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Supply Chain Games

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Many supply chains offer goods and services in time-based competition with other chains, but there are few normative results concerning competitive dynamics. We formulate and analyze normative models of competition between supply chains and among the firms comprising a supply chain. The assumptions are those for which an echelon base-stock policy is known to be optimal if the chain is a centrally managed monopolist (or perfect competitor). However, we show that generally there is no equilibrium point in echelon base-stock policies when supply chains compete with each other, whether centralized or not. In contrast, there is an equilibrium point in echelon base-stock policies when a monopolistic supply chain is decentralized.

Key words: supply chain; competition; decentralized; centralized; base-stock policy; equilibrium
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1. Introduction

The profitability of many firms depends on the competitive effectiveness of supply chains in which they are members. Glitches at a single supplier or a supplier's supplier affect the chain's overall ability to meet customer commitments. For example, Firestone tire distri-butors were hurt badly by the manufacturer's pricing policy following the Ford Explorer-Firestone tire adverse publicity a few years ago. However, their competitors, such as Goodyear, benefited from the mishap (White et al. (2001)).

Little is known about the operational dynamics of supply chain competitive behavior. In the economics of industrial organizations, an ample normative literature on competing supply chains has few operational details (Greenhut and Ohta (1979), Grossman and Hart (1986), and Ziss (1995)). In contrast, most of the normative results in the operations literature are based on either perfect competition or monopoly. This literature during the past decade seems to have focused on the effects of echelon base-stock policies. That is, orders are placed at each stage in the supply chain in order to raise echelon inventory to a target level. However, it seems to us that imperfect competition characterizes the interactions among most supply chains. In other words, little is known about the normative operational dynamics of supply chains in their arguably most prevalent environment. This study investigates the extent to which echelon base-stock policies are appropriate

for these environments.

The necessary attributes of a model of operational dynamics in supply chain competition include two (or more) supply chains, two (or more) echelons in each supply chain, and decisions made in numerous time periods. Several previous studies nearly meet these criteria. Boyaci and Gallego (2004) model competing two-echelon supply chains which attract Poisson demands that are proportional to their service rates. We say that a supply chain is *coordinated* or *centralized* if it has, in effect, one decision maker. The authors analyze the static strategic game in which each player (supply chain) has two alternatives, namely to coordinate or to remain uncoordinated. The payoffs in the resulting bimatrix game stem from the subsequent interactions of the firms. Coordination turns out to be a dominant strategy for both supply chains, but the aggregated expected profits of the chains are smaller under the coordinated scenario than under the uncoordinated scenario. This paper differs from Boyaci and Gallego (2004) in two important ways. First, decisions are made every period, and second, the supply chains interact through product availability, i.e., each retailer's demand depends both on her own and the competing retailer's inventories. This inter-dependency of demand on inventory may be caused by a substitution effect or a demand stimulation effect of inventory. Netessine et al. (2005) review the literature in which customers substitute one product with another or switch from one retailer to another when their first-choice product or source is out of stock.

Several normative studies investigate interactions among the echelons in a single uncoordinated multi-echelon supply chain. These studies implicitly assume either monopoly or perfect competition, but they are relevant here due to their multiplicity of decision makers. Lee and Whang (1999) assume that the lower stage incurs backorder penalties, and the upper stage incurs holding costs. They design performance mechanisms to induce each member to choose a system-optimal base-stock level. In a similar model, Porteus (2000) analyzes responsibility tokens as the coordination scheme. Chen (1999) designs an accounting scheme which induces each member to minimize its own cost without compromising the system-wide performance. Cachon and Zipkin (1999) consider a static strategic game in which the wholesaler and retailer each make once-and-for-all choices of a base-stock level policy. A pair of payoffs, the long-run average profits per period, is associated with each pair of choices. They show that the resulting bimatrix game has a unique equilibrium point that differs from the system-optimal solution. Hence, they develop a linear transfer scheme that coordinates the supply chain. In contrast to these references, the firms in this paper make decisions each period and they are not assumed *ex ante* to use base-stock policies.

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There are several game theoretic analysis of supply chain "front ends," i.e., of models with only a single echelon (the "retailer"). Kirman and Sobel (1974) analyze a dynamic oligopoly model in which competing firms set prices and inventory levels each period. They characterize an equilibrium point when demand depends on prices but (unlike the present study) not on inventories. Netessine et al. (2005) analyze the same model except that demand depends on retailers' inventories but not on prices. They investigate the effects of different representations of customer backlogging behavior and the impacts of retailers' retention incentives on customers when stockouts occur. Bernstein and Federgruen (2004) model a dynamic inventory and pricing game for a distribution system with one wholesaler and two retailers engaged in price competition. They improve on Kirman and Sobel (1974) by obtaining sufficient conditions for the existence of a unique unrandomized equilibrium point.

The next section specifies a model with dynamic stochastic interactions between and within two supply chains who compete through product availability. Each chain consists of a wholesaler and a retailer with an established relationship. If a customer at one retailer encounters a stockout, she might switch her patronage to the other retailer. As a result, each retailer's realized demand depends on both retailers' inventory levels which in turn depend on the wholesalers' inventories. Hence, the supply chains interact through all firms' inventory policies.

We analyze four scenarios: a single monopolistic decentralized supply chain whose retailer and wholesaler are separate decision-makers, two competing decentralized supply chains, two competing centralized supply chains, and a hybrid scenario in which one supply chain is centralized but the other is not. Mimicking the well-known result that an echelon base-stock policy is optimal for a centralized serial supply chain, we prove that the firms in a decentralized supply chain have equilibrium points in echelon base-stock policies. The result is valid for chains with more than two members. However, when two or more supply chains compete, centralized or not, we find that generally there is no equilibrium point in echelon base-stock policies.

Section 2 presents the model, and §3 shows that generally there is no equilibrium point in echelon base-stock policies when (a) centrally managed chains compete with each other, and (b) when a centrally managed chain competes with a decentralized chain. The interests of the firms in a decentralized chain are not congruent and, therefore, they are essentially competing with each other even if they all share the same information. In the resulting dynamic game in a monopolistic decentralized chain, §4 proves that there is an equilibrium point at which each firm uses an echelon base-stock policy. Then in §5 analyzes a dynamic game among the firms comprising two decentralized chains in which we assume that the wholesalers expedite shipments to the retailers

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whenever shortages occur. As a result, there is an equilibrium point at which all the firms employ echelon base-stock policies. Conclusions and unanswered questions are in §6.

