Technical Memorandum Number 800

Are Hub-and-Spoke Networks Better than Point-to-Point Networks?

by

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August 2005

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Abstract

This paper investigates how airlines might determine their network structures through a three-stage duopoly game in which two airlines serve a three-city network under demand uncertainty. At the first stage, the airlines choose their network structures; at the second stage, while route demands are still uncertain, they construct capacities; at the third stage, after demands are known, they allocate seats to each route given capacity constraints imposed by the second stage decision.

We characterize a subgame perfect equilibrium in the capacity and quantity games and show that at equilibrium, either both airlines use *hub-and-spoke* networks, or both use *point-to-point* networks. We also identify critical factors that determine airlines' network choices. Even if fixed investment costs are ignored, a hub-and-spoke network does not necessarily dominate a point-to-point network. However, a high demand variance or a low mean demand generally favors a hub-and-spoke network. This paper sheds some lights into the success of low-cost carriers exemplified by Southwest.

(*Nonlinear programming, Hub-and-Spoke, Point-to-Point, Economies of density, Flexibility*)

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1. Introduction

During the first ten years of U.S. airline deregulation (in the 1980s), major airlines (e.g., Northwest) shifted dramatically from *point-to-point* networks to *hub-and-spoke* networks. Hub-and-spoke networks not only create *economies of density* by flying passengers from different cities to and from a few hub cities but also yield higher flight frequency and broader geographic coverage. The role of traffic density in the airline industry has been widely studied both theoretically (Bailey, Graham and Kaplan (1985), Brueckner and Spiller (1991), Hendricks, Picciione and Tan (1995a)) and empirically (Caves, Christensen and Tretheway (1984), Brueckner, Dyer and Spiller (1992)).

However, some airlines moved back to point-to-point networks recently (e.g. Southwest). The growing level of congestion at major hub airports in the 1980s created opportunities for low-fare, no-frills, and point-to-point services exemplified by Southwest airline. Shunning congested airports and direct competition with major airlines, low-cost carriers carved out a thriving market niche by reviving point-to-point services. In response, several major airlines (Continental, Delta, United, and US Airways) created subsidiaries offering similar services using a single type of aircraft (to reduced aircraft maintenance costs) and lower-paid crews.

Major airlines are now experiencing financial difficulties: US Airways filed for bankruptcy for a second time, and Delta is near bankruptcy. Excess capacity, cut-throat competition, oil price surges, powerful labor unions, and terrorist threats contributed to this situation. Carey and McCartney (2004) suggest that major airlines should de-emphasize hub-and-spoke networks and add direct flights between non-hub cities following the low-cost carriers' strategy.

This paper investigates how airlines might determine their network structures through a three-stage duopoly game in which two airlines serve a three-city network under demand uncertainty. At the first stage, the airlines choose their network structures; at the second stage, while route demands are still uncertain, they construct capacities; at the third stage, after demands are known, they allocate seats and flights to each route given capacity constraints imposed by the second stage decision.

Besides economies of traffic density, a hub-and-spoke network provides an airline the flexibility of allocating capacities among markets after uncertainty is resolved. Hence, if a hub-and-spoke network does not incur excess extra investment costs in the hub, it might seem to be a better choice than a point-to-point network for a monopolistic airline. However, we show later in §4 that this intuition could be wrong.

We address the following questions:

- Is a hub-and-spoke network always better than a point-to-point network? If not, when is it dominated by a point-to-point network?

- Under what conditions are there subgame perfect equilibrium points for the three-stage duopoly game? What are the equilibrium network structures?

There is a stream of papers by economists studying network selection and strategic interactions between airlines. Oum, Zhang and Zhang (1995) study a duopoly model with three cities. Considering economies of density which affect both costs and demand, they show that strategic interaction reinforces the tendency towards hubbing because hubbing reduces airlines' marginal costs and increases product quality thus forcing their competitors to cut output. The authors further show that even if hubbing increases total costs, strategic considerations may lead airlines to adopt this network structure. However, hubbing causes the prisoner's dilemma: both airlines adopt hub-and-spoke networks, but the competitive advantage cancels out, so both may be worse off. Hendricks, Picciione and Tan (1995b) identify conditions under which an equilibrium point with competing hub-and-spoke networks exists for an n-city network. Barla (1999) studies a three-stage duopoly game in which the airlines have different hubs but the same non-hub cities thus competing only on the non-hub route. He concludes that the airlines determine their network structures through balancing the flexibility value versus the committed advantage in the non-hub route. In contrast, we assume that the airlines compete across a collection of three cities of which one is a hub city.

The rest of the paper is organized as follows. §2 presents the model. §3 characterizes a subgame perfect equilibrium point in the quantity and capacity games for all possible cases:
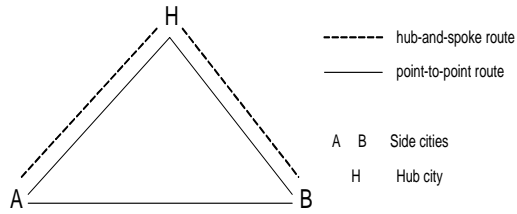
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Figure 1: A Three-City Network

both airlines employ hub-and-spoke networks, both airlines employ point-to-point networks, and they employ different network structures. §4 studies the network game and identifies factors that determine the airlines' network structures. §5 concludes the paper.

2.   Model and Assumptions

Two risk-neutral airlines compete across a collection of three cities: a hub city H and two non-hub cities, A and B (Figure1). Markets AH and BH are addressed as the *local markets* and AB as the *connecting market*. A passenger travelling from A to B has to take two flights, AH and HB, if he or she flies with a hub-and-spoke network, but the same passenger take only one flight (AB) with a point-to-point network. We assume that the markets from and to the hub city are identical, i.e., markets AH and BH have a common demand function. We also assume that all three markets AH, BH, and AB are bidirectionally symmetric, e.g., traffic from A to B has the same characteristics as the traffic from B to A. As a result, the third-stage game is a two-product *Cournot* competition, that is, the airlines compete for the connecting and local passengers. Relaxing above two symmetric assumptions adds the number of markets but will not change the qualitative results.

If an airline has chosen a hub-and-spoke network at the first stage, then at the second stage while demands are still unknown, it decides on the aggregate capacity (e.g., aircraft size and maximum number of flights to offer per week) for the local and connecting markets; if it has chosen a point-to-point network initially, then it chooses capacities for the local and connecting markets, respectively. At the third stage, after demands are known, the airline

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allocates seats between the local and connecting markets if it has chosen a hub-and-spoke network. If it has chosen a point-to-point network, it simply determines upon the number of seats and flights to offer in each market subject to the capacities. Each stage in the model is a simultaneous-move non-cooperative game with complete information.

