. In Run (1), we show how the share expenditure effectiveness of the rival affects whether that rival increases end-of-horizon sales and profit when the firm increases growth expenditure effectiveness. In the first half of Run (1), the rival possesses a share expenditure effectiveness advantage compared to the firm (.030 to .010). If the firm increases its growth expenditure effectiveness, the rival exhibits higher end-of-horizon sales and profit. The firm responds by increasing its own share expenditures because of its rival's sales growth. Since the rival is skilled at acquiring the firm's sales, it benefits from the firm's improvement. In the second half of Run (1), instead, the firm possesses a share expenditure effectiveness advantage compared to its rival (.030 to .010). If growth expenditure effectiveness for the firm is .005, .100, and .200, its rival initially exhibits lower end-of-horizon sales and profit and the firm decreases share expenditures. It is only when the firm's advantage in growth expenditure effectiveness (.300 as compared to .005 for the rival) is larger than its advantage in share expenditures effectiveness (.030 to .010) that the rival starts increasing its end-of-horizon sales and profit.

The result in Run (1) suggesting that the rival's profit can be decreased by an increase in firm growth expenditure effectiveness disagrees with the monotonic increase finding of Bass et al. (2005a). The rationale is that the decline in unrealized sales caused by the firm's improved growth expenditure effectiveness benefits its rival only if that rival possesses sufficient effectiveness in share expenditures. In our model, there is not a guaranteed spillover from the firm's growth expenditures to the sales of its rival, as there is in Bass et al. (2005a, 2005b). Thus, we demonstrate that an increase in a firm's ability to attract new sales does not always benefit competing firms. We also note that in the first entry of Run (1), where growth expenditure effectiveness for both firms is .005, both firms exhibit positive first period earnings, but low profit. As growth expenditure effectiveness increases, the firm's first period earnings become more and more negative, although when the firm's growth expenditure effectiveness is .040, this trend begins to reverse itself. Comparing the first to the second half of Run (1), it is also more expensive in the first period if the firm's share expenditure effectiveness is relatively high. Thus, firms with expertise in rapid sales growth may face a higher risk of running out of cash.

Effects of sales decay ($1-\delta_i$). As per Run (2), an increase in ($1-\delta_i$) yields decreased end-of-horizon sales and profit for the firm, which supports the work of Deal (1979), and lower growth and share expenditures, in contrast to Wang and Wu (2001) who argue for an increase in expenditures. Further, if the firm experiences higher sales decay, its rival decreases share expenditures but increases growth expenditures. However, unlike Deal (1979) who did not have separate effectiveness measures for unrealized and realized customers, higher sales decay for the firm may cause a decrease in end-of-horizon sales and profit for its rival. The rationale is that if the firm's rival prefers to acquire the firm's sales rather than to attract unrealized customers, a drop in the firm's sales will hurt it. We note that the reduction in expenditures caused by a higher sales decay means that first period earnings increase, but in the long run the firm loses profit.

Graphical illustrations of sales, profit, and expenditures over time for the firm provide additional insights; these illustrations are for the first half of Run (2). As per Figure 1(a), low sales decay leads to higher sales at every time period. By the end of the first period, reduced growth expenditures begin to harm the firm's profit, as per Figure 1 (b), and by $T = 6$ the firm is most profitable and has the most sales with lower sales decay. Share expenditure curves, as in Figure 1(c), keep their shape, but are lowered as sales decay increases. The inverted U-shape is commonly observed when the rival increases sales, but when, instead, it decreases sales or grows them slowly a monotonically decreasing convex curve is most often obtained. For growth expenditures, Figure 1 (d) suggests that high sales decay lowers initial spending on growth expenditures. However, after $t = 3$, the reverse is true; the higher the sales decay, the more the growth expenditures. In experimenting with this and other parameter sets over longer time horizons, we find that growth expenditures and in some cases total expenditures may increase over time if the firm experiences a loss in sales. This is in contrast to the majority of the studies that use differential games where they show a pattern of initial high expenditures to rapidly acquire sales, followed by a monotonic decrease in expenditures over time (e.g. Erickson 1995). Studies that show non-monotonic expenditure behavior include Erickson (1985) when a market is rapidly expanding and Bass et al. (2005a, 2005b) for brand advertising (similar to our share expenditure) when both firms increase sales over time. We complement these studies by showing that firm growth expenditures can increase over time, even when the firm's sales decrease.