2. Model

The model is a dynamic supply chain generalization of Cournot oligopoly. Each supply chain consists of a wholesaler and a retailer with complete information. The market determines prices and quantities actually sold. If the chains are centralized, these decisions are made by the chains' coordinators. At the beginning of each period t $(t = 1, 2, \cdots)$ the retailers and wholesalers (or the coordinators if the chains are centralized) review their inventories and make replenishment decisions. In supply chain i $(i = 1, 2)$, let x_t^{ri} and x_t^{wi} denote the retailer's and wholesaler's respective inventory levels at the beginning of period t and z_t^{ri} and z_t^{wi} their order quantities. We indicate a pair of variables for both chains by suppressing chain-identifying superscripts; for example, $x_t^r = (x_t^{r1}, x_t^{r2})$.

For specificity we assume that there is a lag of one period to transfer goods from a supplier to a wholesaler, and that delivery is immediate from a wholesaler to a retailer. Therefore, the total supplies available to satisfy demand at the retailers are their order-up-to levels. However, with minor changes the results are valid for any integer-valued delivery lags between chain members. Let y^{ri} and y^{wi} denote the respective retailer's and wholesaler's inventory positions (i.e. on-hand plus on-order) in supply chain i after purchase orders are processed and transported in period t:

$$y_t^{ri} = x_t^{ri} + z_t^{ri} \qquad \text{and} \qquad y_t^{wi} = x_t^{wi} + z_t^{wi} - z_t^{ri} \tag{1}$$

We assume that each chain member's storage is bounded above, namely that $y_t^{ri} \leq u^{ri}$ and $y_t^{wi} \leq u^{wi}$ with $u^{ri} < \infty$ and $u^{wi} < \infty$ for each i. Consistent with the complete information assumption, we assume that the retailers do not order more than the wholesalers' on-hand inventories. Therefore, $0 \leq z_t^{wi}$ and $0 \leq z_t^{ri} \leq x_t^{wi}$. It can be shown that $z_t^{ri} \leq x_t^{wi}$ is a redundant constraint in a model with expediting and altered information conditions.

When $a = (a_i)$ and $b = (b_i)$ are vectors of the same dimension, $min\{a, b\}$ denotes the vector whose ith component is $min\{a_i, b_i\}$. We preclude planned backlogging and, therefore, constrain $y^{ri} \geq 0$ and $y^{wi} \geq 0$. If the initial conditions satisfy $x_1^{ri} \leq u^{ri}$ and $0 \leq x_1^{wi} \leq u^{wi}$, the bounds correspond to

$$x_t^w \leq y_t^w \leq u^w \qquad x_t^r \leq y_t^r \leq min\{x_t^r + x_t^w, u^r\} \tag{2}$$

Let D_t^i be the nonnegative random demand encountered by retailer i in period t. Since each component of D_t may depend on y_t^r, we sometimes write $D_t(y_t^r)$. This models an array of more

specific customer behaviors (Kirman and Sobel (1974) and Netessine et al. (2005)). For any $\omega_1 \geq 0$ and $\omega_2 \geq 0$, we assume that $D_1(\omega_1, \omega_2), D_2(\omega_1, \omega_2), \cdots$ are independent and identically distributed random vectors; let $D(\omega_1, \omega_2) = (D^1(\omega_1, \omega_2), D^2(\omega_1, \omega_2))$ be a vector with the same probability distribution as $D_1(\omega_1, \omega_2)$.

We let the retailers' revenues and inventory-related costs be random variables whose probability distributions depend on the vector $y = (y^1, y^2)$ of retailers' supply levels. Let $\mathcal{G}_t^i$ be retailer i's revenue net of inventory-related costs in period t and let $\mathcal{G}_t = (\mathcal{G}_t^1, \mathcal{G}_t^2)$. We assume that $\mathcal{G}_t$ is conditionally independent of $\mathcal{G}_1, ..., \mathcal{G}_{t-1}$ given y_t and that the conditional distribution of $\mathcal{G}_t$ given $y_t = y$ is the same for all t. These assumptions are consistent with many specific models in operations and economics.

We assume that the wholesalers incur linear holding costs and that all firms incur proportional ordering costs and are risk-neutral profit-maximizers. However, the paper's conclusions remain valid if decision-makers are risk-averse with exponential utility functions (Chung and Sobel (1987), Bouakiz and Sobel (1992)). Let c_i^w, c_i^r, and h_i^w be wholesaler i's respective unit purchasing cost, wholesale price, and unit holding cost; let ρ_i be retailer i's unit penalty cost of excess demand. For each unit of excess demand, if any, the wholesaler pays $\alpha_i \rho_i$ ($0 \leq \alpha_i \leq 1$) and the retailer pays $(1 - \alpha_i)\rho_i$.

We assume that excess demand $(D_t^i - y_t^{ri})^+$ is backlogged and that the following chronology of events occurs during each period t: inventory levels are observed, ordering decisions are made, retailer demands are realized, revenues and costs are credited and debited, and inventory balances are updated. Hence, the dynamics are

$$x_{t+1}^r = y_t^r - D_t(y_t^r) \qquad\qquad x_{t+1}^w = y_t^w \qquad\qquad (3)$$

Let $\Re^+$ denote the nonnegative real numbers. Some of the paper's results remain valid if the backlogging assumption is replaced with $x_{t+1}^r = y_t^r - \theta_t(y_t^r, D_t)$ where $\theta_1(y^r, d)$, $\theta_2(y^r, d)$, $\cdots$ are independent and identically distributed nonnegative random vectors for each y^r and d, and realized $\theta_1(\cdot, D_1)$ is concave on $\Re^+$. In particular, this assumption includes excess demand being lost.

Let β_{ri} and β_{wi} be the single-period discount factors in supply chain i, and define the following echelon variables:

$$s_t^{ri} = x_t^{ri} \qquad a_t^{ri} = y_t^{ri} \qquad s_t^{wi} = x_t^{ri} + x_t^{wi} \qquad a_t^{wi} = y_t^{ri} + y_t^{wi} \qquad a_t^r = (a_t^{r1}, a_t^{r2}) \qquad (4)$$

The constraints and dynamics in (2) and (3) correspond to

$$s_t^{wi} \leq a_t^{wi} \qquad s_t^{ri} \leq a_t^{ri} \leq s_t^{wi} \qquad\qquad (5)$$

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$$s^{wi}_{t+1} = a^{wi}_t - D^i_t \qquad s^{ri}_{t+1} = a^{ri}_t - D^i_t \tag{6}$$