Let c_h and c_p be the unit capacity costs, respectively, of a hub-and-spoke network and of a point-to-point network, q_{yi} be the number of seats that airline i offers in market y, and p_y be the ticket price for market y. We note that some of the results might change if an economy of scale is considered for the capacity cost. Demands for the origin-destination pairs (AH, BH, AB) are independent random variables. We assumed a linear inverse demand function $p_y = M_y - (q_{y1} + q_{y2})$ ($y = l, c$; l represents the *local markets* and c the *connecting market*). Random variable M_y is nonnegative and has a mean μ_y and a variance σ_y^2 with distribution function $F_y(\cdot)$. The qualitative results would remain valid if a more general demand function were used.

We assume that a connecting passenger cannot be accommodated through the hub if he or she chooses to fly through a point-to-point network. Hence, a hub-and-spoke network provides an airline the *flexibility* of allocating capacities between the local and connecting markets after demands are known, but a point-to-point network does not. In addition, we assume that connecting passengers are prevented from arbitrages, i.e., they have to buy a single ticket AB instead of two separate tickets AH and HB. The airlines can enforce such a policy simply by checking passengers' destination cities on their boarding passes. Furthermore, for expository simplicity we examine only the cases in which the realization of markets satisfies $M_c \leq M_l$, a sufficient condition for $p_c \leq 2p_l$ thus eliminating arbitrage opportunities. This simplicity is justified since hub cities usually attract heavier traffic than non-hub cities. We assume $c_h \leq \mu_y$ and $c_p \leq \mu_y$ so that the airlines make profits. The variable costs are assumed to be zero because airlines' operating costs are mostly associated with offering a seat rather than serving a passenger (Barla (1999)).

Network adjustment is infrequent in the airline industry especially when one of the endpoint airports is congested and gates and landing slots are hard to obtain. Capacity

Figure 2: Three-Stage Duopoly Game

investment is also a long-term decision. However, quantity, number of seats to offer, is more flexible (e.g., airlines can adjust number of flights to offer depending on market realization).

Let airline one be the row-player and airline two be the column player, H denote a hub-and-spoke network and P a point-to-point network, and Π_i represent airline i's net profit. Three cases are possible: both airlines use hub-and-spoke networks, both airlines use point-to-point networks, and they use different networks. Figure 2 represents the game by a 2×2 matrix.

Let π_i represent airline i's profit in the *Cournot* competition. We assume that the capacities are finite. Let K_i denote the capacity on either leg when airline i uses a hub-and-spoke network, and K_{yi} be airline i's capacity in market y when it uses a point-to-point network.

point-to-point	hub-and-spoke
$\max_{k_{l1},k_{c1}} \Pi_1 = E\pi_1 - c_p(2K_{l1} + K_{c1})$ $\max_{q_{l1},q_{c1}} \pi_1 = 2p_l q_{l1} + p_c q_{c1}$ $s.t. \ 0 \le q_{y1} \le K_{y1} \ (y = l, \ c)$	$\max_{k_{l1},k_{c1}} \Pi_1 = E\pi_1 - 2c_h K_1$ $\max_{q_{l1},q_{c1}} \pi_1 = 2p_l q_{l1} + p_c q_{c1}$ $s.t. \ q_{l1} + q_{c1} \le K_1$ $0 \le q_{l1}, 0 \le q_{c1}$

Figure 3: Capacity and Quantity Optimization

Working backward, we first obtain equilibrium quantities in the *Cournot* game, then the capacity competition, and finally the network structure game.

At the third stage, given their rival's network structures and capacities, the airlines en-

gage in the quantity game after demand realization. Specifically, the airlines choose numbers of seats q_{li} and q_{ci} to maximize their expected payoffs (π_1, π_2) given capacity constraints imposed by the second stage decision and their network structures. For example, when airline one uses a hub-and-spoke network and airline two uses a point-to-point network, airline one's problem is

$$max_{q_{l1}, q_{c1}} \pi_1 = 2p_l q_{l1} + p_c q_{c1}$$
$$= 2[M_l - (q_{l1} + q_{l2})]q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1}$$
$$s.t. \quad q_{l1} + q_{c1} \leq K_1 \tag{1}$$
$$0 \leq q_{l1} \quad 0 \leq q_{c1}$$

Similarly, airline two's problem is

$$max_{q_{l2}, q_{c2}} \pi_2 = 2p_l q_{l2} + p_c q_{c2}$$
$$= 2[M_l - (q_{l1} + q_{l2})]q_{l2} + [M_c - (q_{c1} + q_{c2})]q_{c2} \tag{2}$$
$$s.t. \quad 0 \leq q_{l2} \leq K_{l2} \quad 0 \leq q_{c2} \leq K_{c2}$$

At the second stage, in the capacity game the airlines determine their capacities with payoffs (Π_1, Π_2), given their network structures. Continuing the example above, airline one's problem is

$$max_{K_1} \Pi_1 = E\pi_1 - 2c_h K_1 \tag{3}$$

and airline two's problem is

$$max_{K_{l2}, K_{c2}} \Pi_2 = E\pi_2 - c_p(2K_{l2} + K_{c2}) \tag{4}$$

Figure 3 summarizes airline one's capacity and quantity optimization problems given its network structure. Finally, at the first stage, given its rival's network structure, airline i selects its network structure depending on the expected payoff Π_i.

3. Quantity and Capacity Games

We now characterize a subgame perfect equilibrium point in the quantity and capacity games. In this section we assume that the airlines do not necessarily deplete their capacities, so the quantity and capacity games are different. Because the calculations are tedious and lengthy,

we attach the details in Appendices A, B and C and outline the solutions and illustrate the calculation through the case in which both airlines adopt hub-and-spoke networks.

3.1 Hub-and-Spoke Networks

Label the airlines so that $K_1 \leq K_2$. For the quantity competition, the airlines' problems are defined as (1), a concave nonlinear maximization problem to which the Karush-Kuhn-Tucker (KKT) conditions apply. For example, airline one's KKT conditions are

$$
\begin{aligned}
2(M_l - 2q_{l1} - q_{l2}) - u_1 + v_{l1} &= 0 \\
v_{l1} q_{l1} &= 0 \\
M_c - 2q_{c1} - q_{c2} - u_1 + v_{c1} &= 0 \\
v_{c1} q_{c1} &= 0 \\
u_1(q_{c1} + q_{l1} - K_1) &= 0 \\
q_{c1} + q_{l1} &\leq K_1 \\
u_1, v_{l1}, v_{c1} &\geq 0 \\
q_{c1}, q_{l1} &\geq 0
\end{aligned}
\tag{5}
$$

In (5), v_{l1} and v_{c1} are slack variables, and u_1 is a Lagrange multiplier. The realization of demands defines binding conditions of the capacities and the values of the Lagrange multiplier and slack variables. There are six combinations of the variables that define the regions in Figure 4. The equilibrium quantities can be derived by solving (5) and its analogues for airline two for each region.

In area 1, neither airline is capacity-constrained; in area 2, only airline one is capacity-constrained; in area 3, both airlines are capacity-constrained; in area 4, only airline one is capacity-constrained, and it serves only the local markets; in area 5, both airlines are capacity-constrained, and airline one serves only the local markets; in area 6, both airlines are capacity-constrained and serve only the local markets; the connecting market is not served. Areas 4, 5, or 6 occur when the local markets are significantly larger than the connecting market.