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(Insert Figure 1 about here)

..…………………………..

Effects of unit profit margin (π_i). As per Run (3), an increase in π_i yields increased end-of-horizon sales and profit for the firm, which is in contrast to our single-period findings where profit decreases. The rationale appears to be that when there is a carryover effect in sales, the firm can increase its profit no matter what the rival does. An increase in unit profit margin for the firm also increases expenditures for the firm, a result that is also confirmed by Wang and Wu (2001). We were able to construct a few runs (not shown) where the rival decreased its share expenditures, but Run (3) represents the more typical rival reaction with increased share expenditures and decreased growth expenditure. Arguably, the rival's reactions could be because the firm's increase in share expenditures causes investing in growth expenditures to be less profitable for the rival, but the firm's increase in end-of-horizon sales causes investing in share expenditures to be more profitable for the rival. Surprisingly, in contrast to our single-period model, an increase in unit profit margin for the firm may increase the end-of-horizon sales and profit of the rival, as seen in the second half of Run (3). The rationale is that the rival may benefit because, in a multiple-period context, it can leverage its greater expertise in acquiring the firm's sales rather than attracting unrealized sales.

Our numerical findings for the rival thus contrast those of Bass et al. (2005a), who find that a firm's increase in unit profit margin decreases share expenditures and steady-state market share for its rival. Therefore, we offer an altered recommendation. When the firm's unit profit margin increases, rather than reacting by cutting expenses, as in the Bass et al. model, its rival should typically increase share expenditures. That higher unit profit margin for the firm results in higher end-of-horizon sales for that firm and thus its rival's increase in share expenditures may actually allow it to take away enough of the firm's sales to increase its own end-of-horizon sales and profit despite the firm's increase in unit profit margin.

Last, when unit profit margin increases in Run (3), first period earnings tend to decrease and profits increase. For further analysis, we focus on the case where both firms have a unit profit margin of 100. If the firm is relatively skilled at acquiring realized customers then its first period earnings are positive, whereas if the firm is relatively skilled at attracting unrealized customers then it absorbs negative first period earnings (but its profit is more than triple that of the rival). Thus, it appears that the firm that specializes in targeting its rival is less likely to face negative earnings early, but is less profitable in the long term than the firm that specializes in attracting new customers. We also note that a firm can grow quickly enough in the first period to experience the double benefit of increased first period earnings and profit (e.g. we compare with Run (1) when growth expenditure effectiveness for the firm increases from .030 to .040).

Effects of share expenditure effectiveness (ρ_i). As per Run (4), an increase in ρ_i yields increased end-of-horizon sales and profit for the firm, which is in contrast to Bass et al. (2005a) who argue that decreased profit may also occur. As in Bass et al. (2005a), however, the firm increases share expenditures and the rival decreases growth expenditures. The rationale is that the rival, realizing that growth expenditures become less valuable when newly realized sales are targeted more intensely by the firm, should diminish its growth expenditures. Run (4) shows non-monotonic responses in growth expenditures for the firm and share expenditures for the rival. It appears that the firm's increase in sales at its rival's expense is eventually so rapid that its rival increases share expenditures to gain some sales back. Bass et al. (2005a) also find ambiguous results for the firm's growth expenditures, but analytically found that the rival should decrease share expenditures when the firm increases its share expenditure effectiveness. Our result in Run (4) seems to indicate that when there are few initial realized sales, the rival's share expenditures are minimally affected by changes in the firm's share expenditure effectiveness. Thus, advertising wars appear to be rare in our framework.