Let b_{ri} and b_{wi} denote the expected present values of the retailer's and wholesaler's profits, and let b_{ri} and b_{wi} denote the corresponding expected values. Let $a^r = (a^{r1}, a^{r2})$ denote a generic value of the pair $a^r_t = (a^{r1}_t, a^{r2}_t)$ and define the following functions:

$$L_i(a^r) = E[\{\mathcal{G}^i_1 - (1-\alpha_i)\rho_i(a^{ri} - D^i_1)^+ - [(1-\alpha_i)\rho_i + \beta_{ri}c^r_i]D^i_1\}|a^r_1 = a^r] \tag{7}$$
$$- [c^r_i(1-\beta_{ri}) - (1-\alpha_i)\rho_i]a^{ri}$$

$$H_i(a^r, a^{wi}) = [\alpha_i\rho_i + h^w_i + c^r_i(1-\beta_{wi})]a^{ri} - [h^w_i + c^w_i(1-\beta_{wi})]a^{wi} \tag{8}$$
$$- E\{\alpha_i\rho_i(a^{ri} - D^i_1)^+ - [\beta_{wi}(c^r_i - c^w_i) + \alpha_i\rho_i]D^i_1|a^r_1 = a^r, a^{wi}_1 = a^{wi}\}$$

Then

$$b_{ri} = \sum_{t=1}^{\infty} \beta^{t-1}_{ri}\left\{\mathcal{G}^i_t - c^r_i z^r_t - (1-\alpha_i)\rho_i(D^i_t - y^{ri}_t)^+\right\}$$
$$= \sum_{t=1}^{\infty} \beta^{t-1}_{ri}\left\{\mathcal{G}^i_t - c^r_i(1-\beta_{ri})y^{ri}_t - \beta_{ri}c^r_i D^i_t - (1-\alpha_i)\rho_i(D^{ri}_t - y^{ri}_t)^+\right\} + c^r_i x^{ri}_1$$

So

$$b_{ri} = E\left\{\sum_{t=1}^{\infty} \beta^{t-1}_{ri}L_i(a^r_t)\right\} + c^r_i s^{ri}_1 \tag{9}$$

The present value of the wholesaler's profits is

$$b_{wi} = \sum_{t=1}^{\infty} \beta^{t-1}_{wi}\left[c^r_i z^{ri}_t - c^w_i z^{wi}_t - h^w_i y^{wi}_t - \alpha_i\rho_i(D^i_t - y^{ri}_t)^+\right]$$
$$= \sum_{t=1}^{\infty} \beta^{t-1}_{wi}\left\{(c^r_i - c^w_i)(1-\beta_{wi})y^{ri}_t + \beta_{wi}(c^r_i - c^w_i)D^i_t - [h^w_i + c^w_i(1-\beta_{wi})]y^{wi}_t\right.$$
$$\left. - \alpha_i\rho_i(D^i_t - y^{ri}_t)^+\right\} + c^w_i x^{wi}_1 - c^r_i x^{ri}_1$$

So

$$b_{wi} = E\left\{\sum_{t=1}^{\infty} \beta^{t-1}_{wi}H_i(a^r_t, a^{wi}_t)\right\} + c^w_i s^{wi}_1 - (c^r_i + c^w_i)s^{ri}_1 \tag{10}$$

An echelon base-stock policy, loosely speaking, selects order quantities to move each echelon's inventory position as close as possible to a target level. Let a^{ri}_* and a^{wi}_*, respectively, denote the retailer's and wholesaler's targets (decision variables) in supply chain i. In view of (5), an echelon base-stock policy in an infinite-horizon model stipulates $a^{ri}_t = min\{max\{a^{ri}_*, s^{ri}_t\}, s^{wi}_t\}$ and $a^{wi}_t = max\{a^{wi}_*, s^{wi}_t\}$. If $s^{ri}_1 \leq a^{ri}_*$ and $s^{wi}_1 \leq a^{wi}_*$ as we typically assume, then $a^{ri}_t = min\{a^{ri}_*, s^{wi}_t\}$ and

$a_t^{wi} = a_*^{wi}$ for all t. The same is true in a finite-horizon model except that a_*^{ri} and a_*^{wi} acquire time indices.

Although the foregoing model is a sequential game in which the players' time streams of rewards are discounted over an infinite horizon, for simplicity in the following sections we analyze the corresponding sequential game with a finite horizon. We introduce some notation to explain a solution concept (Heyman and Sobel (2004), p. 452) that slightly generalizes the standard notion of an equilibrium point of a sequential game.

In a sequential game let Q be the set of players; at various points in the following sections, the sequential game models have two to four players. For each $q \in Q$, let Π_q be a strategy for player q, that is, a non-anticipative rule for deciding what action to take each period as a function of the elapsed history thus far. Let $\Pi = (\Pi_q, q \in Q)$ be the tuple of all players' strategies. It is common to write $\Pi = (\Pi_q, \Pi_{-q})$ where Π_{-q} consists of the strategies of all the players except player q. Let $v_s^q(\Pi, N)$ be the expected present value of the rewards to player q during an N-period horizon if the players employ strategies in Π and s is the state at the beginning of the first period. For each player q let π_q be a subset of player q's policies, and let $\pi = \times_{q \in Q} \pi_q$. Let S' be a subset of states. We say that $\Pi^* = (\Pi_q^*, q \in Q)$ *is an N-period equilibrium point with respect to initial states $s \in S'$ and policies in π if*

$$v_s^q(\Pi^*, N) \geq v_s^q[(\xi_q, \Pi_{-q}^*), N] \qquad \forall s \in S', \xi_q \in \pi_q, q \in Q$$

Our interest is in supply chain games which have equilibrium points in echelon base-stock policies with respect to the set π of stationary policies and a nonempty set of initial states S'. In game-theoretic terminology, such a Π^* would be Markov-perfect and each player would employ a time-invariant strategy.

3.   Dynamic Competition Between Centralized Supply Chains

The most important finding in the paper, a negative result, concerns centrally managed supply chains. We explain why there is generally no equilibrium point at which each chain employs an echelon base-stock policy. As centralization of an industry's competing supply chains becomes widespread, there is competitive advantage in managing supply levels with policies that are more complex than would be worthwhile if the chains were decentralized.

Competing centralized supply chains lack an equilibrium point among echelon base-stock policies.

