

Technical Memorandum Number 813

Channel Strategies for Durable Goods:  
Coexistence of Selling and Leasing to Individual  
and Corporate Consumers

by

Vera Tilson  
Yunzeng Wang  
Wei Yang

September 2006

Department of Operations  
Weatherhead School of Management  
Case Western Reserve University  
330 Peter B Lewis Building  
Cleveland, Ohio 44106

**Channel Strategies for Durable Goods:  
Coexistence of Selling and Leasing to Individual and  
Corporate Consumers<sup>1</sup>**

Vera Tilson  
William E. Simon Graduate School of Business  
University of Rochester  
Rochester, NY 14627  
e-mail: [VeraT@alum.mit.edu](mailto:VeraT@alum.mit.edu)

Yunzeng Wang  
Weatherhead School of Management  
Case Western Reserve University  
Cleveland, OH 44106  
e-mail: [Yunzeng.Wang@case.edu](mailto:Yunzeng.Wang@case.edu)

Wei Yang  
Department of Management  
Long Island University  
Brookville, NY 11548  
e-mail: [Wei.Yang@liu.edu](mailto:Wei.Yang@liu.edu)

---

<sup>1</sup> The authors wish to thank Dr. Suzhou Huang from Ford Research Lab and Professor Matthew J. Sobel of Case Western Reserve University for useful discussions and comments. All mistakes are, of course, our own.

# **Channel Strategies for Durable Goods: Coexistence of Selling and Leasing to Individual and Corporate Consumers**

## **Abstract**

In durable goods markets, such as those for automobiles or computers, the coexistence of selling and leasing is common as is the existence of both corporate and individual consumers. Leases to the corporate consumers affect the prices of used goods which in turn affect the buying and leasing behavior of individual consumers. The setting of prices (or volume) for sale and lease to individual and corporate consumers is a complicated problem for manufacturers.

We consider a manufacturer who concurrently sells and leases a finitely durable good to both individual and corporate consumers. We construct a model of a dynamic game to capture the interactions among the different distribution channels: (a) sale of new goods to consumers, (b) lease of new goods to both consumers and corporations, and (c) sale of off-lease goods to consumers. Both the manufacturer and consumers seek to maximize their individual payoff over infinite horizon.

Making a number of simplifying assumptions including a two-period lifetime for the finitely durable goods, we construct a Markov Perfect Stationary Equilibrium, where the manufacturer sells and leases the same number of goods every period. We show that in such equilibrium individual consumers strategically separate into four classes: those that lease every period, those that buy new goods and use them for two periods, those that always buy used goods, and those that do not participate in the market. As the number of goods leased to the corporate consumer increases, the individual consumers' markets evolve. For example, if the used goods are very poor substitutes for new goods, then first non-participants disappear – since the price of used goods quickly drops to zero, then the manufacturer stops leasing to individual consumers, so that only two classes of consumers exist. We study how the substitutability of new and used goods, production and transaction costs affect the prices the manufacturer should charge for purchases and leases respectively.

Our findings confirm the observations that the manufacturer must be careful in determining strategically how to set retail prices when there is a corporate lease consumer for his goods. In some cases (when used goods are poor substitutes for new goods) the manufacturer may need to set the one-period consumer lease price to be higher than the sale price, setting a premium for disposing of the used good. We also find that in general consumers derive benefit from the addition of a corporate customer, while not every consumer may be better off, in aggregate consumer surplus increases.

**Keywords:** Channels of Distribution, Game Theory, Market Structure, Retailing and Wholesaling, Segmentation

# 1. Introduction

The production and distribution of durable goods constitutes a large fraction of the economy: in the US personal expenditure on durable goods represents nearly a tenth of gross domestic product. To market durable goods such as automobiles, photocopiers, computers and other electronic devices, manufacturers often adopt a mix of selling and leasing strategies in both individual consumer and business markets. For example, approximately one quarter of GM's automotive production is sold to fleet purchasers such as rental car companies. A great many of these vehicles are "program" cars that GM buys back a year later, making this kind of sale equivalent to a lease. The following quote of (Sawyers 2002) describes automakers' dilemma: "When large numbers of program vehicles return to the market place, used vehicles prices drop. That drags down residual values of new cars. Depressed residual values erode brand image and make it difficult for automakers to offer competitive lease deals." The balance between sales and leases to businesses and consumers is an important driver of profitability and the question of how to strike the right balance has troubled automakers for years.

The longevity of durable goods is what distinguishes them from perishables, and generally makes both leasing and sales viable. The longevity often leads to second-hand markets – particularly where consumers differ in their valuation of used goods (Bulow 1982). Competition between new and used goods creates a complex dynamic problem space for producers in terms of capacity planning, selection of distribution channels and pricing.

In this paper we investigate the dynamic interactions between the corporate and retail markets for durable goods – specifically, where the manufacturer leases his product to corporate and individual consumers and also sells new goods to individuals. Our goal is to answer the following strategically important questions: First, how should the manufacturer determine the selling price of new goods for individual consumers, and leasing prices to both individual and corporate consumers? Off-lease goods impact the used goods price, which in turn must affect the choices made by individual consumers. That leads to our second question: how is the behavior of individual consumers affected by the presence of the corporate consumer? Clearly, the behavior of the manufacturer and consumers is affected by other parameters, such as substitutability of new and used goods from the consumers' point of view. Our final question is: how should the manufacturer coordinate both the retail and corporate markets as a function of varied substitutability of new and used goods in order to maximize his overall profitability?

To contribute to the understanding of the strategic interactions between a manufacturer and his corporate and individual consumers, we construct a model where both retail and corporate markets exist. There is a single corporate consumer who leases new goods according to its demand function. Individual consumers use no more than one unit of good in any particular time period. All consumers prefer using

newer goods, but the strength of the preference varies between consumers. The used goods markets are not frictionless and sellers incur transaction costs on the secondhand markets. The transaction costs for the manufacturer are lower than the transaction costs for the individual consumers.

The interaction between the manufacturer and consumers is modeled as a dynamic game with alternating moves. Both the manufacturer and the consumers seek to maximize discounted profit/utility over an infinite horizon. We demonstrate that there exists an equilibrium solution where individual consumers fall into four groups: those that do not participate in the market, those that only use used goods, those that buy goods when they are new and then use them for the lifetime of the good, and consumers that lease new goods every period. We show that (1) as the manufacturer increases the number of goods leased to the corporate consumer, he should reduce the number of new goods on the retail market, (2) as long as there are individual consumers who do not participate in the market, aggregate surplus of the individual consumers is increased by the addition of the fleet consumer; (3) if there are consumers that lease goods every period, there will also be consumers that buy new goods and use them for a lifetime; (4) if used goods are poor substitutes for new goods the lease price may be higher than the sale price.

The paper proceeds as follows. We provide a brief review of the related literature in the rest of this section. In Section 2, we describe the model settings and formulate the problem as a dynamic gaming problem. In Section 3, we characterize consumers' behavior in terms of their individual consumption strategy in equilibrium. In Section 4, we provide an explicit solution of the model and draw managerial insights. Section 5 concludes the paper.

## 1.1 Literature Review

Economists were first to highlight that a number of issues faced by producers and consumers of durable goods are distinct from those associated with perishables. Coase (1972) noted that a monopolist producer of durable goods is unable to extract monopoly rents because of the *time-inconsistency* in the monopolist's commitment to future prices. When a product is durable, its demand decreases with every period and the manufacturer has an incentive to lower the price over time to attract the remaining consumers. Strategic consumers anticipate price decreases and, assuming they are sufficiently patient, delay purchasing durable goods until the price drops to a competitive (non-monopoly) level. Coase (1972) conjectured that a durable goods producer preserves monopoly power by leasing the goods and controlling the second-hand market. Through the control of the second-hand market, the monopolist internalizes the effect of future decisions on the value of units that have already been produced. A number of papers that followed examined the assumptions under which Coase's conjecture does or does not hold or focused on alternative strategies for dealing with the problem of time inconsistency. For, example

Bulow (1986) showed that the manufacturer can alleviate the time inconsistency problem by reducing the durability of its products -- thus durability and selling/leasing decisions are equivalent choices.

Since Coase's conjecture several alternative explanations of leasing have been proposed in the academic literature. Leasing has been seen as a mechanism for improving the efficiency of used goods markets and increasing consumers' willingness to pay (Hendel and Lizzeri 2002; Johnson and Waldman 2003; Waldman 2003). Information asymmetry is frequently observed in the second-hand markets: a seller knows the quality of the used goods he is selling, but the buyer is not able to verify their quality. As a result a buyer is not willing to pay the price for a high quality used good and owners of such goods are reluctant to bring them to market. This phenomenon, referred to as adverse selection, leads to a "market for lemons" (Akerlof 1970). With leasing it is the original producer who brings all the used goods to market, and information asymmetry is eliminated. The efficiency of the used goods market is increased since consumers are willing to pay higher prices and the higher quality used goods find their way to market.