Of the parameters in Table 3, share expenditure effectiveness is the only one whose increase suggests that the rival exhibits lower end-of-horizon sales and profit. The profit decrease is in contrast to the ambiguous result Bass et al. (2005a) derive. We also demonstrate in Run (4) that increases in share expenditure effectiveness are not as expensive for the firm in the first period as increases in the other parameters are. In fact, an increase in share expenditures for the firm has negligible negative effect on its first period earnings and increases its rival's first period earnings in the first half of Run (2), where the firm is stronger in growth expenditure effectiveness. However, increases in profit for Run (4) tend to be less than that of Run (2) because sales are only added at the expense of the other firm.

Effects of Initial Sales (S_i^0). As per Run (5), a higher S_i^0 yields an increased profit for the firm. We chose a relatively higher unit profit margin and lower sales decay to showcase a surprising result: end-of-horizon sales may decrease if the firm exhibits higher initial sales. The decrease occurs if the firm faces a rival with sufficiently high share expenditure effectiveness, as seen in the second half of Run (5). In contrast to our single-period model, the multiple-period framework does not suggest less profit from an increase in initial sales. This observation provides one of the reasons why the multi-period framework improves on the logic of the single-period one: the latter could not unconditionally state that having a high level of initial sales was advantageous to the firm.

Furthermore, an increase in the firm's initial sales appears to decrease its growth expenditures and that of the rival (because there are now fewer unrealized sales), but increase the share expenditures of the rival. In similar fashion to earlier results, the firm may slightly increase its share expenditures if the rival's increase in share expenditures results in firm sales loss. We note that if the rival exhibits relatively high share expenditure effectiveness, the rival uses share expenditures to increase its end-of-horizon sales and profit. But if that rival exhibits relatively low share expenditure effectiveness, it does not benefit in end-of-horizon sales or profit.

6. Implications and Future Work

In this article we synthesize the Lanchester (inspired by Kimball 1957) and Vidale Wolfe (1957) models for a nascent industry to show how firms should choose between share expenditures (customer acquisition at the expense of a rival firm) and growth expenditures (attraction of unrealized customers). In a single-period setting, we showed that firms should increase expenditures as their effectiveness in those expenditures increase, and that a firm may benefit from the sales growth of its rival. Therefore, firms in the midst of a growing industry should not necessarily view an increase in a rival's sales as a threat to their profit. Spillover benefits, as in the RFID growth example mentioned in the introduction, may provide a net benefit to the firm.

Based on numerical analysis, we also showed in a multi-period setting that an increase in growth expenditure effectiveness, a decrease in sales decay, and an increase in unit profit margin or initial sales level for the firm may have positive spillover effects for its rival's end-of-horizon sales or profit. From the numerical analysis, this is most likely to be true if the rival exhibits high share expenditure effectiveness. Furthermore, Figure 1 suggested that when the firm's sales decay increases, the firm initially decreases growth expenditures and experiences an increased profit, but over time it eventually increases growth expenditures and exhibits a decreased profit.

For the management team of a nascent firm, the general practical implications of our numerical analysis agree with those of Bass et al. (2005a). That is, a firm should pay close attention to unrealized customers at the beginning of the nascent industry and not worry so much about the realized customers of its rival. In Table 2, the firm with the greater effectiveness in reaching unrealized customers tended to be the most profitable as long as it was not too far behind the rival in other key areas. However, if a firm is gaining expertise in reaching unrealized customers and its rival is not skilled at acquiring customers from the firm, the rival's performance is likely to suffer. As the market continues to expand, the nascent firm should increase expenditures that target the sales of its rival and gradually decrease expenditures that lead to acquiring unrealized customers. In the introduction, it was noted that MSN and other ISP's took this step against AOL as the nascent industry was growing. Essentially, the firm makes the transition from a growth and decay focus to a sales competitive environment.