Let b_i be the sum of the retailer's and wholesaler's expected present values of profits in supply chain i and define

$$\gamma_{ri}(a^r) = [c_i^w(1 - \beta_i) + h_i^w + \rho_i]a^{ri} + E[\mathcal{G}_1^i - \rho_i D^i - \rho_i(a^{ri} - D^i)^+]$$

$$\gamma_{wi}(a^{wi}) = -[c_i^w(1-\beta_i) + h_i^w]a^{wi}$$

Summing (9) and (10) yields

$$b_i = E\Big\{ \sum_{t=1}^{\infty} \beta_i^{t-1} \Big[\gamma_{ri}(a_t^r) + \gamma_{wi}(a_t^{wi})\Big] + c_i^w(s_1^{wi} - s_1^{ri}) \Big\}$$

As in Sinha and Sobel (1997), if supply chain two uses a base-stock policy with target echelon supply levels a_*^{r2} and a_*^{w2} at the retailer and the wholesaler, respectively, and $a_*^{w2} \geq s_1^{w2}$ and $a_*^{r2} \geq s_1^{r2}$, then supply chain one's best response corresponds to the following dynamic program:

$$f_t^1(s^{r1}, s^{w1}, s^{w2}) = sup_{a^{r1}, a^{w1}}\{\gamma_{r1}(a^{r1}, min\{a_*^{r2}, s^{w2}\}) + \gamma_{w1}a^{w1} + \beta_1 E[f_{t-1}^1(a^{r1} - D_1,$$
$$a^{w1} - D_1, a_*^{w2} - D_2)] : s^{r1} \leq a^{r1} \leq s^{w1} \leq a^{w1}\}$$

That is, chain one's best response depends on the inventory levels in chains one *and* two. An echelon base-stock policy at chain one would depend on inventory levels *only* in chain one and, therefore, it could not be a best response.

This conclusion is consistent with the following intuition. Suppose that both supply chains use echelon base-stock policies at an equilibrium point. When retailer two's order is constrained by his wholesaler's inventory while wholesaler one's inventory is ample, because inventory information is available to all players, the coordinators of chain one can exploit his competitor's shortage by raising his retailer's supply in that period to a level that is higher than his target level due to demand substitutability. So supply chain one reaps extra profit by deviating from echelon base-stock policies. Similarly, knowing chain one's response, chain two should sometimes deviate from his echelon base-stock policy. Therefore, competing centralized supply chains generally lack an equilibrium point in echelon base-stock policies.

A similar argument shows that competition between centralized and decentralized chains lacks an equilibrium point among echelon base-stock policies.

4.   Decentralized Monopolistic Supply Chain
4.1.   The General Case

Here we suppress the chain label i and consider the interactions between a wholesaler and a retailer in a monopolistic supply chain. If the wholesaler employs an echelon base-stock policy with base-stock level $a_*^w \geq s_1^w$, then $a_t^w = a_*^w$ and $s_{t+1}^w = a_*^w - D_t$ for all t. So the retailer's decision problem corresponds to the following dynamic program with $V_0(\cdot, \cdot) \equiv 0$, $s^r \leq s^w$ and $n = 1, 2, \cdots$:

$$V_n(s^r, s^w) = max\Big\{W_n(a) : s^r \leq a \leq s^w\Big\} \tag{11}$$
$$W_n(a) = L(a) + \beta_r E\{V_{n-1}(a - D, a_*^w - D)\}$$

It follows from (7) and the exogenous distribution of demand that $L(\cdot)$ is concave on its domain if $E(\mathcal{G}_1|a_1^r = a)$ is a concave function of a. Many specific examples yield concavity including $\mathcal{G}_1 = p \cdot min\{D_1, a_1^r\} - h^r(a_1^r - d_1)^+$. The first term is revenue that is proportional to the lesser of supply and demand, and the second term is an inventory cost that is proportional to the excess supply.

An induction starting with $n = 0$ establishes the following conclusion.

LEMMA 1. *If $E(\mathcal{G}_1|a_1^r = a)$ is a concave function of a, then $V_n(\cdot, \cdot)$ and $W_n(\cdot)$ are concave functions on their domains $(n = 1, 2, ...)$.*

In a monopolistic decentralized chain, a base-stock policy is the retailer's best response to the wholesaler's use of an echelon base-stock policy.

THEOREM 1. *A base-stock policy is optimal in (11). That is, there is a scalar a_{*n}^r such that $a = max\{s^r, a_{*n}^r\}$ is optimal in (11).*

Proof. If the wholesaler employs an echelon base-stock policy, the retailer faces a Markov decision process in which the retailer's best response must be an optimal policy. Select $a_{*n}^r \in argmax\, W_n(\cdot)$; then $a^* = min\{s^w, max\{a_{*n}^r, s^r\}\}$ is optimal in (11). There are three cases:

$$a^* = \begin{cases} a_{*n}^r & \text{Case A: } s^r \leq a_{*n}^r \leq s^w; \\ s^r, & \text{Case B: } a_{*n}^r < s^r \leq s^w \text{ (transient case)}; \\ s^w, & \text{Case C: } s^r \leq s^w \leq a_{*n}^r. \end{cases}$$

Let $\tilde{a} = max\{a_{*n}^r, s^r\}$ and observe that $L(min\{\tilde{a}, s^w\}) = L(min\{max\{a_{*n}^r, s^r\}, s^w\}) = L(a^*)$. Therefore, $\tilde{a} = a_{*n}^r = a^*$ and $\tilde{a} = s^r = a^*$ in cases A and B, respectively. In case C, $\tilde{a} = a_{*n}^r$ and $a^* = s^w$, so $L(a^*) = L(s^w)$ and $L(min\{\tilde{a}, s^w\}) = L(min\{a_{*n}^r, s^w\}) = L(s^w)$. Therefore, $\tilde{a} = max\{s^r, a_{*n}^r\}$ is optimal. $\square$

When the retailer uses a base-stock policy with target level $a_*^r \geq s_1^r$, then $a_t^r = min\{s_t^w, a_*^r\}$ because the retailer cannot obtain more than the wholesaler's on-hand inventory. So the wholesaler's decision problem corresponds to the following dynamic program with $v_0(\cdot, \cdot) \equiv 0$, $s^r \leq s^w$, and $n = 1, 2, \cdots$:

$$v_n(s) = max\{J_n^w(a, s) : a \geq s\} \tag{12}$$
$$J_n^w(a, s) = H(min\{s, a_*^r\}, a) + \beta_w E\{v_{n-1}[a - D_t(min\{s, a_*^r\})]\}$$

LEMMA 2. *For each n, $v_n(\cdot)$ and $J_n(\cdot, \cdot)$ are concave functions on their domains.*

Proof. In order to begin a straightforward induction starting with $n = 0$ and $v_0 \equiv 0$ yield

$$\begin{aligned} J_1^w(a, s) = H(min\{s, a_*^r\}, a) &= \beta(c^r - c^w)d - \alpha\rho E[D_t - min\{s, a_*^r\}]^+ \\ &+ [c^r(1 - \beta) + h^w]min\{s, a_*^r\} - [c^w(1 - \beta) + H^w]a \end{aligned} \tag{13}$$

which is a sum of concave terms. $\square$

There is a pure strategy equilibrium point at which the firms in a monopolistic decentralized supply chain employ the same kind of policy as if they were in a monopolistic centralized supply chain.