Concurrent leasing and selling have become commonplace in the automotive and IT industries. Several authors (Desai and Purohit 1998; Hendel and Lizzeri 2002) demonstrate that selling and leasing can be employed as a mechanism to differentiate among consumers. Porter and Sattler (1999) note that when consumers are heterogeneous in their preferences for new and used goods then secondary markets benefit both producer and consumers. Consumers who value newer goods the most trade-in their older goods and purchase new ones. The producer benefits from the additional demand. Consumers who place lower value on the good can purchase from the used goods market.

Desai and Purohit (1999) study how product reliability and market competitiveness affect the relationship between leasing and selling. Huang et al. (2001) focus on the role of transaction costs in second-hand markets. They show that if the transaction costs for the consumer are higher than for the manufacturer, the manufacturer chooses to control the second-hand market, and that both leasing and selling take place. The ratio of selling to leasing decreases with decreasing transaction costs. Recently, Bhaskaran and Gilbert (2005) examined the impact of a complementary product on the manufacturer's strategy, and studied the adoption of the concurrent leasing and selling to balance the manufacturer's commitment across its own market and the complementary market.

Most literature focuses on analyzing the marketing strategy when the manufacturer deals only with individual consumers. However, there are a few exceptions. Using a two-period model, Purohit and Staelin (1994) consider a manufacturer who manages two independent sales channels: dealers and rental agencies. Consumers purchase goods from dealers and rent goods from rental agencies via short-term leasing contracts. The two distribution channels interact through the second-hand market. The rental agencies are treated as exogenous. The focus is on the quantities that should be sold to the dealer and the

impact of different channel structures on the dealer. Later Purohit (1997) extends the model by endogenizing the rental agency and analyzing the effect of market structures on the profitability of the manufacturer and its intermediaries. While considering two channels, these models do not consider the coexistence of leasing and selling to individual consumers.

## 2. The Model

In this section, we describe our model and lay out the assumptions regarding the product, the manufacturer as well as the individual and corporate consumers.

Time is measured discretely. The product is durable but has finite life. To capture the dynamic interaction among market participants while retaining tractability, we restrict longevity of a good to two periods: in period 1 it is new, in period 2 it is used, and after two periods the goods is not usable. The product deteriorates with time, and the difference between a new and used good is discernable to all participants.

The manufacturer is assumed to be a monopolist<sup>2</sup> who produces a single type of product and has no capacity constraints. A constant marginal cost  $c$  is incurred in production and marketing. New goods can be sold or leased to individual consumers. Each individual consumer owns or leases at most one unit in each period. The manufacturer leases multiple new goods to a corporate consumer. Both types of lease contract last one period, after which off-lease goods are returned to the manufacturer who resells them in the second-hand market with a unit disposal cost  $\beta$ . While the lease contract does not contain a buy-back option we are not precluding any individual consumer from purchasing the good at lease expiration at the prevailing used good price. The relationship among the market participants is illustrated in Figure 1.

There is one corporate consumer that leases new goods from the manufacturer. The lease quantity is determined by the corporate consumer's own utility objective. The corporate consumer's reaction function  $R(v)$  is defined as the number of goods leased in a particular period in response to lease price  $v$ <sup>3</sup>.

---

<sup>2</sup> We justify this assumption with a quote from (Waldman 2003): "Even though most durable goods producers are not monopolist most do have market power, and monopoly analyses should provide useful insights".

<sup>3</sup> In reality, a manufacture can face multiple corporate consumers. In that case, the one corporate consumer in our model is the aggregation of all such consumers; and so the lease quantity  $R(v)$  is the aggregated lease quantity of all corporate consumers.



**Figure 1: Model Framework**

We assume that individual consumers have infinite lives and the size of the population is a constant  $P$ . In each period consumers can either purchase a new or used good, lease a new good, or stay out of the market. They can also resell used goods in the second-hand market with a unit disposal cost  $\alpha$ . Since the manufacturer benefits from economies of scale, we assume that  $\alpha > \beta$ . Consumers are heterogeneous in their valuation of goods, consumer preferences are exogenous and do not change with time. We use parameter  $\theta \in [0,1]$  to describe consumer type. Consumers of different types are distributed in the population according to a density distribution function  $f(\theta)$ . The value placed by a consumer of type  $\theta$  on using a good of vintage  $k$  for one period is  $u_k(\theta)$ , where  $k=0$  indicates a new good and  $k=1$  a used good. We assume that every individual consumer prefers newer goods to older goods, that is  $u_0(\theta) > u_1(\theta)$ , and that consumers with higher  $\theta$  value both new and used goods higher than the consumers with low  $\theta$ , which implies that both  $u_0(\theta)$  and  $u_1(\theta)$  are increasing, and that the preference for using the new good rather than a used good,  $u_0(\theta) - u_1(\theta)$ , increases with  $\theta$  as well.

The interaction of the manufacturer with the corporate consumer and individual consumers is modeled as a sequential game with complete information. We assume all players are rational and their objectives are to maximize discounted profits or utilities over an infinite horizon with a common discount factor  $0 < \gamma \leq 1$ . The manufacturer moves first and sets the purchase and lease prices of new goods. The consumers move next. Their collective actions determine the number of new goods that will be leased and purchased, as well as the price of used goods.

The decision variables for the manufacturer are the sale price of a new good  $q_0^t$ , individual lease price  $r^t$  and corporate lease price  $v^t$ , where the superscript  $t$  denotes the time period. For notational



convenience, we define the price vector as  $p^t \equiv \{q_0^t, r^t, v^t\}$ . At the beginning of period  $t$  the manufacturer announces the three prices and places the off-lease goods from the previous period for sale on the second-hand market. We assume that the second-hand market is competitive, and that the price of used goods,  $q_1^t$ , is resolved in such way that the second-hand market clears. All consumers are price-takers. Based on the prices announced by the manufacturer and the used goods price determined by the second-hand market, the consumers decide to buy, lease or do nothing.

## 2.1 Corporate Consumer

The corporate consumer leases new goods from the manufacturer. As an independent economic entity, the corporate consumer determines the optimal lease quantity in each period to maximize her discounted profit over an infinite horizon. Specifically, we assume that the corporate consumer has a profit function of  $\pi^t(R^t, v^t)$  when she chooses to lease  $R^t$  units at the price  $v^t$  in period  $t$ . We further assume that the corporate consumer faces a stable production or profit-generating technology over time, meaning that she has the same profit function of the form  $\pi^t(R^t, v^t) = \pi(R^t, v^t)$  for all  $t$ . Thus, her total discounted profit is  $\sum_{t=0}^{\infty} (\gamma)^t \pi(R^t, v^t)$ .

We assume that the corporate consumer does not play strategically against the manufacturer or individual consumers. Since all corporate leases last exactly one period, a decision on lease quantity made in the current period has no effect on any future decisions or profits. Consequently, maximizing the total discounted profit amounts to maximizing the one-period profit  $\pi(R^t, v^t)$  at each period  $t$ . Let  $R(v^t)$  be the optimal quantity leased in response to manufacturer's price  $v^t$ , that is,  $R(v^t) = \arg \max_R \pi(R, v^t)$ <sup>4</sup>.

We further assume that the corporate consumer's response function  $R(v^t)$  is known to the manufacturer. Since there is no capacity constraint for the manufacturer, the corporate consumer's demand can always be satisfied.

## 2.2 Individual Consumers

At the beginning of each period a consumer is in one of two states: either owning a one-period old good or not. We use  $o^t \in \{0,1\}$  to designate consumer state at the start of period  $t$ :  $o^t = 1$  indicates ownership

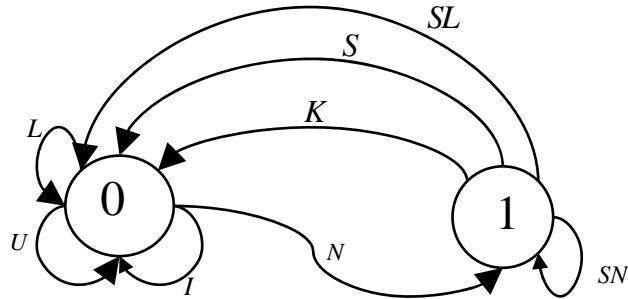
---

<sup>4</sup> We assume that  $\pi(\cdot, v^t)$  is such that  $R(v^t)$  is unique.

of a used good,  $o^t = 0$  indicates otherwise. Depending on his state the consumer has a number of actions available. If he does not possess a good he can either do nothing ( $I$ ), lease a new good for one period ( $L$ ), buy a new good ( $N$ ), or buy a used good ( $U$ ). If he possesses a one-period old good he can choose to keep it ( $K$ ), sell it and lease a new good ( $SL$ ), sell it and buy a new good ( $SN$ ) or just sell the good and not replace it ( $S$ ). Let  $A(o^t)$  denote the feasible action set in state  $o^t$ . Each consumer action,  $a^t \in A(o^t)$ , is associated with (1) a consumer type-specific reward obtained in the current period  $\pi_\theta(o^t, a^t, p^t, q_1^t)$ , and (2) the state transition function  $T(o^t, a^t)$  specifying next period's state given the consumer's current state and the action taken. The reward function  $\pi_\theta(o^t, a^t, p^t, q_1^t)$  matrix is detailed in Table 1; the state transition diagram representing  $T(o^t, a^t)$  is shown in Figure 2.