M_c (a) $K_1 < K_2$

$K_1 + 2K_2$

$2M_l - M_c = 6K_1$

3

$3K_1$

5

$2M_l - M_c = 4K_2 + 2K_1$

2

6

1

4

$3K_1$ $K_1 + 2K_2$ M_l

M_c (b) $K_1 = K_2 = K$

$2M_l - M_c = 6K^h$

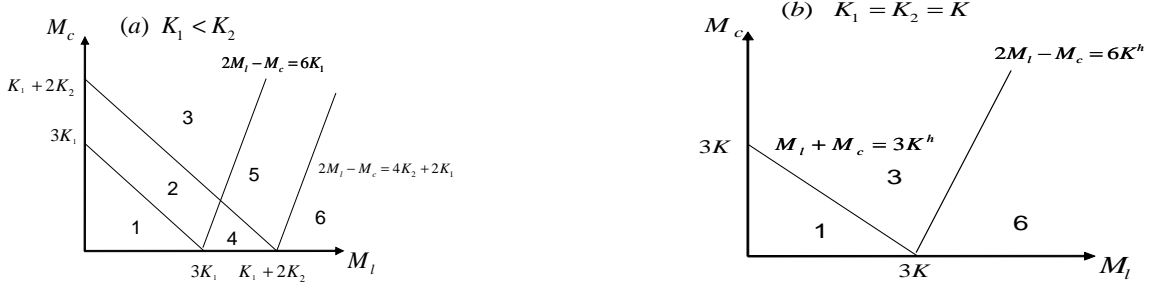$3K$ $M_l + M_c = 3K^h$

3

1

6

$3K$ M_l

Figure 4: Hub-and-Spoke Networks

We now illustrate the procedure to calculate equilibrium quantities for some areas and relegate the details to Appendix A. In area 3, since both airlines are capacity-constrained, $u_i > 0$ and $v_{yi} = 0$ $(i = 1, 2; y = c, l)$. Solving (5) and its analogues for airline two yields

$$q_{li} = (2M_l - M_c + 3K_i)/9 \quad q_{ci} = (M_c - 2M_l + 6K_i)/9$$

$$u_i = 2(M_c + M_l - 2K_i - K_j)/3 > 0 \quad (i \neq j; i, j = 1, 2)$$

Similarly, in area 4, airline two is not capacity-constrained, so $u_2 = v_{c2} = v_{l2} = 0$; airline one serves only the local markets.

$$v_{c1} = M_l - M_c/2 - 3K_1 > 0 \quad q_{c1} = 0 \quad q_{c2} = M_c/2 \quad q_{l1} = K_1 \quad q_{l2} = (M_l - K_1)/2$$

The other areas can be analyzed similarly, and the details are in Appendix A. In summary, the areas can be defined as follows:

$$\Delta_1 = \{(x_l, x_c) : x_l + x_c \leq 3K_1\}$$

$$\Delta_2 = \{(x_l, x_c) : x_l + x_c \geq 3K_1, x_l + x_c \leq K_1 + 2K_2, 2x_l - x_c \leq 6K_1\}$$

$$\Delta_3 = \{(x_l, x_c) : x_l + x_c \geq K_1 + 2K_2, 2x_l - x_c \leq 6K_1\}$$

$$\Delta_4 = \{(x_l, x_c) : 2x_l - x_c \geq 6K_1, x_l + x_c \geq K_1 + 2K_2\}$$

$$\Delta_5 = \{(x_l, x_c) : 2x_l - x_c \geq 6K_1, 2x_l - x_c \leq 2K_1 + 4K_2, x_l + x_c \geq K_1 + 2K_2\}$$

$$\Delta_6 = \{(x_l, x_c) : 2x_l - x_c \geq 2K_1 + 4K_2\}$$

Notice that (x_l, x_c) are restricted to the Northeastern quadrant. When the airlines have identical capacities, areas 2, 4, and 5 disappear as shown in Figure 4(b). Note that $E\pi_i$ in

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(3) or (4) is the sum of the product of the profit in each area and the probability of that area.

Having solved the quantity game, we now characterize an equilibrium point of the capacity game. The proof of the next result is in Appendix A.

Proposition 1. *When both airlines employ hub-and-spoke networks, the capacity game has a pure strategy equilibrium point. Furthermore, if $K_1 = K_2 = K$, the equilibrium point is unique, and K satisfies*

$$\frac{1}{3} \iint_{\Delta_3} (x_l + x_c - 3K) dF_l dF_c + 2 \iint_{\Delta_6} (x_l - 3K) dF_l dF_c = c_h \tag{6}$$

3.3 Point-to-Point Networks

This section studies the quantity and capacity games when both airlines use point-to-point networks. We assume that one airline has larger capacities than the other in all routes. There are other cases (e.g., one airline has larger capacity in one market but smaller capacity in the other market than the other airline). However, as we focus on the symmetric case in the succeeding sections, this assumption is only for expository convenience. Label the airlines so $K_{y2} \geq K_{y1}$ ($y = l, c$). Figure 5(a) shows nine areas that are defined by the binding conditions of the capacities.

The quantity game can be solved for each area as illustrated in §3.2. The details are in Appendix B. In area 1, neither airline is capacity-constrained; in area 2, airline one is capacity-constrained in the connecting market; in area 3, airline one is capacity-constrained in all markets; in area 4, airline one is capacity-constrained in the local markets; in area 5, both airlines are capacity-constrained in the local markets; in area 6, airline one is capacity-constrained in all markets, and airline two is capacity-constrained in the local markets; in area 7, both airlines are capacity-constrained in the connecting market; in area 8, airline one is capacity-constrained in all markets, and airline two is capacity-constrained in the connecting market; in area 9, both airlines are capacity-constrained in all markets. For the same reason as in the proof of Proposition 1, the capacity game has a pure strategy equilibrium point. The proof of the following result is relegated to Appendix B.

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Figure 5: Point-to-Point Networks

Proposition 2. *When both airlines adopt point-to-point networks, the capacity game has a pure strategy equilibrium point. If $K_{y1} = K_{y2} = K_y$ $(y = l, c)$, the equilibrium point is unique and K_c and K_l satisfy*

$$\iint_{\Delta_{7,9}} (x_c - 3K_c)dF_l dF_c = c_p \tag{7}$$

$$\iint_{\Delta_{5,9}} (x_l - 3K_l)dF_l dF_c = c_p \tag{8}$$

3.4 Different Network Structures

Let airline one employ a hub-and-spoke network and airline two a point-to-point network. Figure $6(a)$ represents the case in which $K_1 > K_{y2}(y = l, c)$. Note that there are other cases (e.g., $K_{l2} > K_1 > K_{c2}$), so the capacity game might have multiple equilibria. Figure $6(a)$ shows nine areas that are defined by the binding conditions of the capacities. The quantity game for each area can be solved explicitly. The details are in Appendix C. In area 1, neither airline is capacity-constrained; in area 2, airline two is capacity-constrained in the local markets; in area 3, airline two is capacity-constrained in the connecting market; in area 4, airline one is capacity-constrained; in area 5, airline one is capacity-constrained, and airline two is capacity-constrained in the local markets; in area 6, airline one is capacity-constrained, and airline two is capacity-constrained in the connecting market; in area 7, airline one is capacity-constrained and serves only the local markets where airline two is capacity-constrained; in area 8, airline one is capacity-constrained and serves only the local markets and airline two is capacity-constrained in all markets; in area 9, both airlines are
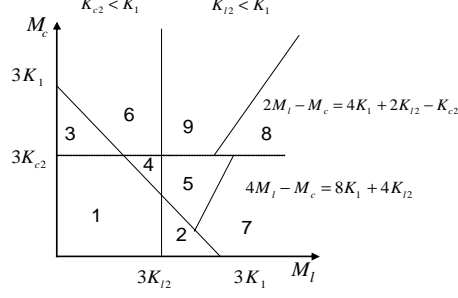
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Figure 6: Different Network Structures

capacity-constrained in all markets.