Furthermore, a firm's sales decay due to product imperfections and market downturns has a severe effect on that firm's sales and profit that cannot be entirely canceled out by optimal investments in growing its industry (i.e. the pie) and competing on market share (i.e. the slice). We suggest that the firm takes all appropriate measures, including collaboration with another firm, to slow down or reverse sales decay trends. High sales decay for a firm may harm its rival's sales and profit as well and, as a result, that rival may be willing to collaborate. In some situations it may be necessary to increase growth expenditures after the firm has been competing in the industry for some time. Such a step was advocated in the action pursuit games example highlighted in the introduction.

There are also implications from the multi-period setting for those investing in new firms, including venture capitalists and business angels. Investors may benefit from exercising patience with firms in nascent industries that show favorable growth potential (e.g. high effectiveness in attracting unrealized customers, low sales decay, or high profit margins). Although our numerical analysis showed that initial profit in such industries tends to be lower than in nascent industries with less favorable growth potential, the firm becomes quite profitable with the passing of time. We also noted that a firm that excels in acquiring the sales of competition may be initially less costly to run than one that excels in attracting unrealized customers. However, if the firm has adequate funding to enter the nascent industry, it will earn higher profits and have more sales in the long run by focusing on improving its growth expenditure effectiveness. Finally, advisors of a well-funded firm with low initial sales levels should encourage them to focus on the large numbers of unrealized customers rather than trying to acquire the customers of their rivals.

This work is not without limitations, which open doors for further research. We showed how a decision should be made with respect to promotional expenditures by dividing the firm's expenditures by market segment into realized and unrealized sales. Other possibilities exist in ways to segment the market, expenditures, and firms, such as modeling expenditure on customer retention. For instance, Fruchter and Zhang (2004) offer an in-depth treatment of offensive and defensive marketing strategies, but they assume that customer retention is done in a fixed (and thus mature) market. Sophisticated customer retention policies have been mostly associated with mature firms. We could refine our modeling of customer acquisition by introducing a parameter for the effectiveness of share expenditures in retaining customers.

Finally, we do not keep track of a customer's prior purchase history. That is, there is no distinction made between an unrealized sales unit that may have passed several times between the unrealized sales group to the realized sales group of a given firm and an unrealized sales unit that has never been acquired by any firm (see e.g. Mueller 1983). Because we consider finite horizons for an industry in its nascent period, we assume that there would not be enough time for this lack of distinction to affect our results. However, modeling the group of customers who leave due to sales decay as a separate customer segment may change the expenditure patterns as the expenditures near the end of the planning horizon. Far from closing the debate on this area of research, our study and models lend themselves to several research extensions and we hope that others follow us in this quest.

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Table 1. Sensitivity Analysis for the Single-Period Model

Increases in		The Firm				Its Rival			
Description	Notation	Optimal share expenditures a_i^*	Optimal growth expenditures w_i^*	End-of-period sales S_i	Profit P_i	Optimal share expenditures a_j^*	Optimal growth expenditures w_j^*	End-of-period sales S_j	Profit P_j
Effectiveness of growth expenditures	β_i	–	↑	↑	↑	–	–	–	–
Sales decay	$1-\delta_i$	–	-	↓	↓	–	–	–	–
Unit profit margin	π_i	↑	↑	↑	NM	–	–	↓	↓
Effectiveness of share expenditures	ρ_i	↑	–	↑	↑	–	–	↓	↓
Initial sales level	S_i^0	–	↓	NM	NM	↑	↓	NM	NM