THEOREM 2. *If $E(\mathcal{G}_1|a_1^r = a)$ is a concave function of a, then there are scalars a_*^r and a_*^w such that the decentralized monopoly supply chain game has an equilibrium point relative to $[0, a_*^r] \times [0, a_*^w]$ at which $(a_t^r, a_t^w) = (min\{a_*^r, s_t^r\}, a_*^w)$, $t = 1, 2, ...$*

Proof. Consider the one-period two-player strategic game Γ in which the retailer and wholesaler, respectively, choose $a^r \in [0, u^r]$ and $a^w \in [0, u^w]$ and receive payoffs $L(a^r)$ and $H(a^r, a^w)$. Then Γ, termed the *reduced game*, has a pure strategy equilibrium point (a_*^r, a_*^w) due to the concavity of $L(\cdot)$ and $H(\cdot, \cdot)$ and Debreu (1952). Because of the additive separability of (9) in (a_t^r, s_1^r) and (10) in $(a_t^w, a_t^r; s_1^w, s_1^r)$, the nonnegativity of demands, (5), and (6), following Sobel (1981), an induction establishes that the dynamic game with payoffs (9) and (10) has an equilibrium point relative to $[0, a_*^r] \times [0, a_*^w]$ consisting of $(a_t^r, a_t^w) = (min\{a_*^r, s_t^r\}, a_*^w)$ for all $t = 1, 2, \cdots$. $\square$

4.2. Decentralized Supply Chain with Expedited Shipment

This sub-section is related to Cachon and Zipkin (1999) in which each firm's criterion is its long-run average profit per period (no discounting) and each firm chooses an echelon target inventory level. Thus the dynamic game becomes a static strategic game in which each firm's decision is its target echelon supply level. Cachon and Zipkin (1999) analyze this strategic game and conclude that the wholesaler has little influence on the retailer's strategy. We aim to garner additional insight into the wholesaler's role in a decentralized supply chain.

We assume that whenever the wholesaler cannot completely fill the retailer's order, she expedites the shortfall from her supplier to the retailer at unit cost λc^w ($\lambda > 1$). Expediting may occur because the retailer has more power, or because the wholesaler wants to retain the retailer's goodwill. In addition, if the retailer incurs large losses upon stockout, he may require the wholesaler to provide this premium service. We note that as λ rises, the wholesaler should raise her base-stock level to reduce expediting cost.

The retailer's expected profit per period remains the same as (9) but $s_t^r \leq a_t^r \leq s_t^w$ in (5) is replaced with $s_t^r \leq a_t^r$. For simplicity, let $\alpha = 0$, i.e., the wholesaler does not share the retailer's backorder cost. This simplification affects the numerical values of the players' stock levels but not the structure of equilibrium points. Moreover, since the wholesaler fulfills the retailer's order in each period, it is reasonable to hold the retailer fully accountable for excess demand. Since the retailer's ordering quantity is not constrained by the wholesaler's inventory, an argument that is

similar to the proof of Theorem 2 yields the optimality (relative to $[0, a_*^r]$) of the retailer's use of a base-stock policy consisting of $a_t^r = a_*^r$ for all $t = 1, 2, \cdots$. Now we address the effect of this policy on the wholesaler.

The expediting feature of the model alters the expected present value of the wholesaler's profit from (10). Under the expediting assumption, $a_t^w = (s_t^w - a_*^r)^+ + z_t^w + a_*^r$. The dynamics remain the same as in §2, i.e., $s_{t+1}^w = a_t^w - D_t$ and $s_{t+1}^r = a_*^r - D_t$, but the wholesaler's ordering constraint $z_t^w \geq 0$ corresponds to $a_t^w \geq a_*^r$ for all $t = 1, 2, \cdots$, which is satisfied (as assumed in §2) because planned backlogging is precluded. The resulting expected present value of the wholesaler's profit is

$$
\begin{aligned}
b_w &= E \sum_{t=1}^{\infty} \beta_w^{t-1} \left\{ c^r(a_*^r - s_t^r) - c^w z_t^w - \lambda c^w(a_*^r - s_t^w)^+ - h^w(a_t^w - a_*^r) \right\} \\
&= E \sum_{t=1}^{\infty} \beta_w^{t-1} \left\{ \beta_w(c^r - c^w) D_t(a_*^r) + \left[h^w + c^r(1 - \beta_w)\right] a_*^r - \left[h^w + c^w(1 - \beta_w)\right] a_t^w \right. \\
&\quad \left. - \beta_w c^w(\lambda - 1) \left[a_*^r - a_t^w + D_t(a_t^r)\right]^+ \right\} - (\lambda - 1) c^w(a_*^r - s_1^w)^+ - c^r s_1^r + c^w s_1^w \\
&= E \left[\sum_{t=1}^{\infty} \beta_w^{t-1} M(a_*^r, a_t^w) \right] - (\lambda - 1) c^w(a_*^r - s_1^w)^+ - c^r s_1^r + c^w s_1^w
\end{aligned}
\tag{14}
$$

with the definition

$$
\begin{aligned}
M(a^r, a^w) &= \beta_w(c^r - c^w) E[D_1(a^r)] + [h^w + c^r(1 - \beta_w)] a^r - [h^w + c^w(1 - \beta_w)] a^w \\
&\quad - \beta_w c^w(\lambda - 1) E[a^r - a^w + d(a^r) + \eta]^+
\end{aligned}
\tag{15}
$$

In the braces on the first line of (14), the first term is the wholesaler's revenue in period t, the second is the purchasing cost for a regular order, the third term is the purchasing cost of an expedited order, and the last term is the holding cost. We observe that the wholesaler's single-period measure of effectiveness, $M(a, \cdot)$, has economies of scale, i.e., for any $a \in \Re$, $M(a, \cdot)$ is concave on $\Re^+$.

The wholesaler faces a Markov decision process with payoff (14) which is parameterized by the wholesaler's target base-stock level. Let $a_*^w \in argmax M(a_*^r, a^w)$. If $a_*^w \geq s_1^w$, then $a_*^w \geq s_t^w = a_*^w - D_{t-1}$ for $t \geq 2$ because D_t is nonnegative. So $a_t^w = a_*^w$ is optimal for all $t = 1, 2, \cdots$. In conclusion, the dynamic game with payoffs (9) and (14) has an equilibrium point relative to $[0, a_*^r] \times [0, a_*^w]$ consisting of $(a_t^r, a_t^w) = (a_*^r, a_*^w)$ for all $t = 1, 2, \cdots$.