**Table 1: Immediate Reward Matrix  $\pi_\theta(o^t, a^t, p^t, q_1^t)$  for Individual Consumers**

At State $o^t = 0$		At State $o^t = 1$	
Action	Reward	Action	Reward
$I$	0	$K$	$u_1(\theta)$
$N$	$u_0(\theta) - q_0^t$	$S$	$q_1^t - \alpha$
$U$	$u_1(\theta) - q_1^t$	$SN$	$q_1^t - \alpha + u_0(\theta) - q_0^t$
$L$	$u_0(\theta) - r^t$	$SL$	$q_1^t - \alpha + u_0(\theta) - r^t$



**Figure 2: State Transition Diagram for Individual Consumers**

At the beginning of every period each consumer chooses an action  $a^t$  to maximize his discounted reward over an infinite horizon

$$\sum_{t=0}^{\infty} (\gamma)^t \pi_\theta(o^t, a^t, p^t, q_1^t). \quad (1)$$

Let  $V_\theta^t(o^t, p^t, q_1^t)$  denote the total discounted payoff for a type  $\theta$  customer when he follows an

optimal policy  $\sigma_\theta^*$  starting at period  $t$ . Then, an individual consumer's problem can be described by the following dynamic programming equation:

$$V_\theta^t(o^t, p^t, q_1^t) = \max_{a^t \in A(o^t)} \left\{ \pi_\theta(o^t, a^t, p^t, q_1^t) + \gamma V_\theta^t \left[ T(o^t, a^t), p^{t+1}, q_1^{t+1} \right] \right\}. \quad (2)$$

Let  $a_\theta^t(o^t, p^t, q_1^t)^*$  be the action that policy  $\sigma_\theta^*$  specifies for consumer  $\theta$  at period  $t$ , given the customer state  $o^t$ , manufacturer-set prices  $p^t$  and the used goods price  $q_1^t$ .

### 2.3 Manufacturer's Problem

The manufacturer's profit is determined by the sale and lease prices he charges for new goods, the number of new goods sold and leased as well as by the price of used goods, since he sells off-lease goods on the used goods market. The manufacturer's reward is composed of his profit from several channels. Let  $B^t$  be the number of new goods sold by the manufacturer in period  $t$ . The profit from sales is

$$(q_0^t - c)B^t. \quad (3)$$

Let  $L^t$  be the total number of leased goods, which includes those leased to the corporate consumer, namely,  $R(v^t)$ . The number of goods leased to individual consumers is  $L^t - R(v^t)$ . The one-period profit from leases in period  $t$  is given by

$$(r^t - c) \left[ L^t - R(v^t) \right] + (v^t - c)R(v^t). \quad (4)$$

In addition there is a profit stream from selling the off-lease goods which were produced and leased out last period. The profit from the sale of off-lease goods is

$$(q_1^t - \beta)L^{t-1}. \quad (5)$$

The manufacturer sets prices  $q_0^t$ ,  $r^t$  and  $v^t$  each period to maximize his discounted profit over an infinite horizon:

$$\sum_{t=0}^{\infty} (\gamma)^t \cdot \pi_M(L^{t-1}, B^t, L^t, p^t, q_1^t), \quad (6)$$

where  $\pi_M(L^{t-1}, B^t, L^t, p^t, q_1^t)$  is the sum of (3), (4) and (5). At the beginning of each period  $t$ , the state of the system relevant to the manufacturer's decision includes: 1) the number of used goods in his possession to be sold in the current period, and 2) the collective state of individual consumers. The number of used goods the manufacturer possesses in the current period equals the number of new goods leased last period, namely,  $L^{t-1}$ . The collective state of individual consumers is more involved, and additional notation is needed to describe it.

We define a function  $g^t(\theta)$  as the fraction of individual consumers in the interval  $[\theta, \theta + d\theta]$  that own a used good at the start of period  $t$ . The fraction of consumers in the same interval that do not own a used good at the start of period  $t$  is  $\bar{g}^t(\theta) = 1 - g^t(\theta)$ . We are now ready to make more concrete statements about the dynamics of the system. When the manufacturer offers a specific set of prices  $p^t = \{q_0^t, r^t, v^t\}$ , the corporate consumer chooses to lease  $R(v^t)$  units and each individual consumer chooses an action  $a_\theta^t(o^t, p^t, q_1^t)^*$  to maximize his payoff function (2). Individual consumers' decisions interact with the used goods market through the following market clearing condition:

$$L^{t-1} + P \cdot \int_0^1 g^t(\theta) \cdot IN_{\{a_\theta^t(1, p^t, q_1^t)^* \neq K\}} f(\theta) d\theta = P \cdot \int_0^1 \bar{g}^t(\theta) \cdot IN_{\{a_\theta^t(0, p^t, q_1^t)^* = U\}} f(\theta) d\theta, \quad (7)$$

where the indicator function is defined as

$$IN_{\{condition\}} \equiv \begin{cases} 1, & \text{if } condition \text{ is satisfied;} \\ 0, & \text{otherwise.} \end{cases}$$

The left-hand side of (7) gives the total supply of used goods: the number of goods that were leased in the previous period plus the number of used goods that are being sold by the individual consumers who own a used good at the start of the period. The right-hand side the total demand: the number of used goods that are being bought by individual consumers who do not own a used good at the start of the period.

Given a state  $\{L^{t-1}, g^t(\theta)\}$  and manufacturer's prices  $p^t$ ,  $a_\theta^t(o^t, p^t, q_1^t)^*$ , the optimal action for each individual consumer and used good price,  $q_1^t$ , are determined *jointly* by solving equations (2) and (7). As a result, the used goods price can be expressed as  $q_1^t[L^{t-1}, g^t(\theta), p^t]$ .

The number of new goods sold in period  $t$  can be calculated as

$$B^t(L^{t-1}, g^t(\theta); p^t) = P \cdot \int_0^1 \bar{g}^t(\theta) \cdot IN_{\{a_\theta^t(0, p^t, q_1^t)^* = N\}} f(\theta) d\theta + P \cdot \int_0^1 g^t(\theta) \cdot IN_{\{a_\theta^t(1, p^t, q_1^t)^* = SN\}} f(\theta) d\theta, \quad (8)$$

and the number of new goods leased in period  $t$  is

$$L^t(L^{t-1}, g^t(\theta); p^t) = P \cdot \int_0^1 \bar{g}^t(\theta) \cdot IN_{\{a_\theta^t(0, p^t, q_1^t)^* = L\}} f(\theta) d\theta + P \cdot \int_0^1 g^t(\theta) \cdot IN_{\{a_\theta^t(1, p^t, q_1^t)^* = SL\}} f(\theta) d\theta + R(v^t). \quad (9)$$

Equation (9) provides the transition function for the number of used goods to be sold in period  $t + 1$ .

The fraction of individual consumers in the interval  $[\theta, \theta + d\theta]$  that own a used good at the start of period  $t + 1$  is equal to the sum of (1) the fraction of consumers who owned nothing at the start of period  $t$  and chose to buy a new good and (2) the fraction of consumers who owned a used good and

chose to sell it and buy a new good. The transition of  $g^{t+1}(\theta)$  is governed by

$$\{g^{t+1}(\theta)\}(L^{t-1}, g^t(\theta); p^t) = \bar{g}^t(\theta) \cdot IN_{\{a_\theta^t(0, p^t, q_1)^* = N\}} + g^t(\theta) \cdot IN_{\{a_\theta^t(1, p^t, q_1)^* = SN\}} \quad (10)$$

The manufacturer's problem can be described by the following Bellman equation:

$$V_M^t[L^{t-1}, g^t(\theta)] = \max_{p^t} \left\{ \pi_M(L^{t-1}, B^t, L^t, p^t, q_1^t) + \gamma V_M^{t+1}[L^t, g^{t+1}(\theta)]^{t+1} \right\} \quad (11)$$

Let  $\sigma_M^*$  denote an optimal policy for the manufacturer, and  $p^t[L^{t-1}, g^t(\theta)]^*$  the action that policy  $\sigma_M^*$  specifies in period  $t$  when the number of used goods leased in the previous period is  $L^{t-1}$  and the aggregate state of consumers is described by  $g^t(\theta)$ .