A proof (attached in Appendix C) that is similar to those of Propositions 1 and 2 establishes the following result.

Proposition 3. *When the airlines use different network structures, the capacity game has a pure strategy equilibrium point which satisfies*

$$c_h = \frac{1}{3} \iint_{\Delta_4} (x_l + x_c - 3K_1)dF_l dF_c + \frac{2}{11} \iint_{\Delta_5} (2x_c + 3x_l - 6K_1 - 3K_{l2})dF_l dF_c \quad (9)$$

$$+ \frac{1}{5} \iint_{\Delta_6} (3x_c + 2x_l - 6K_1 - 3K_{l2})dF_l dF_c + 2 \iint_{\Delta_{7,8}} (x_l - 2K_1 - K_{l2})dF_l dF_C$$

$$+ \frac{2}{3} \iint_{\Delta_9} (x_c + x_l - 2K_1 - K_{c2} - K_{l2})dF_l dF_c$$

$$c_p = \iint_{\Delta_2} (x_l - 3K_{l2})F_l dF_c + \frac{2}{11} \iint_{\Delta_5} (x_c + 7x_l - 3K_1 - 18K_{l2})dF dG \quad (10)$$

$$+ 2 \iint_{\Delta_{7,8}} ((x_l - 2K_{l2} - K_1)dF_l dF_c + \frac{1}{3} \iint_{\Delta_9} (4x_l + x_c - 2K_1 - K_{c2} - 10K_{l2})dF_l dF_c$$

$$c_p = \frac{1}{5} \iint_{\Delta_6} (4x_c + x_l - 3K_1 - 9K_{c2})dF_l dF_c + \iint_{\Delta_8} (x_c - 2K_{c2})dF_l dF_c$$

$$+ \frac{1}{6} \iint_{\Delta_9} (5x_c + 2x_l - 4K_1 - 11K_{c2} - 2K_{l2})dF_l dF_c \quad (11)$$

The equations in Propositions 1-3 provide little information about the subgame perfect equilibrium points. In order to investigate the network structure game further, we make two important assumptions in §4.

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4.   Network Structure Game

Having analyzed the quantity and the capacity games, we address the two questions raised in the introduction: what factors affect airlines' network choices? What are the airlines' network structures at equilibrium? §4.1 states two additional assumptions. Following the logic of §3, §4.2 derives equilibrium capacities, prices, and profits for the three cases: both airlines use hub-and-spoke networks, both use point-to-point networks, and they use different network structures. §4.3 compares the cases. §4.4 investigates a monopolist's network choice and a duopoly network game when the two network structures have the same unit capacity costs. Then §4.5 addresses the same questions when the unit capacity costs are different.

Henceforth, we assume that the airlines are identical, so the airline identity i is suppressed.

4.1   Assumptions

As shown in §3, the quantity and capacity games have a subgame perfect equilibrium point. However, the equilibrium capacities lack closed-forms. In order to analyze the network game, we have to compare the profits. So we make the following assumptions.

Assumption 1. *Both the local and connecting markets are always served by the airlines.*

This assumption excludes the cases in which the local markets are so much larger than the connecting market that serving the connecting market is uneconomical. Specifically, when both airlines use hub-and-spoke networks, areas 4, 5, and 6 disappear in Figure 4(a), and areas 7 and 8 disappear in Figure 6.

Assumption 2. *If the airlines use a hub-and-spoke networks, $q_{li} + q_{ci} = K_i$; if they use point-to-point networks, $q_{yi} = K_{yi}$ $(y = l, c; i = 1, 2)$.*

Reflecting characteristics of the U.S. airline industry, this assumption forces the airlines to deplete their capacities regardless of market realization. Because most expenses for a flight occur at departure and landing, and the marginal cost of an passenger is minuscule (Barla (1999)), it is beneficial for airlines to sell up to the maximum number of seats available per

flight even at a sub-optimal price. For example, because demands decreased dramatically after September 11, U.S. airlines had to cut prices to fill more seats in the flights.

Assumptions 1 and 2 eliminate the complexity caused by different demand realization and binding conditions of the capacities. Furthermore, the quantity and capacity games are equivalent.

4.2 Equilibrium Capacities and Prices

Let superscripts h, p, and m represent, respectively: both airlines use hub-and-spoke networks, both airlines use point-to-point networks, and one airline uses a hub-and-spoke network and the other a point-to-point network. The proof of the following result is relegated to Appendix A.

Proposition 4. *When both airlines employ hub-and-spoke networks, the capacity game has a subgame perfect equilibrium point at which*
(a) the optimal capacity is $K^h = (\mu_l + \mu_c)/3 - c_h/2$,
(b) the expected prices are $p_c^h = (\mu_c + 2c_h)/3$ and $p_l^h = (\mu_l + c_h)/3$;
(c) the expected profit is

$$\Pi^h = (4\sigma_l^2 + \sigma_c^2)/27 + (2\mu_l^2 + \mu_c^2)/9 - 5c_h(\mu_l + \mu_c)/9 + 2c_h^2/3 \tag{12}$$

It can be verified that $K^h \geq 0$ and $\Pi^h \geq 0$ because $c_h \leq \mu_l$ and $c_h \leq \mu_c$ by assumption.

For the remainder of the paper, we use "increase" and "decrease" for "nondecreasing" and "nonincreasing", respectively. From Proposition 4, it can be derived that the optimal capacity increases as mean demands increase or the unit capacity cost decreases and that the expected prices increase as the respective mean demand or the unit capacity cost increases. Moreover, (12) indicates that the airlines' expected profits increase if demand variances increase, but they are not monotone in c_h, μ_l, or μ_c.

We now study the case in which both airlines adopt point-to-point networks. The proof of the following results is in Appendix B.