Key: ↑ for increase, ↓ for decrease, – for unchanged, NM for non-monotonic

Table 2. Numerical Solutions

$$\left(m=1,\ r_i=r_j=0,\ \gamma_i=\gamma_j=\theta_i=\theta_j=1/2\right)$$

Parameter Values	(1) Growth Effectiveness β_i , β_j	Share Exp Firm *i*	Share Exp Firm *j*	Growth Exp Firm *i*	Growth Exp Firm *j*	Final Sales Firm *i*	Final Sales Firm *j*	First Period Profit Firm *i*	First Period Profit Firm *j*	Total Profit Firm *i*	Total Profit Firm *j*
$\rho_i=.01$ $\pi_i=300$	.005, .005	$0.09	$0.73	$14.77	$15.25	.028	.030	$1.60	$1.50	$45.84	$47.85
$\rho_j=.03$ $\pi_j=300$	.010, .005	0.09	2.81	49.13	14.20	.073	.032	-1.13	1.48	84.48	49.02
$S_i^0=.03$ $1-\delta_i=.20$	.020, .005	0.11	13.39	118.97	11.56	.176	.047	-5.74	1.37	188.58	58.68
$S_j^0=.03$ $1-\delta_j=.20$	.030, .005	0.16	27.72	161.24	9.27	.255	.071	-7.01	1.13	290.12	74.62
	.040, .005	0.24	41.54	181.65	7.56	.311	.094	-6.00	0.82	376.98	91.74
$\rho_i=.03$ $\pi_i=300$	.005, .005	0.73	0.09	15.25	14.77	.030	.028	1.50	1.60	47.85	45.84
$\rho_j=.01$ $\pi_j=300$	.010, .005	0.71	0.37	52.37	13.75	.080	.027	-1.59	1.62	90.33	45.08
$S_i^0=.03$ $1-\delta_i=.20$	.020, .005	0.67	2.08	134.04	11.18	.204	.026	-7.76	1.69	214.68	44.21
$S_j^0=.03$ $1-\delta_j=.20$	.030, .005	0.67	4.87	187.19	8.99	.308	.027	-10.50	1.73	344.62	44.86
	.040, .005	0.69	7.89	214.18	7.37	.385	.030	-10.34	1.74	459.48	46.37
Parameter Values	**(2) Sales Decay $1-\delta_i$, $1-\delta_j$**	**Share Exp Firm *i***	**Share Exp Firm *j***	**Growth Exp Firm *i***	**Growth Exp Firm *j***	**Final Sales Firm *i***	**Final Sales Firm *j***	**First Period Profit Firm *i***	**First Period Profit Firm *j***	**Total Profit Firm *i***	**Total Profit Firm *j***
$\beta_i=.03$ $S_i^0=.03$	.05, .05	$1.33	$54.58	$175.82	$47.73	.332	.217	-$12.15	-$5.48	$354.25	$177.26
$\beta_j=.01$ $S_j^0=.03$	.10, .05	1.12	47.76	164.77	50.87	.288	.204	-9.90	-5.60	317.84	169.84
$\rho_i=.01$ $\pi_i=300$	.15, .05	0.96	41.76	154.43	53.80	.251	.194	-8.04	-5.70	285.98	163.90
$\rho_j=.03$ $\pi_j=300$	.20, .05	0.83	36.49	144.71	56.51	.219	.186	-6.49	-5.77	257.99	159.14
	.25, .05	0.72	31.87	135.54	59.02	.193	.179	-5.20	-5.84	233.33	155.36
$\beta_i=.01$ $S_i^0=.03$	.05, .05	54.58	1.33	47.73	175.82	.217	.332	-5.48	-12.15	177.26	354.25
$\beta_j=.03$ $S_j^0=.03$	.10, .05	49.25	1.02	39.48	181.52	.176	.346	-3.63	-12.63	152.45	365.98
$\rho_i=.03$ $\pi_i=300$	.15, .05	44.55	0.79	32.90	186.61	.145	.358	-2.25	-13.09	132.09	376.64
$\rho_j=.01$ $\pi_j=300$	.20, .05	40.40	0.61	27.62	191.17	.120	.369	-1.21	-13.53	115.26	386.33
	.25, .05	36.72	0.48	23.36	195.25	.100	.379	-0.43	-13.95	101.25	395.15
Parameter Values	**(3) Profit Per Unit Sold π_i , π_j**	**Share Exp Firm *i***	**Share Exp Firm *j***	**Growth Exp Firm *i***	**Growth Exp Firm *j***	**Final Sales Firm *i***	**Final Sales Firm *j***	**First Period Profit Firm *i***	**First Period Profit Firm *j***	**Total Profit Firm *i***	**Total Profit Firm *j***
$\beta_i=.03$ $S_i^0=.03$	100, 100	$0.01	$1.36	$36.74	$5.38	.163	.036	-$1.96	$0.40	$57.32	$17.51
$\beta_j=.01$ $S_j^0=.03$	200, 100	0.05	2.92	103.26	4.59	.240	.040	-5.69	0.40	173.72	18.38
$\rho_i=.01$ $1-\delta_i=.20$	300, 100	0.12	4.27	178.36	4.08	.289	.045	-9.35	0.39	325.29	19.34