## 2.4 Concept of Model Solution

We seek a steady-state equilibrium, where the manufacturer leases the same number of units every period and the aggregate state of the consumers is constant. In such an equilibrium, the time index (i.e., superscript  $t$ ) on various quantities satisfying (1) – (11) is dropped. Under the assumption that a price vector  $p^*$  maximizes the manufacturer's long-term discounted profit, quantities  $L$ ,  $g(\theta)$  and  $p^*$  satisfy

$$V_M[L, g(\theta)] = \pi_M(L, B, L, p^*, q_1) + \gamma V_M[L, g(\theta)] \quad (12)$$

and the price vector  $p^*$  maximizes one-period profit:

$$p^* = \arg \max_{\{q_0, r, v\}} \left\{ (q_0 - c)B + (r - c)[L - R(v)] + (v - c)R(v) + (q_1 - \beta)L \right\}. \quad (13)$$

The Bellman equation for individual consumers (2) reduces to

$$V_\theta(o, p^*, q_1) = \pi_\theta \left[ o, a(o, p^*, q_1)^*, p^*, q_1 \right] + \gamma V_\theta \left\{ T \left[ o, a(o, p^*, q_1)^* \right], p^*, q_1 \right\}.$$

Since  $p^*$  and  $q_1$  are constants for all consumers, we simplify the notation by defining

$V_\theta(o) \equiv V_\theta(o, p, q_1)$  and  $a_\theta(o)^* \equiv a_\theta(o, p^*, q_1)^*$  to write the above Bellman equation as

$$V_\theta(o) = \pi_\theta \left[ o, a_\theta(o)^*, p^*, q_1 \right] + \gamma V_\theta \left\{ T \left[ o, a_\theta(o)^* \right] \right\}. \quad (14)$$

## 3. Strategic Behaviors in Equilibrium

In a steady-state equilibrium, all the prices are constant, including the lease price  $v$  for the corporate consumer. Each period the corporate consumer chooses her lease quantity based only on the lease price for that period. In equilibrium she chooses the same lease quantity  $R(v)$  period after period.

Identifying the optimal consumption strategies for individual consumers of different types is more involved. First, there are many different consumption options available for an individual consumer: leasing, buying new or used, staying idle, etc. As the main focus of this section, we will establish that from the point of view of any individual consumer, in steady-state there are at most four optimal stationary policies, or strategies: 1) lease every period; 2) repeatedly buy a new good and use it for two periods; 3) buy a used good every period; and 4) stay idle or never use the good. Furthermore, following their optimal policies, individual consumers form four clusters according to their types: high  $\theta$  individuals choose strategy 1; mid-high  $\theta$  individuals choose strategy 2; mid-low  $\theta$  individuals take strategy 3; and low  $\theta$  individuals adopt strategy 4. We shall also partially characterize the manufacturer's behavior in choosing his optimal pricing policies.

### 3.1 Individual Consumer Behavior in Equilibrium

In equilibrium each individual consumer chooses a consumption strategy to maximize his own discounted payoff over an infinite horizon. We restrict ourselves to examining stationary consumption strategies since individual reward and transition functions are time-independent, prices are constant over time, and the consumer action space is finite (Blackwell 1965). Stationary strategies depend *only* on the consumer state  $o = 0$  or  $1$ . We denote a stationary strategy by a vector  $[a(1), a(0)]$ , specifying the action prescribed by the strategy in each state. As shown in Figure 2, there are four actions available in each of the two states, therefore there are sixteen ( $4^2$ ) possible stationary consumption strategies. An optimal strategy  $\sigma_\theta^* \equiv [a_\theta(1)^*, a_\theta(0)^*]$  for a consumer of type  $\theta$  satisfies the Bellman equation of (14). To find an optimal strategy  $\sigma_\theta^*$  we first establish two Lemmas that allow us to eliminate those strategies that are dominated by others.

**Lemma 1.** *At most seven consumer strategies dominate:  $(S, I)$ ,  $(K, I)$ ,  $(K, U)$ ,  $(K, N)$ ,  $(K, L)$ ,  $(SL, L)$  and  $(SN, N)$ .*

**Proof:** We show that the strategies  $(SL, I)$ ,  $(SL, U)$  and  $(SL, N)$  are always dominated by  $(SL, L)$ ; strategies  $(SN, I)$ ,  $(SN, U)$  and  $(SN, L)$  are always dominated by  $(SN, N)$ , and  $(S, U)$ ,  $(S, L)$  and  $(S, N)$  are always dominated by  $(S, I)$ .

Recall that  $V_\theta(0)$  and  $V_\theta(1)$  represent the discounted payoffs a consumer of type  $\theta$  derives when starting in states 0 and 1, respectively. Consider a consumer that strictly prefers action  $SL$  in state 1. For such a consumer the following inequalities must hold:

$$q_1 - \alpha + u_0(\theta) - r + \gamma V_\theta(0) > q_1 - \alpha + \gamma V_\theta(0), \quad (15)$$

$$q_1 - \alpha + u_0(\theta) - r + \gamma V_\theta(0) > u_1(\theta) + \gamma V_\theta(0), \quad (16)$$

$$q_1 - \alpha + u_0(\theta) - r + \gamma V_\theta(0) > q_1 - \alpha + u_0(\theta) - q_0 + \gamma V_\theta(1), \quad (17)$$

indicating that in state 1 action  $SL$  is preferable to actions  $S$ ,  $K$ , and  $SN$ , respectively. Subtracting the quantity  $q_1 - \alpha$  from both sides of (15)-(17), leads to inequalities which imply that the same consumer chooses to lease in state 0 since action  $L$  is preferable to action  $I$ ,  $U$  and  $N$ . That is, a consumer, who chooses to sell a used good and lease a new one when he owns the used good, will also choose to lease a new good when he owns nothing.

Next we consider a consumer who strictly prefers action  $SN$  in state 1. Using the argument analogous to the one used in the previous paragraph, we can show that strategies  $(SN, I)$ ,  $(SN, U)$  and  $(SN, L)$  are always dominated by  $(SN, N)$ . A consumer, who chooses to sell a used good and buy a new one when he owns the used good, also chooses to buy a new good when he owns nothing. Similarly we can show that a consumer that strictly prefers action  $S$  in state 1 prefers action  $I$  in state 0. That is, a consumer who prefers to own nothing when he already owns a used good chooses to keep on owning nothing. Strategies  $(S, U)$ ,  $(S, N)$  and  $(S, L)$  are suboptimal.  $\square$

We will show later that for maximizing his own profit, the manufacturer always offers his sale price  $q_0$  and lease price  $r$  to individuals such that the following relationship holds:

$$r \leq q_0 - \gamma(q_1 - \alpha). \quad (18)$$

Under such a condition, strategy  $(SL, L)$  dominates  $(SN, N)$  for any individual consumer, as shown in the following Lemma:

**Lemma 2.** *If (18) holds, strategy  $(SL, L)$  weakly dominates  $(SN, N)$  for all individual consumers.*

**Proof:** Table 2 presents the discounted surpluses of a consumer of any type  $\theta$  following the  $(SN, N)$  and  $(SL, L)$  strategies. The conclusion then follows by simply comparing the surpluses under the two strategies in each initial state.

**Table 2: Discounted Surplus for any Consumer Using  $(SN, N)$  and  $(SL, L)$  Strategies**

Strategy	Initial State: 0	Initial State: 1
$(SN, N)$	$\frac{u_0(\theta) - q_0 + \gamma(q_1 - \alpha)}{1 - \gamma}$	$\frac{u_0(\theta) - q_0 + \gamma(q_1 - \alpha)}{1 - \gamma} + q_1 - \alpha$
$(SL, L)$	$\frac{u_0(\theta) - r}{1 - \gamma}$	$\frac{u_0(\theta) - r}{1 - \gamma} + q_1 - \alpha$

Combining Lemmas 1 and 2, we conclude that there are at most six dominating stationary strategies for any individual consumer:  $(S, I)$ ,  $(K, I)$ ,  $(K, U)$ ,  $(K, N)$ ,  $(K, L)$  and  $(SL, L)$ . For consumers

following strategies  $(S, I)$ ,  $(K, I)$ ,  $(K, U)$ ,  $(K, L)$  and  $(SL, L)$ , state 0 is absorbing. Focusing on a consumer's consumption pattern in steady-state, strategies  $(S, I)$  and  $(K, I)$  lead to the identical consumption pattern: to stay idle in every period. Similarly, strategies  $(SL, L)$  and  $(K, L)$  imply leasing every period. A consumer following strategy  $(K, U)$  buys a used good every period. Those that follow the strategy  $(K, N)$  repeatedly buy new goods and use them for two periods. In summary, we have

**Corollary 1.** *In steady-state, there are at most four dominating stationary consumption strategies for any individual consumer: 1) to stay idle every period; 2) to buy a used good every period; 3) to repeatedly buy a new good and use it for two periods; and 4) to lease a good every period.*

In steady-state, an individual chooses the strategy that delivers him the highest (total discounted) surplus pertaining to his specific type  $\theta$ . Without loss of generality, assume that the individual starts out possessing nothing, and let  $V_{\theta,I}$ ,  $V_{\theta,U}$ ,  $V_{\theta,N}$  and  $V_{\theta,L}$  denote the discounted surplus received from following the strategies of staying Idle, buying a Used good every period, repeatedly buying a New good and using it for two periods, and Leasing a good every period, respectively. It is straightforward to derive the values of these surpluses as

$$V_{\theta,I} = 0, \quad V_{\theta,U} = \frac{1}{1-\gamma} \cdot (u_1(\theta) - q_1), \quad V_{\theta,L} = \frac{1}{1-\gamma} \cdot (u_0(\theta) - r),$$

and

$$V_{\theta,N} = \frac{1}{1-\gamma} \cdot \left( \frac{1}{1+\gamma} u_0(\theta) + \frac{\gamma}{1+\gamma} u_1(\theta) - \frac{1}{1+\gamma} q_0 \right)$$

Furthermore, these value functions have the following easily verifiable properties:

**Lemma 3.** *If  $u_1(\theta)$  and  $[u_0(\theta) - u_1(\theta)]$  each increase in  $\theta$ , then*

- 1)  $V_{\theta,U}$ ,  $V_{\theta,N}$  and  $V_{\theta,L}$  each increase in  $\theta$ ; and
- 2)  $(V_{\theta,U} - V_{\theta,I})$ ,  $(V_{\theta,N} - V_{\theta,U})$  and  $(V_{\theta,L} - V_{\theta,N})$  each increase in  $\theta$ .