Proposition 5. *When both airlines employ point-to-point networks, the capacity game has a subgame perfect equilibrium point at which*

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(a) the equilibrium capacities are $K_y^p = (\mu_y - c_p)/3$ $(y = l, c)$;

(b) the expected prices are $p_y^p = (\mu_y + 2c_p)/3$;

(c) the expected profit is

$$\Pi^p = (2\mu_l^2 + \mu_c^2)/9 - 2c_p(2\mu_l + \mu_c)/9 + c_p^2/3 \tag{13}$$

It can be verified similarly that $K_y^p \geq 0$ and $\Pi^p \geq 0$ since $c_p \leq \mu_l$ and $c_p \leq \mu_c$ by assumption. From Proposition 5, it can be verified that the monotonicity of capacities and prices with respective to μ_y or c_h are the same as that from Proposition 4. However, demand variances no longer affect the expected profit. Furthermore, the expected profit increases as either demand mean increases, or as the unit capacity cost decreases, whereas this monotonicity does not exist when both airlines use hub-and-spoke networks.

Finally, we study the case in which the airlines employ different network structures. Let subscripts h and p represent the airline's network structures, and superscript m represent the case. The results are summarized as follows, and its proof is attached in Appendix C.

Proposition 6. *When the airlines employ different network structures, the capacity game has a subgame perfect equilibrium point at which*

(a) the optimal capacities are $K_h^m = (2\mu_c + 2\mu_l + 3c_p - 6c_h)/6$ and $K_c^m = (\mu_c + c_h - 2c_p)/3$ and $K_l^m = (2\mu_l + c_h - 2c_p)/6$;

(b) the expected prices are $p_l^m = (2\mu_l + c_p + c_h)/6$ and $p_c^m = [(\mu_c + c_h + c_p)/3$;

(c) the expected profits are

$$\Pi_h^m = (4\sigma_l^2 + \sigma_c^2)/18 + [(2\mu_l^2 + \mu_c^2) - (\mu_l + \mu_c)(7c_h - 2c_p)]/9 \tag{14}$$
$$+ (10c_h^2 + c_p^2 - 7c_p c_h)/6$$

$$\Pi_p^m = (2\mu_l^2 + \mu_c^2)/9 + \mu_l(2c_h - 7c_p)/9 + 2\mu_c(c_h - 2c_p)/9 - 5c_h c_p/6 + c_h^2/6 + c_p^2 \tag{15}$$

From Proposition 6(a), it is straightforward to derive that the hubbing airline's capacity increases as either mean market increases, and the equilibrium capacities for the local and connecting markets of the airline with a point-to-point network increase as the corresponding mean market increases. Furthermore, if hubbing becomes more expensive, the airline with a point-to-point network should raise its capacities, while the other airline should reduce its

capacity and vice versa. The expected prices increase as the corresponding mean demands increase, or as either unit capacity cost increases. Demand variances positively affect the hubbing airline's profit but has no impacts on the other airline's profit. More interestingly, comparing (14) and (12), one observe that demand variances have larger impacts on the hubbing airline's expected profit in this scenario than when both airlines adopt hub-and-spoke networks because the flexibility value is shared by the airlines in the latter case.

Notice the wording in Propositions 4 to 6, "equilibrium capacity" and "expected prices". Because capacities are determined before demands are known, so they are deterministic. However, prices are determined after uncertainty is resolved, so actual prices depend on demand realization. Therefore, "expected" is used to restrict "price".

Table 1 summarizes each airline's equilibrium capacities in the local and connecting markets at a subgame equilibrium point. Since the airlines deplete their capacities, the capacity

	p	h	m
Local	$\frac{2}{3}(\mu_l - c_p)$	$\frac{1}{3}(2\mu_l - c_h)$	$\frac{2}{3}\mu_l - \frac{1}{6}(c_p + c_h)$
Connecting	$\frac{2}{3}(\mu_c - c_p)$	$\frac{2}{3}(\mu_c - c_h)$	$\frac{2}{3}\mu_c - \frac{1}{3}(c_p + c_h)$

Table 1: Equilibrium Capacities

decision and pricing decision are equivalent, and the orderings of prices and capacities are opposite. The following results are immediate from Table 2 or Propositions 4 to 6.

Corollary 1. If $c_h > c_p$, then $p_c^h > p_c^m > p_c^p$. If $c_h > 3c_p$, then $p_l^h > p_l^m > p_l^p$. If $2c_p > c_h > c_p$, then $p_l^p > p_l^h > p_l^m$. If $3c_p > c_h > 2c_p$, then $p_l^m > p_l^p > p_l^h$.

Corollary 1 claims that if a hub-and-spoke network is more expensive than a point-to-point network, scenario h provides the lowest capacity thus the highest price in the connecting market. However, if the unit capacity cost is the same regardless of the network structures, then all three cases offer the same price and capacity for the connecting market. In reality, low-cost carriers' cost per mile seat could be 40% lower than that of a major hub-and-spoke airline (Borenstein (1992)), so $2c_p > c_h > c_p$ probably best reflects the reality. Hence, from Corollary 1 local passengers benefit most from the airlines' network differentiation.

Furthermore, if $\mu_l = \mu_c$, the total capacities of the local markets is larger than that of the connecting market as long as at least one airline uses a hub-and-spoke network. Consequently, local flights are cheaper than connecting flights.

4.3 Network Choice with Uniform Capacity Cost

This subsection studies the airlines' equilibrium network structures when $c_p = c_h = c$. This assumption is relaxed in §4.4.

4.3.1 Monopoly

As conjectured in §1, if fixed investment costs are ignored, a hub-and-spoke network might seem to dominate a point-to-point network for a monopolist because it possesses a flexibility value and economies of density. However, the following result shows that a monopolist might favor a point-to-point network over a hub-and-spoke network.

Let Π_p^M and Π_h^M be a monopolistic airline's expected profit when it uses a point-to-point network and a hub-and-spoke network, respectively. Let $\Delta^M = \Pi_h^M - \Pi_p^M$. It can be verified that optimal capacities are $K_y = (\mu_y - c_p)/2$ $(y = l, c)$ if the monopolist uses a point-to-point network, and $K^h = (\mu_c + \mu_l - 3c_h)/2$ if it uses a hub-and-spoke network. So

$$\Pi_p^M = (2\mu_c^2 + \mu_l^2 - 4\mu_l c_p - 2\mu_c c_p + 3c_p^2)/4$$

$$\Pi_h^M = [(\sigma_c^2 + 2\sigma_l^2) + (\mu_c - 2c_h)^2 + 2(\mu_l - c_h)^2]/4$$

$$\Delta^M = (\sigma_c^2 + 2\sigma_l^2 - \mu_c^2 + \mu_l^2 + 6c_h^2 - 3c_p^2 - 4\mu_c c_h + 2\mu_c c_p - 4\mu_l c_h + 4\mu_l c_p)/4 \qquad (16)$$

For expository simplicity, let $\sigma_l = \sigma_c = \sigma$ and $\mu_l = \mu_c = \mu$, then $\Delta^M = (3c^2 - 2\mu c + 3\sigma^2)/4$. Let $\underline{c}$ and $\bar{c}$ are the respective smaller and larger roots of c by setting Δ^M to zero. Then $\bar{c} = (\mu + \sqrt{\mu^2 - 9\sigma^2})/3$ and $\underline{c} = (\mu - \sqrt{\mu^2 - 9\sigma^2})/3$. So if $\sigma/\mu \geq 3$, $\Delta^M > 0$; otherwise, if $c \in (\underline{c}, \bar{c})$, then $\Delta^M < 0$, and if $c \in (0, \underline{c}) \cup (\bar{c}, \mu)$, then $\Delta^M > 0$. The results are summarized as follows.