More specific assumptions regarding the structures of revenues, costs, and demand lead to explicit solutions and comparative statics results. These assumptions correspond to an exogenous retail price, a linear retail revenue function, and concavity of the mean demand as a function of the retail supply level. In part (b) of the following result, d' denotes the right-hand derivative of d, and setting $dL(a^r)/da^r$ and $\partial M(a^r, a^w)/\partial a^w$ to zero yields (16) and (17), respectively.

THEOREM 3. *(a) The decentralized monopolistic supply chain game under expedited shipment has an equilibrium point relative to $[0, a_*^r] \times [0, a_*^w]$ consisting of $(a_t^r, a_t^w) = (a_*^r, a_*^w)$ for all $t = 1, 2, \cdots$.*
(b)If

(i) $\mathcal{G}_1 = p \min\{a_1^r, D_1\} - h^r (a_1^r - D_1)^+$

(ii) $D_1(a_1^r) = d(a_1^r) + \eta_t$ *with $\eta_1, \eta_2, \cdots$ independent and identically distributed random variables with mean zero and distribution function $F(\cdot)$,*

(iii) $d(\cdot)$ *concave and $d(a) \leq d(a + \delta) < d(a) + \delta$ $(0 \leq a, 0 < \delta)$,*

then

$$a_*^r = d(a_*^r) + F^{-1}\Big[\frac{p - c^r (+ (\rho + \beta c^r)(1 - d'))}{(\rho + p + \rho)(1 - d')}\Big] \tag{16}$$

$$a_*^w = a_*^r + d(a_*^r) + F^{-1}\Big[\frac{h^w + c^w(1 - \beta_w)}{\beta_w c^w (\lambda - 1)}\Big] \tag{17}$$

The assumptions regarding expected demand as a function of supply level, $d(\cdot)$, imply that the right-hand derivative satisfies $0 \leq d'(a) < 1$. The hypothesis of part (b) is presumed throughout the remainder of this section.

From (16) and (17), the chain members should set their target supply levels by considering the inventory effect on demand. However, the degree of this effect differs for the retailer and the wholesaler. The second term of (17) depends on several parameters but it does not depend on $d(\cdot)$ because the wholesaler should set a supply level such that her expected overstock cost equals her expected shortage cost and these costs depend only on the parameters in the second term and $F(\cdot)$. However, d' appears in the second term of (16) because the retailer should adjust his inventory both for the deterministic part of demand (d) and for the random part of demand. His inventory level affects demand and, consequently, his profits.

The numerator of (17), $c^w(\lambda \beta_w - 1) - h^w$, is the wholesaler's unit shortage cost, while the denominator, $\beta_w c^w(\lambda - 1)$, which can be rewritten as $[(\beta_w \lambda - 1)c^w - h^w] + [c^w(1 - \beta_w) + h^w]$, is the sum of the unit shortage cost and the unit overstocking cost. Hence, the wholesaler should set her echelon base-stock level as if she were an independent dynamic newsvendor.

We now characterize the dependence of echelon base-stock levels on cost and revenue parameters. We say "increasing" and "decreasing" for nondecreasing and nonincreasing, respectively. The results are summarized as follows.

PROPOSITION 1. *The retailer's target supply level a_*^r increases as as p or α increase, it decreases as c^r increases, and if $d(x + \delta) \leq d(x) + \delta(\delta > 0)$, then it also increases as h^r decreases or β^r increases. The wholesaler's target echelon level a_*^w increases as λ, c^w or a^r increase or as h^w or c^r decrease. If $\lambda < 2$, then a_*^w increases as β_w increases.*

Proof. Under the assumptions in Theorem 3(b), $L(\cdot)$, $d(\cdot)$ and $M(\cdot,\cdot)$ are concave, so their derivatives from the right exist.

$$\frac{\partial^2 L(a^r)}{\partial p \partial a^r} = 1 - [1 - d'(a^r)]F[a^r - d(a^r)] \geq 0$$

$$\frac{\partial^2 L(a^r)}{\partial c^r \partial a^r} = -\beta_r d'(a^r) - (1 - \beta_r) \leq 0$$

$$\frac{\partial^2 L(a^r)}{\partial a^w \partial a^r} = 0$$

If $d(x+\delta) \leq d(x) + \delta(\delta > 0)$, then $d'(a^r) \leq 1$ and

$$\frac{\partial^2 L(a^r)}{\partial \alpha \partial a^r} = \rho[1 - d'(a^r)] + d'(a^r)\rho \geq 0$$

$$\frac{\partial^2 L(a^r)}{\partial h^r \partial a^r} = -[1 - d'(a)]F[a^r - d(a^r)] \leq 0$$

$$\frac{\partial^2 L(a^r)}{\partial \beta_r \partial a^r} = c^r[1 - d'(a^r)] \geq 0$$

Also,

$$\frac{\partial^2 M(a^r, a^w)}{\partial h^w \partial a^w} = -1 < 0$$

$$\frac{\partial^2 M(a^r, a^w)}{\partial \lambda \partial a^w} = \beta_w c^w[1 - F(a^w - a^r - d(a^r))] \geq 0$$

$$\frac{\partial^2 M(a^r, a^w)}{\partial \beta_w \partial a^w} = c^w(2 - \lambda)\{1 - F[a^w - a^r - d(a^r)]\} \geq 0$$

$$\frac{\partial^2 M(a^r, a^w)}{\partial a^r \partial a^w} = \beta_w c^w(\lambda - 1)\partial F[a^w - a^r - d(a^r)]/\partial a^r \geq 0$$

$$\frac{\partial^2 M(a^r, a^w)}{\partial c^r \partial a^w} = 0$$

$$\frac{\partial^2 M(a^r, a^w)}{\partial a^w \partial c^w} = -(1 - \beta_w) - \beta_w(\lambda - 1)F[a^w - a^r - d(a^r)] \leq 0$$

□

The assumption $d(x + \delta) < d(x) + \delta(\delta > 0)$ means that a unit increase in inventory induces an increase of less than one unit of demand. When p, α or β_r increase, the retailer's expected backorder costs increase, so he should raise his base-stock level. Conversely, when c^r or h^r increase, his expected holding costs increase, so he should lower his base-stock level. Similarly, when λ or α increase, the wholesaler should raise her echelon base-stock level to reduce the expected backorder cost. If expediting is not too expensive $\lambda < 2$, then the wholesaler should increases a_*^w as β_w increases to reduced the expected backorder cost. Conversely, she should lower her echelon base-stock level to reduce the expected inventory cost when c^w or h^w increase. We note that because a_*^w increases as a_*^r increases, and a_*^r decreases as c^r increases, a_*^w decreases as c^r increases. This means that knowing that the retailer will lower his supply level if she raises the wholesale price, the wholesaler lowers her echelon supply level accordingly.