Now let's define three break points  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  for the customer type parameter  $\theta$  such that

$$V_{\theta_1,L} = V_{\theta_1,N} \Rightarrow u_0(\theta_1) - u_1(\theta_1) = r - \frac{q_0 - r}{\gamma}, \quad (19)$$

$$V_{\theta_2,N} = V_{\theta_2,U} \Rightarrow u_0(\theta_2) - u_1(\theta_2) = q_0 - (1+\gamma)q_1, \quad (20)$$

and.

$$V_{\theta_3,U} = V_{\theta_3,I} \Rightarrow u_1(\theta_3) = q_1 \quad (21)$$

Obviously, the relative magnitudes of the three such defined points depends on the equilibrium prices  $r$ ,  $q_0$  and  $q_1$ . Lemma 3, however, leads to the following Proposition, regarding optimal consumption strategies of individuals of different types:

**Proposition 1.** *In steady-state, if the equilibrium prices  $r$ ,  $q_0$  and  $q_1$  lead to  $0 < \theta_3 < \theta_2 < \theta_1 < 1$ , then all individuals in  $[0, \theta_3]$  stay idle every period; all those in  $[\theta_3, \theta_2]$  buy a used good every period; all those*



in  $[\theta_2, \theta_1]$  repeatedly buy a new good and use it for two periods; and all those in  $[\theta_1, 1]$  lease a good every period.

Of course, when the monopolistic manufacturer prices her products optimally, it may not always lead to the situation described above. For example, depending on system parameters, it might be optimal for the manufacturer to dictate  $0 < \theta_3 < \theta_2 < \theta_1 = 1$ , which implies that no individual will choose to lease a good.

### 3.2 Manufacturer Behavior in Equilibrium

Here we show that in any steady-state equilibrium, the manufacturer's optimal prices satisfy (18). Complete characterizations of the manufacturer's optimal pricing policies will be illustrated and discussed in greater detail in Section 4.

**Lemma 4.** *In steady-state, the manufacturer's optimal pricing strategy satisfies  $r \leq q_0 - \gamma \cdot (q_1 - \alpha)$ .*

**Proof.** Assuming to the contrary that the manufacturer sets his prices such that

$$r > q_0 - \gamma \cdot [q_1(r, q_0, v) - \alpha]. \quad (22)$$

Then, an individual of any type  $\theta$  will find that the strategy  $(SN, N)$  of “repeatedly buying a new good and selling it after using it for one period” dominates the strategy  $(SL, L)$  of leasing a good every period. Both strategies generate the same per-period utility of  $u_0(\theta)$ , yet the former strategy incurs the per-period cost of  $q_0 - \gamma \cdot [q_1(r, q_0, v) - \alpha]$  which is smaller than the per-period cost  $r$  associated with the latter strategy. Thus, no individual actually uses the strategy  $(SL, L)$ , while there possibly are individuals using the  $(SN, N)$  strategy. We next argue that under this situation, it is possible for the manufacturer to increase his profits by reducing the leasing price to  $r'$  such that

$$q_0 - \gamma \cdot [q_1(r, q_0, v) - \beta] < r' < q_0 - \gamma \cdot [q_1(r', q_0, v) - \alpha]. \quad (23)$$

As a consequence, any pricing strategy satisfying (22) cannot be optimal.

First of all, individuals who preferred strategy  $(SN, N)$  under the old policy (22), switch to strategy  $(SL, L)$  under the new price policy satisfying (23). Consequently, under the new policy the manufacturer receives per-period profit

$$r' - c + \gamma \cdot [q_1(r', q_0, v) - \beta] \quad (24)$$

from each individual that uses the  $(SL, L)$  strategy (recall that  $c$  is the manufacturer's marginal production cost). (24) is larger than the per-period profit of  $q_0 - c$  under the old policy.

What remains to be shown is that starting with any policy  $\{q_0, r, v\}$  satisfying (22), we can generate a corresponding new policy  $\{q_0, r', v\}$  that satisfies (23). To this end, note that if the prices  $q_0$  and  $v$  are unchanged, then the manufacturer can reduce the leasing price to  $r' = q_0 - \gamma \cdot [q_1(r', q_0, v) - \alpha]$  and no individuals, other than those who originally used the strategy  $(SN, N)$ , would be induced to switch their consumption strategy. This last condition ensures that both the supply and demand sides of the used goods market are unchanged, and therefore, used good price  $q_1$  will remain unchanged, and consequently, the new pricing policy can, indeed, be feasibly generated.  $\square$

## 4. Model Solution and Managerial Insights

### 4.1 Methodology for Finding a Steady-State Solution

Under the assumptions of the Lemmas in the previous section, consumers are partitioned, in steady-state, into four types along the  $\theta$  continuum: those in the interval  $[0, \theta_3]$  that do not participate in the market, those in  $[\theta_3, \theta_2]$  that buy used goods every period, those in  $[\theta_2, \theta_1]$  that buy new goods and use them for two periods and those in  $[\theta_1, 1]$  that lease goods every period. The three break points,  $\theta_1, \theta_2$  and  $\theta_3$ , depend on the manufacturer's pricing policy and are determined through equations (19) – (21).

We consider a steady-state where the state variable  $g(\theta)$  takes values of

$$g(\theta) = \begin{cases} 0, & \text{for } \theta \in (\theta_1, 1]; \\ 1/2, & \text{for } \theta \in [\theta_2, \theta_1]; \\ 0, & \text{for } \theta \in [0, \theta_2). \end{cases}$$

Let  $P_L$  be the number of consumers that lease goods every period, then  $P_L$  can be calculated as

$$P_L(p, q_1) = P \cdot \int_{\theta_1}^1 f(\theta) d\theta. \quad (25)$$

Similarly, the number of consumers in  $[\theta_3, \theta_2]$  that buy used goods every period can be calculated as

$$P_U(p, q_1) = P \cdot \int_{\theta_3}^{\theta_2} f(\theta) d\theta. \quad (26)$$

Since all the off-lease goods must be sold to individual consumers, we have the following “conservation of goods” equation, which corresponds to (7):

$$P_U(p, q_1) = P_L(p, q_1) + R(v). \quad (27)$$

Combining (25), (26) and (27) we can find the price of used goods  $q_1$ , the number of individual consumers who lease goods  $P_L$  and the number of consumers who buy used goods  $P_U$ , as functions of the

prices set by the manufacturer.

The number of consumers in  $[\theta_2, \theta_1]$  that buy new goods in any one period, denoted by  $P_N$ , equals the number of consumers who keep their used goods, denoted by  $P_K$ , is

$$P_N(p) = P_K(p) = P \cdot \int_{\theta_2}^{\theta_1} g(\theta) f(\theta) d\theta = \frac{P}{2} \cdot \int_{\theta_2}^{\theta_1} f(\theta) d\theta.$$

The number of consumers in  $[0, \theta_3]$  who do not participate in the market or stay idle is

$$P_I(p) = P \cdot \int_0^{\theta_3} f(\theta) d\theta.$$

The manufacturer sets the price vector  $p = \{q_0, r, v\}$  to maximize his one-period profit

$$\pi_M(p) = (q_0 - c)P_N(p) + (r - c)P_L(p) + (v - c)R(v) + [q_1(p) - \beta][P_L(p) + R(v)], \quad (28)$$

subject to

$$P_L(p), P_N(p), P_U(p), P_I(p), R(v) \geq 0 \quad (29)$$

and

$$P_L(p) + P_N(p) + P_K(p) + P_U(p) + P_I(p) = P. \quad (30)$$

## 4.2 Explicit Model Solution and Managerial Insights

To gain managerial insights, we make several simplifying assumptions about system parameters. These assumptions allow us to obtain an explicit model solution. First, we assume that customers value new and used goods according to the following simple linear functions:

$$u_0(\theta) = \theta \quad \text{and} \quad u_1(\theta) = \delta \cdot \theta \quad \text{with} \quad 0 < \delta < 1.$$

Here, without loss of any generality, we normalize the maximum value placed by any customer on using a new good for one period to be \$1, and the maximum value for a used good to be  $\delta$ . The parameter  $\delta$  obviously measures how close of a substitute used goods are to new ones. The closer  $\delta$  is to 1, the closer the substitution.

We assume that consumers of different types are distributed uniformly on  $[0,1]$ . That is,  $f(\theta) = 1$ , for  $0 \leq \theta \leq 1$ . Without loss of generality, we normalize the individual consumer population to  $P = 1$ . Finally, to reduce the clutter in algebraic expressions without losing significant insights, we show the results when the discount factor  $\gamma = 1$ .