$\rho_j=.03$ $1-\delta_j=.20$	400, 100	0.23	5.43	256.28	3.70	.324	.050	-12.51	0.38	501.48	20.26
	500, 100	0.38	6.43	334.87	3.41	.350	.053	-15.06	0.37	696.37	21.09
$\beta_i=.01$ $S_i^0=.03$	100, 100	1.36	0.01	5.38	36.74	.036	.163	0.40	-1.96	17.51	57.32
$\beta_j=.03$ $S_j^0=.03$	200, 100	4.70	0.02	20.35	34.55	.059	.151	-0.16	-1.77	48.04	53.89
$\rho_i=.03$ $1-\delta_i=.20$	300, 100	9.28	0.04	43.42	32.70	.080	.141	-1.48	-1.61	90.22	51.01
$\rho_j=.01$ $1-\delta_j=.20$	400, 100	14.66	0.05	73.42	31.10	.099	.133	-3.43	-1.47	142.97	48.54
	500, 100	20.56	0.07	109.38	29.69	.116	.126	-5.89	-1.35	205.40	46.38
Parameter Values	**(4) Share Effectiveness ρ_i , ρ_j**	**Share Exp Firm *i***	**Share Exp Firm *j***	**Growth Exp Firm *i***	**Growth Exp Firm *j***	**Final Sales Firm *i***	**Final Sales Firm *j***	**First Period Profit Firm *i***	**First Period Profit Firm *j***	**Total Profit Firm *i***	**Total Profit Firm *j***
$\beta_i=.03$ $\pi_i=300$	.005, .005	$0.06	$1.24	$185.43	$33.78	.300	.050	-$11.14	-$0.88	$338.18	$69.64
$\beta_j=.01$ $\pi_j=300$	.010, .005	0.23	1.24	185.43	33.59	.301	.050	-11.14	-0.85	338.41	69.30
$S_i^0=.03$ $1-\delta_i=.20$	.020, .005	0.87	1.23	185.43	32.87	.302	.049	-11.16	-0.73	339.28	67.99
$S_j^0=.03$ $1-\delta_j=.20$	.030, .005	1.79	1.20	185.44	31.80	.303	.047	-11.18	-0.56	340.62	66.01
	.040, .005	2.87	1.18	185.46	30.49	.305	.045	-11.21	-0.36	342.29	63.59
$\beta_i=.01$ $\pi_i=300$	.005, .005	1.24	0.06	33.78	185.43	.050	.300	-0.88	-11.14	69.64	338.18
$\beta_j=.03$ $\pi_j=300$	.010, .005	4.69	0.06	33.75	182.03	.056	.293	-0.95	-10.65	73.35	331.12
$S_i^0=.03$ $1-\delta_i=.20$	.020, .005	15.34	0.07	33.69	170.73	.076	.271	-1.17	-9.05	85.93	307.81
$S_j^0=.03$ $1-\delta_j=.20$	.030, .005	26.48	0.09	33.67	157.04	.098	.244	-1.38	-7.22	101.61	279.95
	.040, .005	35.42	0.11	33.72	143.70	.118	.219	-1.53	-5.54	117.29	253.20
Parameter Values	**(5) Initial Sales S_i^0 , S_j^0**	**Share Exp Firm *i***	**Share Exp Firm *j***	**Growth Exp Firm *i***	**Growth Exp Firm *j***	**Final Sales Firm *i***	**Final Sales Firm *j***	**First Period Profit Firm *i***	**First Period Profit Firm *j***	**Total Profit Firm *i***	**Total Profit Firm *j***
$\beta_i=.03$ $\pi_i=500$	.00, .00	$3.45	$115.34	$288.45	$95.34	.301	.235	-$17.38	-$15.88	$578.60	$297.40
$\beta_j=.01$ $\pi_j=500$	.02, .00	3.49	118.08	281.97	93.24	.301	.235	-13.59	-15.88	594.32	299.35
$\rho_i=.01$ $1-\delta_i=.05$	.04, .00	3.54	120.93	275.58	91.15	.300	.236	-9.85	-15.92	609.80	301.59
$\rho_j=.03$ $1-\delta_j=.05$	.06, .00	3.59	123.90	269.29	89.04	.300	.238	-6.16	-15.98	625.04	304.12
	.08, .00	3.65	126.97	263.08	86.94	.300	.239	-2.51	-16.06	640.04	306.93
$\beta_i=.01$ $\pi_i=500$	.00, .00	115.34	3.45	95.34	288.45	.235	.301	-15.88	-17.38	297.40	578.60
$\beta_j=.03$ $\pi_j=500$	.02, .00	111.89	3.90	92.86	284.80	.238	.298	-12.03	-17.34	331.59	568.99
$\rho_i=.03$ $1-\delta_i=.05$	.04, .00	108.48	4.40	90.41	281.10	.242	.295	-8.19	-17.30	365.77	559.38
$\rho_j=.01$ $1-\delta_j=.05$	.06, .00	105.12	4.98	87.97	277.34	.246	.293	-4.36	-17.26	399.94	549.78
	.08, .00	101.82	5.62	85.56	273.52	.250	.290	-0.55	-17.21	434.10	540.18