4.3. Pricing and Inventory Games

Since the wholesaler expedites any inventory shortfalls, the retailer always obtains the quantity that he orders. That is, the wholesaler's inventory policy does not influence him directly ($\partial L/(\partial a^r \partial a^w) = 0$). However, the price that the wholesaler charges the retailer certainly does affect the retailer. Proposition 1 asserts that the retailer should lower his base-stock level when the wholesale price increases. Since wholesale price is a vital element of contracts between wholesalers and retailers, in this subsection we enlarge the dynamic game to include the wholesale price decision by the wholesaler.

The enlarged model has two stages. At the first stage, the wholesaler fixes the wholesale price, c^r, which remains constant thereafter. At the second stage, the retailer and wholesaler play the infinite-horizon game described in §4.2. However, expediting implies that the retailer optimizes without regard to the inventory choices made by the wholesaler. If the wholesaler chooses wholesale price c^r at the first stage, let $a^r_*(c^r)$ make explicit the dependence of an optimal a^r on c^r in (7). Anticipating this choice by the retailer, the wholesaler's second-stage decisions are selected to maximize (14), i.e., the wholesaler selects a^w to maximize $M[a^r_*(c^r), a^w]$ subject to $a^w \geq a^r_*(c^r)$. Let $a^w_*(c^r)$ be a maximizing value of a^w. It follows from (14) that the wholesaler's first-stage problem is to choose c^r to maximize

$$M[a^r_*(c^r), a^w]/(1-\beta_w) - (\lambda-1)c^w(a^r_* - s^w_1)^+ c^r - s^r_1 + c^w s^w_1$$

The assumptions in Theorem 3(b) lead to a characterization of the resulting wholesale price.

THEOREM 4. *Under the hypothesis of Theorem 3(b), there is a pure strategy equilibrium point relative to $[0, a^r_*] \times [0, a^w_*]$ for the dynamic game preceded by the pricing decision in which the retailer and wholesaler employ echelon base-stock policies, and the wholesale price satisfies*

$$c^r_* = c^w + \frac{(h^w + c^w - \beta_w c^w)d'(a^r_*)}{\beta_w d'(a^r_*) + 1 - \beta_w} + \frac{(s^r_1 - a^r_*)(1-\beta_w) - \beta_w d(a^r_*)}{a^{r'}_*[\beta_w d'(a^r_*) + 1 - \beta_w]} \tag{18}$$

Setting the derivative of (14) with respect to c^r to zero yields (18). From (18), the equilibrium wholesale price does not depend on the unit expediting cost but depends only on the wholesaler's purchasing cost and holding cost, β_w, demand's marginal rate in inventory, the sensitivity of the retailer's target supply level to the wholesaler price, and the retailer's initial inventory!

We now characterize the impacts of c^w, h^w, , s^r_1 and β_w on c^r_*. A proof similar to that of Proposition 1 establishes the following result.

PROPOSITION 2. *The equilibrium point wholesale price, c^r_*, increases as c^w or h^w increases, and decreases as β_w increases.*

Proposition 2 indicates that the wholesaler passes her higher purchasing and holding costs on to the retailer by raising the wholesale price. This transfer enhances "double marginalization", the phenomenon that a decentralized retailer often sets a lower base-stock level than a centralized decision maker would select. A smaller discount factor β_w makes the present more valuable than the future; therefore, the wholesaler raises the wholesale price. Higher s_1^r means that the retailer buys less in the first period, so the wholesaler increases the wholesale price.

5. Dynamic Competition Between Decentralized Supply Chains

This section studies dynamic inventory competition between two decentralized supply chains. We continue to assume that the wholesalers expedite orders to the retailers at unit cost $\lambda_i c_i^w (\lambda_i > 1)$ when they are out of stock.

It follows from (7) and the proof of Theorem 3 that the two retailers play an infinite horizon game having an embedded strategic game in which the payoffs to the retailers one and two, when retailers one and two select respective supply levels a and b, is

$$L_1(a,b) = E\{\mathcal{G}_1^1 - \rho_1[a - D_1^1(a,b)]^+ - (\rho_1 + \beta_{r1}c_1^r)D_1^1(a,b)\} - c_1^r(1-\beta_{r1})a$$
$$L_2(a,b) = E\{\mathcal{G}_1^2 - \rho_2[b - D_1^2(a,b)]^+ - (\rho_2 + \beta_{r2}c_2^r)D_1^2(a,b)\} - c_2^r(1-\beta_{r2})b$$

Each retailer's decision is affected directly by the other retailer because each one's demand depends on both retailers' supply levels. The concavity of $L_1(\cdot, b)$ and $L_2(a, \cdot)$ and the result in Debreu (1952) imply that the retailers' imbedded game has an equilibrium point relative to $\times_{i=1}^2 [0, a_*^{ri}]$ consisting of $(a_t^{r1}, a_t^{r2}) = (a_*^{r1}, a_*^{r2})$ for all $t = 1, 2, \cdots$.

Now we turn to the wholesalers. wholesaler one faces a Markov decision process with payoff

$$M_i(a_*^{r1}, a_*^{r2}, a^{wi})/(1 - \beta_{wi}) + \lambda_i - 1)c_i^w(a_*^{ri} - s_1^{wi})^+ - c_i^r s_1^{ri} + c_i^w s_1^{wi}$$

where

$$\begin{aligned}M_i(a_*^{r1}, a_*^{r2}, a^{wi}) = {} & \beta_{wi}(c_i^r - c_i^w)E[D_1^i(a_*^{r1}, a_*^{r2})] + [h_i^w + c_i^r(1 - \beta_{wi})]a_*^{ri} - [h_i^w \\ & + c_i^w(1 - \beta_{wi})]a^{wi} - \beta_{wi}c_i^w(\lambda_i - 1)E[a_*^{ri} - a^{wi} + d(a_*^{r1}, a_*^{r2}) + \eta_i]^+\end{aligned} \qquad (19)$$

for $i = 1, 2$. Above equation has the same form as (15) with a^r replaced by (a_*^{r1}, a_*^{r2}). We observe that the retailers' decisions directly affect the wholesalers, but the wholesalers do not interact with each other. The concavity of $M_i(\cdot, \cdot, a^{wi})$ ensures the existence of $a_*^{wi} \in argmax M_i(a_*^{r1}, a_*^{r2}, a^{wi})$ for $i = 1, 2$. In summary, the dynamic decentralized supply chain game with payoffs defined by (9) and (14) has an equilibrium point relative to $\times_{i=1}^2 [0, a_*^{ri}] \times [0, a_*^{wi}]$ consisting of $(a_t^{ri}, a_t^{wi}) = (a_*^{ri}, a_*^{wi})$ for all $t = 1, 2, \cdots$. More specific assumptions regarding the structures of revenues, costs and demand lead to an explicit solution and comparative results. Let $d_i^i = \partial d_i(a^{r1}, a^{r2})/\partial a^{ri}$ $(i = 1, 2)$. The results are summarized as follows.