With these assumptions, the three break points are

$$\theta_1(p) = \frac{2r - q_0}{1 - \delta}, \quad \theta_2(p) = \frac{q_0 - 2q_1}{1 - \delta} \quad \text{and} \quad \theta_3(p) = \frac{q_1}{\delta} \quad (31)$$

The four possible subgroups of the consumer population are given by

$$P_L(p) = 1 - \frac{2r - q_0}{1 - \delta}, \quad P_N(p) = P_K(p) = \frac{r + q_1 - q_0}{1 - \delta}, \quad P_U(p) = \frac{\delta q_0 - (1 + \delta)q_1}{\delta(1 - \delta)}, \quad P_I(p) = \frac{q_1}{\delta} \quad (32)$$

The market clearing price for used goods is

$$q_1(p) = \frac{\delta(2r + \delta - 1)}{1 + \delta} - \frac{\delta(1 - \delta)}{1 + \delta} \cdot R(v) \quad (33)$$

Substituting the above quantities into (28), we rewrite manufacturer's profit function as

$$\pi_M(p) = \frac{(q_0 - c)(r + q_1 - q_0) - (2r - q_0 + \delta - 1)(r + q_1 - c - \beta)}{1 - \delta} + (q_1 - c - \beta + v) \cdot R(v) \quad (34)$$

where,  $q_1$  is given by (33).

The manufacturer chooses prices  $q_0$ ,  $r$  and  $v$  *jointly* to maximize his profit function (34), subject to the system constraints (29) and (30). In the sections that follow, the joint optimization problem is solved sequentially in two steps. In step 1, we consider the corporate leasing price  $v$ , or equivalently the corporate leasing quantity  $R = R(v)$ , to be fixed, and find the corresponding optimal consumer selling price  $q_0 = q_0^*(R)$  and consumer leasing price  $r = r^*(R)$ . In step 2, we substitute  $q_0 = q_0^*(R)$  and  $r = r^*(R)$  back into the profit function (34), and, once the specific function form of  $R = R(v)$  is known, find the global optimal corporate leasing price  $v^*$  or, equivalently, the optimal leasing quantity  $R^* = R(v^*)$ . The “step 1” analysis, carried out in the next subsection, allows us to examine carefully a number of issues, including how the introduction of a corporate leasing program affects the consumers' market -- from the viewpoints of both the manufacturer and individual consumers. The “step 2” analysis is illustrated in Subsection 4.2.2.

### 4.2.1 Impact of Corporate Leasing on Individual Consumers Market

By leasing goods to the corporate consumer, the manufacturer increases the number of used goods available to individual consumers on the second-hand market. As one would expect, the market price of used goods decreases as the availability of off-lease goods increases. However, as equation (33) indicates the price of used goods is affected not only by the leases to the corporate consumer, but also by the lease price the manufacturer sets for individual consumers. If the manufacturer were to raise the individual consumers' lease price without correspondingly increasing the sales price, the number of individuals who lease would decrease. Decreasing the number of individuals that lease would in turn decrease the total number of used goods available, and that, in turn, would increase the used goods price.

Indeed, our model shows that for a broad range of system parameters, as the manufacturer increases the number of goods leased to the corporate consumer, he raises the individual lease price to maximize his profit. The price paid by individual consumers for purchasing new goods does not change, nor does the number of consumers that purchase new goods. An increase in the lease price for individuals

means that the number of individual consumers who lease decreases. However, the number of used goods available on the market grows faster – so the total number of consumers participating in the market increases. Proposition 1 specifies the conditions on system parameters and the corresponding model solution that lead to these conclusions. Later in Proposition 2 we present model solutions and draw further conclusions for cases that violate the assumptions of Proposition 1.

**Proposition 1:** Assume that  $\delta < \bar{\delta} \equiv \left( c - 2\beta + \sqrt{(c - 2\beta)^2 + 4(1 - c - 2\beta)} \right) / 2$ . For a given quantity  $R$

leased to the corporate consumer within the range

$$0 < R < \frac{c + \beta + 2\delta}{1 - \delta}, \text{ for } 0 < \delta < (1 - c - 2\beta)/3,$$

$$\text{and } 0 < R < \frac{1}{4\delta}(1 + \delta - c - 2\beta \frac{1 + \delta}{1 - \delta}), \text{ for } (1 - c - 2\beta)/3 < \delta < \bar{\delta},$$

the manufacturer's optimal leasing and selling prices to individual consumers are given by

$$r^*(R) = \frac{(1 + \delta)(c + \beta) + (1 + 4\delta - \delta^2)}{2(1 + 3\delta)} + \frac{\delta(1 - \delta)}{(1 + 3\delta)} \cdot R \text{ and } q_0^* = \frac{(1 + \delta) + c}{2}. \quad (35)$$

The resulting market clearing price for used goods is

$$q_1^*(R) = \frac{\delta(c + \beta + 2\delta)}{1 + 3\delta} - \frac{\delta(1 - \delta)}{(1 + 3\delta)} \cdot R, \quad (36)$$

**Proof:** It is straightforward to verify that the profit function (34) is concave in  $r$  and  $q_0$ , and that the first-order conditions lead to the unique solution (35). Furthermore, under the conditions on parameters specified in the Proposition, the solution to the first-order conditions satisfies constraints (29) and (30) and, hence, is optimal for maximizing the profit function.  $\square$

According to (35) as the manufacturer leases more goods to the corporate consumer, he raises the lease price for individuals, but does not change the price for an outright sale. An increase in the leases to the corporate consumer initially increases the supply of used goods thus lowering their price and making leasing to individuals less profitable for the manufacturer. A lower used goods price induces some individual consumers to switch from following the strategy of buying new goods to that of buying used ones. Higher leasing price, on the other hand, pushes other individuals to switch from leasing to buying new goods. Consequently, it is not obvious how the manufacturer should optimally adjust the selling price with increase in leasing to the corporate consumer. Indeed, it turns out that the optimal selling price does not change, as seen from (35).

Note that for large enough values of  $R$ , such that

$$R \geq \bar{R} \equiv \frac{1}{1-\delta} \left( c + 2\delta - \frac{\beta(1+\delta)}{2\delta} \right),$$

the leasing price to individuals  $r^*(R)$  exceeds the selling price  $q_0^*$ . In fact, for  $0 < \delta < (1-c-2\beta)/3$ , the leasing price is always higher than the selling price. For fixed production cost  $c$  and used goods value  $\delta$ , the threshold value  $\bar{R}$  decreases with transaction cost  $\beta$ , implying that the higher the transaction cost, the sooner the manufacturer's optimal leasing price to individuals surpasses the optimal selling price, which is rather intuitive. The higher leasing price can be interpreted as a surcharge for convenience.

Manufacturer's optimal prices induce individuals to take different consumption strategies according to their individual preferences. Substituting (35) into (32), we get the sizes of different subgroups as

$$P_L^*(R) = \frac{1}{2(1+3\delta)} \left[ 1 + \delta - c - 2\beta \frac{1+\delta}{1-\delta} \right] - \frac{2\delta}{1+3\delta} \cdot R, \quad (37)$$

$$P_N^*(R) = P_K^*(R) = \frac{\beta}{2(1-\delta)}, \quad (38)$$

$$P_U^*(R) = \frac{1}{2(1+3\delta)} \left[ 1 + \delta - c - 2\beta \frac{1+\delta}{1-\delta} \right] + \frac{1+\delta}{1+3\delta} \cdot R, \quad (39)$$

and

$$P_I^*(R) = \frac{c + \beta + 2\delta}{1+3\delta} - \frac{1-\delta}{(1+3\delta)} \cdot R.$$

The number of individuals choosing to lease in (37) decreases with  $R$  due to the increased individual leasing price charged by the manufacturer. Due to the lower price of used goods the number of individuals buying them in (39) increases while the number of individuals choosing to stay idle decreases.

As can be seen from (38) the number of consumers buying new goods is not affected. This, however, does not mean that individuals in this subgroup are not affected by the amount of corporate leasing. In fact, due to the lower used goods price, a fraction of the individuals who previously bought new goods switches to buying used goods. On the other hand, due to the increased individual leasing price, a fraction of consumers who previously leased switches to buying new goods. It turns out the two fractions are equal in size, resulting in no net change in the number of consumers who buy new goods.

The surplus of different consumers is affected differently by the increase in corporate leasing. As the individual lease price increases, those consumers that continue to lease are worse off, as are the consumers who switch from leasing to buying new goods. The consumers who continue to buy used goods are better off just as the consumers who switch from buying new to buying used, since the used goods price decreases. The welfare of the consumers that continue to buy new goods is not affected, nor

is the welfare of the consumers who continue to stay idle. The aggregate (net) welfare of all consumers, denoted by  $S^*(R)$ , can be computed as

$$S^*(R) = \frac{(1 + \delta + c + \beta)^2}{8(1 + 3\delta)} - \frac{\beta^2}{8(1 - \delta)} + \frac{\delta(1 - \delta)}{2(1 + 3\delta)} \cdot R^2$$

and always increases as the manufacturer increases its corporate leasing.