Proposition 7. *(a) A monopolist uses a hub-and-spoke network if $\sigma/\mu \geq 3$. Otherwise, it uses a hub-and-spoke network if $c \in (\bar{c}, \mu) \cup (0, \underline{c})$ and a point-to-point network if $c \in (\underline{c}, \bar{c})$; (b) If μ_c increases, or if c_p, σ_c , or σ_l decreases, Δ^M decreases.*

Proposition 7 reveals that only if the demand variation coefficient is larger than 3, a monopolist favors a hub-and-spoke network. Otherwise, it might favor a point-to-point network depending on the parameters. For an extreme case, if $\sigma = 0$, then $\Delta^M = (3c^2 - 2\mu c)/4$. So if $\mu/c < 3/2$, the airline should use a hub-and-spoke network and otherwise a point-to-point network. Although a hub-and-spoke network has economies of density, it requires one seat in each leg to transport a connecting passenger, so a monopolist does not necessarily favor a hub-and-spoke network. When demand/cost ratio is greater than 3/2, this capacity-costly disadvantage dominates economies of density. So a monopolist should use a point-to-point network.

In addition, Proposition 7 implies that under the same market condition, a low-cost airline is more likely to opt for a point-to-point network than a high-cost airline. This coincides with the fact that all the prosperous U.S. direct flight airlines are low-cost. In general a high demand variance or a low demand/ cost ratio favors a hub-and-spoke network.

4.3.2 Best Response

This subsection examines an airline's best response given its rival's network structure. We first analyze the case in which the rival airline employs a hub-and-spoke network. Let Δ^H denote the profit gap of responding with a hub-and-spoke network versus a point-to-point network. From (12) and (15),

$$\Delta^H = \Pi^h - \Pi_p^m = (\sigma_c^2 + 4\sigma_l^2)/27 + (c^2 - \mu_c c)/3 \tag{17}$$

Let $\bar{c}_1 = \{3\mu_c + [9\mu_c^2 - 4(\sigma_c^2 + 4\sigma_l^2)]^{1/2}\}/6$ and $\underline{c}_1 = \{3\mu_c - [9\mu_c^2 - 4(\sigma_c^2 + 4\sigma_l^2)]^{1/2}\}/6$. Following the reasoning as for Proposition 7, if (18) holds, Δ^H in (17) is nonnegative. So the best response is a hub-and-spoke network; otherwise, the sign of Δ^H depends on the value of c: if $c \in (\underline{c}_1, \bar{c}_1)$, $\Delta^H > 0$; if $c \in (\underline{c}_1, c) \cup (\bar{c}_1, \mu)$, $\Delta^H < 0$. The consequence results are summarized as follows.

Proposition 8. *When its competitor employs a hub-and-spoke network, an airline's best response is a hub-and-spoke network if*

$$\frac{\sqrt{\sigma_c^2 + 4\sigma_l^2}}{\mu_c} \geq 3/2 \tag{18}$$

Otherwise, the best response is a hub-and-spoke network if $c \in (0, \underline{c_1}) \cup (\bar{c}_1, \mu_c)$ and a point-to-point network if $c \in (\underline{c_1}, \bar{c}_1)$.

From (17), it can be derived that as μ_c decreases, or as σ_l or σ_c increases, Δ^H increases. A hub-and-spoke response enables the airline to share with the incumbent airline, a flexibility value, $(4\sigma_l^2 + \sigma_c^2)/27$. In contrast, a point-to-point response provides the airline a committed position in the connecting market whose value depends on the size of the mean of the connecting market. The airline chooses its network structure by weighing the flexibility value and the committed value. Specifically, when (18) holds, the flexibility value is larger, so it responds with a hub-and-spoke network. Otherwise, an intermediate capacity cost favors a point-to-point response, while a low or high capacity cost favors a hub-and-spoke response.

If $\sigma_c = \sigma_l = \sigma$, then $c > \bar{c}_1$ and $c < \underline{c_1}$, respectively, correspond to

$$\mu_c/c - \sqrt{(\mu_c/c)^2 - 20/9(\sigma/c)^2} \;\; > \;\; 2 \tag{19}$$

$$\mu_c/c + \sqrt{(\mu_c/c)^2 - 20/9(\sigma/c)^2} \;\; < \;\; 2 \tag{20}$$

Inequalities (19) and (20) imply that if σ/c is higher than μ_c/c by a certain increment, the best response is a hub-and-spoke network and otherwise a point-to-point network. So the airline determines its network structure through weighing μ_c/c versus σ/c, or roughly speaking, the commitment value and the flexibility value. One extreme case is $\sigma_c = \sigma_l = 0$. Then $\Delta^H = (c^2 - \mu_c c)/3 \leq 0$ because $\mu_c \geq c$ by assumption. So an airline should respond with a point-to-point network without uncertainty. The flexibility value diminishes without uncertainty, and economies of density cancel out if the airline responds with a hub-and-spoke network. Therefore, by responding with a point-to-point network, the airline improves its profit. In addition, if $\mu_c/c < (>)2$, Δ^H increases (decreases) as c increases because a hub-and-spoke's economies of density dominate its cost disadvantage when demand/cost ratio is

low. In summary, the demand/cost ratio affects the airline's network choice similarly in a duopoly case as in a monopoly case.

We now consider the airline's best response when its competitor uses a point-to-point network. Let Δ^P be the profit gap of responding with a hub-and-spoke network versus a point-to-point network. From (13) and (14),

$$\Delta^P = \Pi_h^m - \Pi^p = c^2/3 - c(\mu_l + 3\mu_c)/9 + (\sigma_c^2 + 4\sigma_l^2)/18 \tag{21}$$

Let $\underline{c_2} = \{\mu_l + 3\mu_c - [(\mu_l + 3\mu_c)^2 - 6(4\sigma_l^2 + \sigma_c^2)]^{1/2}\}/6$ and $\bar{c}_2 = \{\mu_l + 3\mu_c + [(\mu_l + 3\mu_c)^2 - 6(\sigma_c^2 + 4\sigma_l^2)]^{1/2}\}/6$. Similarly, following the same reasoning as for Propositions 7 and 8, the results below are consequences of (21).

Proposition 9. *When its competitor employs a point-to-point network, an airline's best response is a hub-and-spoke network if*

$$\frac{\sqrt{\sigma_c^2 + 4\sigma_l^2}}{\mu_l + 3\mu_c} > \sqrt{6}/6 \tag{22}$$

Otherwise, the best response is a hub-and-spoke network if $c \in (0, \underline{c_2}) \cup (\bar{c}_2, \mu_l)$ and a point-to-point network if $c \in (\underline{c_2}, \bar{c}_2)$.