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THEOREM 5. *(a) If $E(\mathcal{G}_t^i)$ is concave in a_t^{ri} and $E(D_t^i)$ is concave in a_t^{ri}, $i = 1, 2$, then the decentralized supply chain game under expedited shipment has an equilibrium point relative to $\times_{i=1}^2 [0, a_*^{ri}] \times [0, a_*^{wi}]$ consisting of $(a_t^{ri}, a_t^{wi}) = (a_*^{ri}, a_*^{wi})$ for all $t = 1, 2, \cdots$.*

(b) If

(i) $\mathcal{G}_1^i = p_i \min\{D_1^i(a_1^r), a_1^{ri}\} + h_i^r (a_1^{ri} - D_1^i)^+$

(ii) $D_t^i(a^1, a^2) = d_i(a^1, a^2) + \eta_{it}$ with $\eta_{i1}, \eta_{i2}, \cdots$ independent and identically distributed random variables with mean zero and distribution function $F_i(\cdot)$,

(iii) $d_1(\cdot, a^2)$ and $d_2(a^1, \cdot)$ increasing and concave,

then,

$$a_*^{ri} = d_i(a_*^{r1}, a_*^{r2})] + F_i^{-1}\left[\frac{\rho_i - c_i^r(1 - \beta_i)}{(\rho_i + h_i^r)(1 - d_i^i)} + \frac{(p_i - \rho_i - \beta_i c_i^r)d_i^i}{(\rho_i + h_i^r)(1 - d_i^i)}\right] \tag{20}$$

$$a_*^{wi} = a_*^{ri} + d_i(a_*^{r1}, a_*^{r2}) + F_i^{-1}\left[\frac{h_i^w + c_i^w(1 - \beta_{wi})}{\beta_{wi} c_i^w(\lambda_i - 1)}\right] \tag{21}$$

(c) As c_i^r, β_{ri}, or h_i^r increase, or as c_j^r or ρ_j decrease, a_*^{ri} decreases $(i = 1, 2; i \neq j)$.

The hypothesis in part(a) of Theorem 5 are sufficient for the concavity of $L_1(a, \cdot)$ and $L_2(\cdot, b)$ on their domains.

We note that expediting is essential for the existence of equilibrium points of the decentralized supply chain game. Without the expediting feature, the argument in §3 leads to the conclusion that competing decentralized multi-echelon supply chains generally lack equilibrium points in echelon base-stock policies.

6. Conclusions and Questions

Since Clark and Scarf (1960), sufficient conditions have been known for an echelon base-stock policy to be optimal in a monopolistic *centralized* supply chain. When these same conditions are applied to competing *centralized* chains, the class of echelon base-stock policies does *not* generally contain an equilibrium point. The negative result persists if a centralized chain competes with a decentralized one. Consequently, it is not clear whether a first-mover advantage accrues to the supply chain that invests first in coordination. In addition, it is unclear whether costless coordination is profitable if the competitor chain is centralized. Since we assume that the chains compete through product availability, and that the firms' revenues are functions of goods supply level, perhaps specific forms of the revenue functions $\mathcal{G}^i$ could lead to answers to these questions.

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References

Bernstein, F., A. Federgruen. 2004. Dynamic inventory and pricing models for competing retailers. *Naval Research Logistics* **52** 258–274.

Bouakiz, M., M.J. Sobel. 1992. Inventory control with an exponential utility criterion. *Oper. Res.* **40** 603–608.

Boyaci, T., G. Gallego. 2004. Supply chain coordination in a market with customer service competition. *Production & Operations Management* **13** 3–22.

Cachon, G.P., P.H. Zipkin. 1999. Competitive and cooperative inventory policies in a two-stage supply chain. *Management Sci.* **45** 936–953.

Chen, F. 1999. Decentralized supply chains subject to information delays. *Management Sci.* **45** 1076–1090.

Chung, K.J., M.J. Sobel. 1987. Discounted mdp's: Distribution function and exponential utility maximization. *SIAM J. Control & Optimization* **25** 49–62.

Clark, A.J., H. Scarf. 1960. Optimal policies for a multi-echelon inventory problem. *Management Sci.* **6** 475–490.

Debreu, G. 1952. A social equilibrium existence theorem. *Proceedings of National Academy of Science* **38** 886–893.

Greenhut, M.L., H. Ohta. 1979. Vertical integration of successive oligopolists. *American Economics Review* **69** 1137–141.

Grossman, S.J., O.D. Hart. 1986. The costs and benefits of ownership: a theory of vertical and lateral integration. *J. of Political Econom.* **94** 691–719.

Heyman, Daniel P., Matthew J. Sobel. 2004. *Stochastic Models in Operations Research, Volume II:Stochastic Optimization*. Dover Books, Mineola, NY.

Kirman, A., M. Sobel. 1974. Dynamic oligopoly with inventories. *Econometrica* **42** 279–287.

Lee, H., J. Whang. 1999. Decentralized multi-echelon supply chains: Incentives and information. *Management Sci.* **45** 633–640.

Netessine, S., N. Rudi, Y. Wang. 2005. Inventory competition and incentives to backorder. Unpublished Manuscript. Wharton School of Business, University of Pennsylvania.

Porteus, E.L. 2000. Responsibility tokens in supply chain management. *Manufacturing, Service & Operations Management* **2** 203–219.

Sinha, C., M. Sobel. 1997. Work order release rules in serial manufacturing of a single product. Working Paper. Havuman School, SUNY Stony Brook.

Sobel, M.J. 1981. Myopic solutions of markov decision processes and stochastic games. *Oper. Res.* **29** 995–1009.

White, J.B., T. Aeppel, C. Arsberry. 2001. Ford plans to recall 47,000 year 2002 suvs—car maker may also replace more firestone tires on older explorers. *Wall Street J.* A3.

Ziss, S. 1995. Vertical separations and horizontal mergers. *J. of Industrial Econcom.* **43** 63–75.