The manufacturer's profit as a function of  $R = R(v)$  can be derived by substituting the relevant quantities of (37)-(39) into (34). We have

$$\pi_M [R(v)] = \frac{(1 + \delta + c + \beta)^2}{4(1 + 3\delta)} - \frac{\beta^2}{4(1 - \delta)} + \left( v - \frac{\delta(1 - \delta)(1 + R(v))}{1 + 3\delta} - \frac{(c + \beta)(1 + \delta)}{1 + 3\delta} \right) \cdot R(v) \quad (40)$$

The manufacturer selects the leasing price  $v$  which determines  $R = R(v)$  so as to maximize his profit  $\pi_M [R(v)]$ . We address this issue in subsection 4.2.2.

**Complete Model Solutions:** When the parameter values fall outside the boundaries specified in Proposition 1, the manufacturer's optimal policies and individual consumers' responses can be quite different. The following Proposition summarizes the complete model solutions:

**Proposition 2:** *For a given quantity  $R$ ,  $0 < R < 1$ , leased to the corporate consumer, the manufacturer's optimal policy for the leasing and selling prices to individual consumers forms one of five distinctive types of 'markets', which are separated from each other by the solid boundaries shown in Figure 3:*

*Market 1: All four classes of consumers exist; Proposition 1 describes the corresponding explicit model solution.*

*Market 2: All consumers participate and are divided into three classes: leasing, buying new goods or buying used goods; Table 3 lists the corresponding model solution.*

*Market 3: With no leasing offered to individuals, consumers are divided into three classes: buying new goods, buying used goods or being idle; Table 4 lists this solution.*

*Market 4: With no leasing offered, all consumers participate and are divided into only two classes: buying new goods or buying used ones; Table 5 lists this solution.*

*Market 5: With no leasing or new-goods-selling offered, consumers are divided into only two classes: buying used goods or being idle; Table 6 lists this solution.*

**Proof:** Continuing the arguments for the proof of Proposition 1, as the parameter values deviate away from the ranges specified in Proposition 1, the constraints (29) and (30) become binding one after another, leading to the solution structure as described here in Proposition 2.  $\square$

In markets 3, 4 and 5 optimal policies call for no leasing to individuals. In these markets we

obtain a lower bound on leasing price  $r^*(R)$ , such that if a price at or above this bound is offered, no individual will choose to lease. Similarly, in market 5 where the optimal policy calls for no selling to individuals, we have a lower bound on selling price  $q_0^*(R)$ .

Market transition boundaries shown in Figure 3 can be viewed as a function of  $\delta$ , which is a measure of substitutability between new and used goods. As the corporate lease quantity  $R$  increases, in economies with small  $\delta \leq (1-c-2\beta)/3$ , where used goods are poor substitutes for new goods, market 1 evolves into market 2 and then market 4.

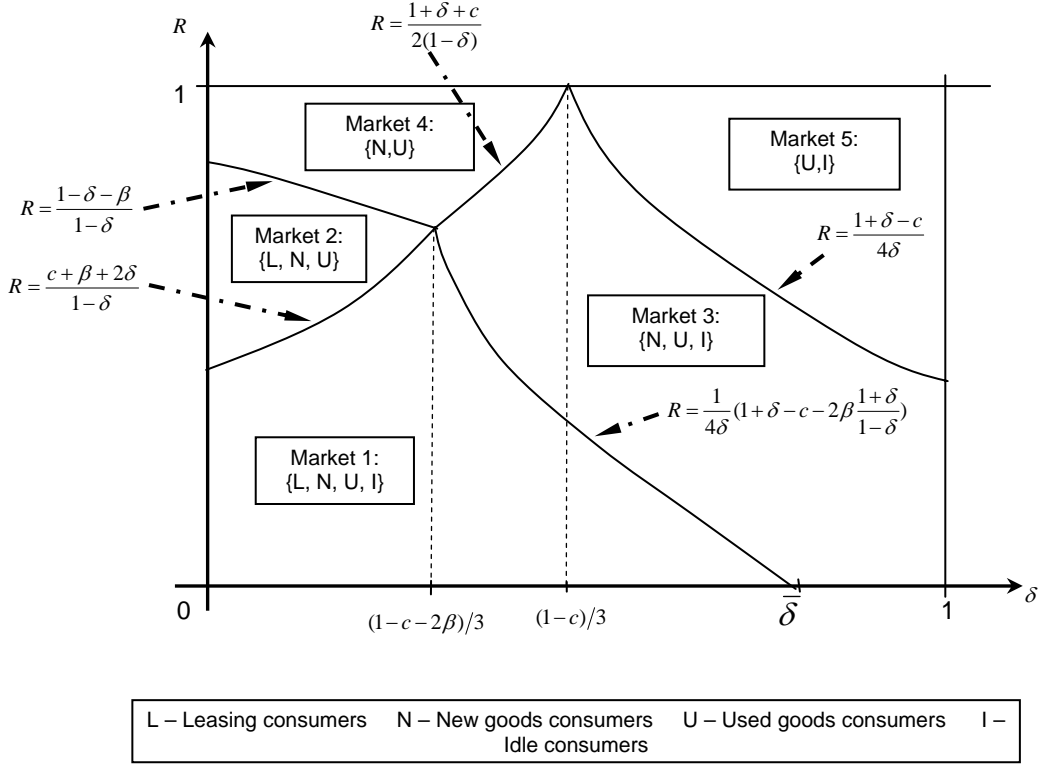
Market 2 is characterized by the absence of non-participants. Market 2 evolves from market 1 when used goods are poor substitutes for new goods. In such a case an increase in corporate leasing causes the price of used goods to decline precipitously. Once the used goods price is zero all the individual consumers participate in the market – essentially used goods are being given away to satisfy the “conservation of goods” constraint<sup>5</sup>. In market 2 the manufacturer still leases and sells new goods to individual consumers, raising both leasing and selling prices as  $R$  increases. In market 2, in contrast to market 1, the manufacturer finds it necessary to adjust both the leasing and the selling prices to increase profitability. The leasing price is always above the selling price. Interestingly the total number of buyers remains unchanged, although who the buyers are changes. The number of leasers decreases with increasing  $R$ . Once there are no more leasers, market 2 transforms into market 4. There are only two types of individual consumers in market 4: those that receive free used goods every period and those that buy new goods. As  $R$  increases, the manufacturer increases purchase price to reduce the number of buyers, since more individual consumers are needed to accept free used goods. In both markets 2 and 4 the aggregate consumer surplus decreases with increasing  $R$ . All consumers participate in the market, but some of those that buy new goods would have preferred to lease, and some of those that receive used goods would have preferred to buy new goods.

When used goods are somewhat better substitutes for new goods  $\delta$  lies between  $(1-c-2\beta)/3$  and  $(1-c)/3$ . As corporate leasing increases market 1 evolves into market 3 and then market 4. The difference between markets 2 and 3 is that in market 3 there are no leasers, while in market 2 there are no non-participants. In market 3, the manufacturer is able to maintain used goods price above zero. As  $R$  increases the used goods price falls, and the number of consumers that choose to buy decreases. However the aggregate consumer surplus grows with  $R$ . Consumers, on the aggregate, are benefiting from increased market participation.

---

<sup>5</sup> In markets 2 and 4 the “conservation of goods” constraint results in the manufacturer reducing its sales of new goods to ensure that it can give away used ones – an unrealistic scenario. A more elaborate model would allow the manufacturer to discard used goods.





**Figure 3. Market Evolution Diagram: Complete Model Solution**

When used goods are fair substitutes for new goods,  $(1-c)/3 < \delta < \bar{\delta}$ , market 1 evolves into market 3 and then into market 5; and when used goods are even closer to new goods, for  $\bar{\delta} < \delta < 1$ , leasing to individuals can never be optimal for the manufacturer, thus market 3 evolves into market 5. Only two types of consumers are present in market 5: those that buy used goods and those that do not participate in the market.

#### 4.2.2 Optimal Leasing to Corporate Consumer

The manufacturer's optimal leasing price  $v^*$  and the corresponding corporate leasing quantity  $R^* = R(v^*)$  is found by maximizing the profit function  $\pi_M^*[R(v)]$ . Once the corporate consumer's demand function  $R = R(v)$  is known, the optimization procedure is straightforward. To illustrate, assume the manufacturer operates in market 1. Then, the relevant profit function  $\pi_M^*[R(v)]$  is given by (40). The leasing price  $v^*$  must then satisfy the following first-order condition:

$$2\delta(1-\delta)R'(v)R(v) - [(1+3\delta)v + \delta(\delta-1) - (1+\delta)(c+\beta)]R'(v) - (1+3\delta)R(v) = 0$$

Assume, for example, that  $R(v) = b - v$ . Substituting  $R(v) = b - v$  into the above equation and solving it

for  $v$ , we get,

$$v^* = \frac{[2\delta(1-\delta) + (1+3\delta)]b + (1+\delta)(c+\beta) + \delta(1-\delta)}{2[\delta(1-\delta) + (1+3\delta)]}$$

which leads to the corresponding optimal leasing quantity of

$$R^* = \frac{(1+3\delta)b - (1+\delta)(c+\beta) - \delta(1-\delta)}{2[\delta(1-\delta) + (1+3\delta)]}$$

As expected, the manufacturer's optimal leasing price  $v^*$  increases with marginal production cost  $c$  and transaction cost  $\beta$ . It should be noted that to assure the optimality of  $v^*$ , the corresponding leasing quantity  $R^*$  computed above needs to fall within the boundaries for market 1, as defined in Proposition 1. Otherwise, it is optimal for manufacturer to operate under one of the other four possible markets, and the optimal corporate leasing price and quantity should be found accordingly.