From (21), Δ^P increases (decreases) as σ_c or σ_l (μ_l or μ_c) increases. In addition, if $(\mu_l + 3\mu_c)/c \leq (>)3$, Δ^P increases (decreases) as c increases. Comparing (22) to (18), one observes that both μ_l and μ_c now affect the airline's response when the incumbent uses a point-to-point network. Moreover, (22) indicates that μ_c weights more than μ_l in determining the airline's response network.

Comparing Propositions 7 to 9, one observes that with or without competition, in general, a high demand variation coefficient or an extremely high or low unit capacity cost favors a hub-and-spoke network, but an intermediate capacity cost favors a point-to-point network.

4.3.3 Equilibrium Network Structures

Before analyzing the network game, we first compare the airline's expected profits in the three cases. From Propositions 4 to 6,

$$\Pi^h - \Pi_h^m = -(\sigma_c^2 + 4\sigma_l^2)/54 \leq 0 \qquad \Pi^p - \Pi_p^m = c\mu_l/9 > 0$$

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So the following results are immediate.

Proposition 10. $\Pi^p > \Pi_p^m$; $\Pi_h^m \geq \Pi^h$.

Proposition 10 reveals that a hubbing airline prefers its rival to use a point-to-point network, while an airline with a point-to-point network prefers its rival to use the same kind of network. These results are intuitive. When the airlines use different network structures, the hubbing airline enjoys not only the flexibility value but also the economies of density thus forcing the other airline to cut capacities. Consequently, the hubbing airline benefits if its rival uses a point-to-point network. For the same reason, if both airlines use point-to-point networks initially, as soon as one of them switches to a hub-and-spoke network, the other suffers. The ordering of the other pairs of expected profits depends on the parameters and demand characteristics.

We now analyze the network game. Let H and P denote a hub-and-spoke network and a point-to-point network, respectively, and let NEP stand for pure strategy Nash equilibrium point. The following result concerns the case in which $\sigma_l = \sigma_c = \sigma$ and $\mu_l = \mu_c = \mu$, and its proof is in Appendix D.

Proposition 11. *The network game is characterized as follows:*
(a) if $\sigma/\mu > 2\sqrt{30}/15$, (H, H) is the NEP,
(b) if $3\sqrt{5}/10 < \sigma/\mu < 2\sqrt{30}/15$, and
    (i) if $c \in (0, \underline{c_2}) \cup (\bar{c}_2, \mu)$, then (H, H) is the NEP,
    (ii) if $c \in (\underline{c_2}, \bar{c}_2)$, then (H, H) and (P, P) are the NEPs;
(c) if $\sigma/\mu < 3\sqrt{5}/10$, then
    (i) if $c \in (0, \underline{c_2}) \cup (\bar{c}_2, \mu)$, then (H, H) is the NEP,
    (ii) if $c \in (\underline{c_1}, \bar{c}_1)$, then (P, P) is the NEP,
    (iii) if $c \in (\underline{c_2}, \underline{c_1}) \cup (\bar{c}_1, \bar{c}_2)$, (H, H) and (P, P) are the NEPs.

Proposition 11 concludes that the airlines adopt the same kind of networks at equilibrium. Specifically, for a high demand variation coefficient, both airlines choose hub-and-spoke networks; for an intermediate or low demand variation coefficient, two cases are possible:

either both airlines choose hub-and-spoke networks, or both airlines choose hub-and-spoke networks. Note that in reality, airlines' networks are much more complicated, so the symmetric equilibrium conclusion of this ideal three-city model does not exclude the coexistence of hub-and-spoke and point-to-point networks in reality.

4.4 Network Choice with Different Unit Capacity Costs

A hub-and-spoke network incurs extra cost such as gaining landing slots and gates, handling passengers' packages in the hub, and coordinating flights. In addition, it contributes to airport congestion and lowers aircraft usage because aircrafts arrive and depart at about the same time for connecting passengers to take connecting flights. Moreover, in reality, low-cost carriers usually have a lower cost per mile seat than major airlines. So we assume $c_p < c_h$.

4.4.1 Monopoly

The following results can be derived by computing the partial derivatives of Δ^M with respect to the corresponding parameters from (16).

Proposition 12. *(a) As σ_l, σ_c, or μ_c decreases, or as c_p increases, Δ^M increases; (b) If $(\mu_l + \mu_c)/c_h \geq (<)3$, Δ^M decreases (increases) as c_h increases.*

Comparing Propositions 12 and 7, one observes that the essence of the results do not change when $c_p < c_h$: larger demand variances favor a hub-and-spoke network, whereas larger market means favor a point-to-point network; if $(\mu_l + \mu_c)/c_h < 3$, as c_h increases, the monopolist should tend to use a hub-and-spoke network because of its economies of density; otherwise, economies of density are dominated by the capacity-costly factor, so a monopolist should instead tend to choose a point-to-point network. In reality, there is an excess capacity in airline industry, namely, $(\mu_l + \mu_c)/c_h$ is relatively small. Thus, loosely speaking, airlines with dominant positions in their hubs should favor hub-and-spoke networks. This explains in part why major airlines, whose costs are high relative to low-cost carriers such as Southwest, favor hub-and-spoke networks.

In conclusion, the essence of the results does not change compared to the case with

uniform capacity costs, so we here do not derive specific criterion for a monopolist's network selection.

4.4.2 Best Response

We now study an airline's best response to a hubbing competitor. From (12) and (15),

$$\Delta^H = \Pi^h - \Pi_p^m = (4\sigma_l^2 + \sigma_c^2)/27 - 7\mu_l(c_h - c_p)/9 - \mu_c(7c_h - 4c_p)/9 + c_h^2/2 + 5c_h c_p/6 - c_p^2 \quad (23)$$

The following results follow by computing the partial derivatives of Δ^H with respect to the corresponding parameters.

Proposition 13. *(a) As σ_l, σ_c, or c_p increases, or as μ_c or μ_l decreases, Δ^H increases; (b) Δ^H is submodular in c_h and c_p.*

Proposition 13 claims that when there is a competing hubbing airline, the larger the demand variances, or the smaller the mean demands, the less an airline should tend to respond with a hub-and-spoke network. This monotonicity is not surprising because a hub-and-spoke network's flexibility value increases when demand variances increase, and because economies of density are larger when mean demand is smaller. The submodularity of Δ^H means that the marginal increasing rate of Δ^H in c_p decreases as c_h decreases.

We now consider the airline's best response when its rival employs a point-to-point network. From (13) and (14),

$$\Delta^P = \Pi_h^m - \Pi^p = 5(4\sigma_l^2 + \sigma_c^2)/18 - [7c_h(\mu_l + \mu_c) - 2c_p(3\mu_l + 2\mu_c)]/9 + (10c_h^2 - c_p^2 - 7c_h c_p)/6 \quad (24)$$

The following results are obtained by deriving partial derivatives of Δ^P with respect to the corresponding parameters.