Table 3. Conjectured Sensitivity Analysis for the Multi-Period Model

Increases in		The Firm				Its Rival			
Description	Notation	Share expenditures $\int_0^T a_i^*(t)dt$	Growth expenditures $\int_0^T w_i^*(t)dt$	End-of-horizon sales $S_i(T)$	Profit P_i	Share expenditures $\int_0^T a_j^*(t)dt$	Growth expenditures $\int_0^T w_j^*(t)dt$	End-of-horizon sales $S_j(T)$	Profit P_j
Effectiveness of growth expenditures	β_i	NM	↑	↑	↑	↑	↓	NM	NM
Sales decay	$1-\delta_i$	↓	↓	↓	↓	↓	↑	NM	NM
Unit profit margin	π_i	↑	↑	↑	↑	NM	↓	NM	NM
Effectiveness of share expenditures	ρ_i	↑	NM	↑	↑	NM	↓	↓	↓
Initial sales level	S_i^0	NM	↓	NM	↑	↑	↓	NM	NM

Key: ↑ for increase, ↓ for decrease, NM for non-monotonic

Figure 1. Sales, Profit, Share and Growth Expenditures for the Firm (first part of Run (2) in Table 2)

$$\beta_i = .03, \beta_j = .01, \rho_i = .01, \rho_j = .03, S_i^0 = .03, S_j^0 = .03, \pi_i = 300, \pi_j = 300$$