## 5. Conclusion

In this paper we investigated the coexistence of selling and leasing of finitely durable goods, when the producer of the goods is a monopolist and there are both corporate and individual consumers. The individual consumers have heterogeneous preferences and each uses no more than one unit of good at any time. The corporate consumer may use multiple units of good. We examined how the addition of the corporate consumer affects the market equilibrium, the pricing decisions of the manufacturer and the individual consumers' surplus.

We modeled the interaction between the producer and the consumers as an infinite horizon dynamic game, where the objectives of the parties, the monopolist and the consumers, are to maximize their discounted profits over an infinite horizon. We assumed that the used goods markets are not frictionless, and that both the monopolist and the consumers incur transaction costs when they sell goods on the second-hand market. Further we assumed that the transaction costs incurred by the monopolist are lower than those incurred by individual consumers. Using a model where the goods last two periods and assuming "conservation of goods", we showed that under very mild and reasonable assumptions about the individual consumer utility function, individual consumers separate into four groups along their utility continuum: those that do not participate in the market, those that buy used goods every period, those that buy new goods and use them for two periods, and those that lease new goods every period.

We measured the substitutability of new and used goods by the ratio of the consumer values for new and used goods. Under additional assumptions about the individual consumer's utility function we showed that as used goods become poorer substitutes for new goods the manufacturer may charge more for a single-period lease than for selling the good outright, since the manufacturer will remove from the

consumer the burden of disposing of a used good. We also found that as long as there are non-participating consumers in the market the addition of a corporate consumer increases the aggregate welfare of individual consumers. Consumers that were previously unable to participate in the market, now do – since used goods become more plentiful and more affordable. We also found that in some situations where it is profitable for the manufacturer to lease more goods to the corporate consumer, the manufacturer controls the individual consumer market by adjusting the lease price and keeping the purchase price constant. If used goods are poor substitutes for new goods, then the manufacturer adjusts both lease and sale prices as he increases the number of goods leased to the corporate consumer.

Opportunities for further research are wide. A possible area to explore is how differences in the terms of individual and corporate leases affect the equilibrium. In our model both corporate and individual leases lasted one term. In a number of industries, corporate leases are shorter than leases to individual consumers. Adding in competition is another area that would be interesting to explore.

## References:

- Akerlof, G. (1970). "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." Quarterly Journal of Economics **84**(3): 488-500.
- Bhaskaran, S. R. and S. M. Gilbert (2005). "Selling and Leasing Strategies for Durable Goods with Complementary Products." Management Science **51**(8): 1278-1290.
- Blackwell, D. (1965). "Discounted Dynamic Programming." The Annals of Mathematical Statistics **36**(1): 226-235.
- Bulow, J. (1986). "An Economic Theory of Planned Obsolescence." Quarterly Journal of Economics **101**(4): 729-750.
- Bulow, J. I. (1982). "Durable-Goods Monopolists." Journal of Political Economy **90**(2): 314-332.
- Coase, R. H. (1972). "Durability and Monopoly." Journal of Law & Economics **15**(1): 143-149.
- Desai, P. and D. Purohit (1998). "Leasing and Selling: Optimal Marketing Strategies for a Durable Goods Firm." Management Science **44**(11): S19-S34.
- Desai, P. S. and D. Purohit (1999). Competition in Durable Goods Markets: The Strategic Consequences of Leasing and Selling. Marketing Science, **18**(1): 42-58.
- Hendel, I. and A. Lizzeri (2002). "The Role of Leasing under Adverse Selection." Journal of Political Economy **110**(1): 113-143.
- Huang, S., Y. Yang and K. Anderson (2001). "A Theory of Finitely Durable Goods Monopoly with Used-Goods Market and Transaction Costs." Management Science **47**(11): 1515-1532.
- Johnson, J. P. and M. Waldman (2003). "Leasing, lemons, and buybacks." RAND Journal of Economics **34**(2): 247-265.
- Porter, R. H. and P. Sattler (1999). Patterns of Trade in the Market for Used Durables: Theory and Evidence. Working Paper W7149. Cambridge, MA, NBER.
- Purohit, D. (1997). "Dual distribution channels: The competition between rental agencies and dealers." Marketing Science **16**(3): 228-245.
- Purohit, D. and R. Staelin (1994). "Rentals, sales, and buybacks: Managing secondary distribution channels." Journal of Marketing Research (JMR) **31**(3): 325-338.
- Sawyers, A. (2002). GM seeks more profit in fleet sales mix. Automotive News, **76**: 20.
- Waldman, M. (2003). "Durable Goods Theory for Real World Markets." Journal of Economic

Perspectives 17(1): 131-154.

## Appendix: Tables of Detailed Model Solutions

**Table 3. Explicit Model Solutions for Market 2**

---


$$r^*(R) = \frac{1-\delta}{2} + \frac{1-\delta}{2} \cdot R; \quad q_0^* = \frac{1-\delta-\beta}{2} + \frac{1-\delta}{2} \cdot R; \quad q_1^*(R) = 0$$

$$P_L^*(R) = \frac{1-\delta-\beta}{2(1-\delta)} - \frac{1}{2} \cdot R; \quad P_N^*(R) = P_K^*(R) = \frac{\beta}{2(1-\delta)}; \quad P_U^*(R) = \frac{1-\delta-\beta}{2(1-\delta)} + \frac{1}{2} \cdot R; \quad P_I^*(R) = 0$$

$$S^*(R) = \frac{(1-\delta)(1+3\delta) + \beta^2}{8(1-\delta)} - \frac{(1-\delta)(2-R)R}{8}$$

$$\pi_M^*[R(v)] = \frac{(1-\delta-\beta)^2 - 2c(1-\delta)}{4(1-\delta)} + \frac{2(2v-c-\beta) - (1-\delta)R(v)}{4} \cdot R(v)$$


---

**Table 4. Explicit Model Solutions for Market 3.**

---


$$r^*(R) \geq \frac{3-\delta+c}{4}; \quad q_0^*(R) = \frac{1+\delta+c}{2}; \quad q_1^*(R) = \frac{\delta(1+\delta+c)}{2(1+\delta)} - \frac{\delta(1-\delta)}{(1+\delta)} \cdot R;$$

$$P_L^*(R) = 0; \quad P_N^*(R) = P_K^*(R) = \frac{1+\delta-c}{2(1+\delta)} - \frac{2\delta}{(1+\delta)} \cdot R; \quad P_U^*(R) = R; \quad P_I^*(R) = \frac{1+\delta+c}{2(1+\delta)} - \frac{1-\delta}{1+\delta} \cdot R;$$

$$S^*(R) = \frac{(1+\delta-c)^2}{16(1+\delta)} + \frac{(1-\delta)\delta}{2(1+\delta)} \cdot R^2; \quad \pi_M^*[R(v)] = \frac{(1+\delta-c)^2}{8(1+\delta)} + \frac{(1+\delta)(v-\beta) - c - (1-\delta)\delta R(v)}{(1+\delta)} R(v)$$


---

**Table 5. Explicit Model Solution for Market 4.**

---


$$r^*(R) \geq \frac{1-\delta}{2} + \frac{1-\delta}{2} \cdot R; \quad q_0^*(R) = (1-\delta) \cdot R; \quad q_1^*(R) = 0;$$

$$P_L^*(R) = 0; \quad P_N^*(R) = P_K^*(R) = \frac{1}{2} - \frac{1}{2} \cdot R; \quad P_U^*(R) = R; \quad P_I^*(R) = 0;$$

$$S^*(R) = \frac{1+\delta}{4} - \frac{(1-\delta)(2-R)R}{4}; \quad \pi_M^*[R(v)] = \frac{[2v-c-2\beta+(1-\delta)(1-R(v))]}{2} R(v) - \frac{c}{2}$$


---

**Table 6. Explicit Model Solution for Market 5.**

---


$$r^*(R) \geq \delta - \delta R; \quad q_0^*(R) \geq 1 + \delta - 2\delta R; \quad q_1^*(R) = \delta - \delta R;$$

$$P_L^*(R) = 0; \quad P_N^*(R) = P_K^*(R) = 0; \quad P_U^*(R) = R; \quad P_I^*(R) = 1 - R;$$

$$S^*(R) = \frac{\delta R^2}{2}; \quad \pi_M^*[R(v)] = [v - c - \beta + \delta(1 - R(v))] R(v)$$


---