Proposition 14. *(a) As σ_l or σ_c increases, or as μ_l or μ_c decreases, Δ^P increases; (b) Δ^P is submodular in c_h and c_p.*

Comparing (17) and (21) with (23) and (24), respectively, one observe that changing $c_h = c_p$ to $c_h > c_p$ does not alter the qualitative results. So we here do not derive specific

conditions that determine the airlines' network structures at equilibrium. Nevertheless, the submodularity of Δ^p or Δ^H in c_h and c_p captures the impacts of unit capacity costs on the airlines' network selection. The submodularity of Δ^P means that the marginal decreasing rate of Δ^P in c_p increases as c_h decreases. Therefore, the lower its competing low-carrier's cost, the greater a hubbing airline's profit deteriorates if it does not follow suit and switch to a point-to-point network as well. So this partly explains major airlines' response strategy, creating airlines with airlines to match low-cost carriers' business model.

5. Conclusion

This paper examines how airlines might determine their network structures through a three-stage duopoly game in which two airlines serve a three-city network with uncertain demand. We show that at equilibrium the airlines employ the same kind of networks. The monopoly case is also investigated to exclude competition's impacts on the airline's network selection. Even if fixed investment costs are ignored, we show that a hub-and-spoke network does not necessarily dominate a point-to-point network. A larger demand variation coefficient or an extreme high or low unit capacity cost generally favors hub-and-spoke networks whether there is competition or not.

This paper sheds some lights into how demand, cost, and competition affect the airlines network selection and capacity investment and why low-cost carriers exemplified by Southwest prospers. In reality, networks are much more complex, airlines use multiple types of aircrafts, passengers are sensitive to flight quality such as number of stops and flight frequency. Future research might consider these factors.

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Appendix A. Hub-and-Spoke Networks

Airline one's problem at the third stage is

$$\pi_1 = \max_{q_{l1},q_{c1}} 2[M_l - (q_{l1} + q_{l2})]q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1} \tag{25}$$

$$s.t. \quad q_{l1} + q_{c1} \leq K_1 \quad 0 \leq q_{l1} \quad 0 \leq q_{c1}$$

This is a concave nonlinear maximization problem which Karush-Kukh-Tucker (KKT) condition applies. Airline one's KKT conditions are

$$
\begin{aligned}
2(M_l - 2q_{l1} - q_{l2}) - u_1 + v_{l1} &= 0 \\
v_{l1}q_{l1} &= 0 \\
M_c - 2q_{c1} - q_{c2} - u_1 + v_{c1} &= 0 \\
v_{c1}q_{c1} &= 0 \\
u_1(q_{c1} + q_{l1} - K_1) &= 0 \\
q_{c1} + q_{l1} &\leq K_1 \\
u_1, v_{l1}, v_{c1}, q_{c1}, q_{l1} &\geq 0
\end{aligned}
\tag{26}
$$

where v_{c1} and v_{l1} are slack variables, and u_1 is a Lagrange multiplier. There are six areas depending on the binding conditions of the capacities. The values of the slack variables or multipliers are specified only when they are positive because they define the various boundaries of the areas in Figure 4. The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and the slack variables from (26) and the analogues of airline two.

Neither airline is capacity-constrained (Δ_1)

$$q_{y1} = q_{y2} = A_y/3 \quad y = l, c$$

Airline one is capacity-constrained; airline two is not (Δ_2)

$$q_{li} = (2M_l - M_c - 3K_i)/9(i = 1, 2) \quad q_{ci} = (4M_c + M_l - 3K_i)/9 \quad u_1 = (M_l + M_c)/3 - K_1 > 0$$

Both airlines are capacity-constrained (Δ_3)

$$q_{li} = (2M_l - M_c + 3K_i)/9 \qquad q_{ci} = (M_c - 2M_l + 6K_j)/9$$

$$\mu_i = 2(M_c + M_l - 2K_i - K_j)/3 > 0 \quad (i \neq j; i, j = 1, 2)$$

Airline one is capacity-constrained and serves only the local markets (Δ_4)

$$q_{c1} = 0 \quad q_{c2} = M_c/2 \quad q_{l1} = K_1 \quad q_{l2} = (M_l - K_1)/2$$

$$u_1 = M_l - 3K_1 > 0 \quad v_{c1} = M_l - M_c/2 - 3K_1 > 0$$

Both airlines are capacity-constrained; airline one serves only the local markets (Δ_5)

$$q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{l2} = (2M_l - M_c - 2K_1 + 2K_2)/6$$

$$q_{c2} = (M_c - 2M_l + 2K_1 + 4K_2)/6 \quad v_{c1} = (2M_l - M_c - 6K_1)/3 > 0$$

$$u_1 = (M_c + 4M_l - 10K_1 - 2K_2)/3 > 0 \quad u_2 = 2(M_c + M_l - K_1 - 2K_2)/3 > 0$$

Both airlines are capacity-constrained and serve only the local markets (Δ_6)

$$q_{li} = K_i \quad q_{ci} = 0 \quad u_i = 2(M_l - 2K_i - K_j) > 0$$

$$v_{ci} = 2M_l - M_c - 4K_i - 2K_j > 0 \quad (i, j = 1, 2; i \neq j)$$

Proof of Proposition 1.

Proof. It can be verified that the airlines' payoff functions Π_i $(i = 1, 2)$ are concave in their own decision variable regardless of their rival's decision. It follows from Debreu (1952) that the capacity game has a pure strategy NEP. From (3), the first-order condition (FOC) of Π_i with respect to K_i in the capacity game is

$$E\frac{\partial \pi_i}{\partial K_i} = 2c_h \tag{27}$$

Using Leibnitz's rule to derive $\partial \pi_i / \partial K_i$ for each area in Figure 4, for airline one, (27) becomes

$$\frac{1}{3} \iint_{\Delta_2} (x_l + x_c - 3K_1)dF_l dF_c + \frac{2}{3} \iint_{\Delta_3} (x_l + x_c - 2K_1 - K_2)dF_l dF_c$$
$$+ \iint_{\Delta_4} (x_l - 3K_1)dF_l dF_c + \frac{1}{3} \iint_{\Delta_5} (x_c + 4x_l - 10K_1 - 2K_2)dF_l dF_c \tag{28}$$
$$+ 2 \iint_{\Delta_6} (x_l - 2K_1 - K_2)dF_l dF_c = c_h$$

for airline two, (27) becomes

$$\frac{2}{3} \iint_{\Delta_{3,5}} (x_l + x_c - K_1 - 2K_2)dF_l dF_c + 2 \iint_{\Delta_6} (x_l - K_1 - 2K_2)dF_l dF_c = c_h \tag{29}$$

When the airlines have the same capacities, areas 4, 5, and 6 disappear, and (28) and (29) can be simplified to (6).

The following condition is sufficient for uniqueness via contraction (Vives (1999)).

$$|\partial^2 \Pi_j / \partial K_i \partial K_j| < |\partial^2 \Pi_j / \partial (K_i)^2| \qquad (i, j = 1, 2; i \neq j) \tag{30}$$

The second-order derivatives are

$$
\begin{aligned}
\left| \frac{\partial^2 \Pi_2}{\partial (K_2)^2} \right| &= \frac{4}{3} \iint_{\Delta_3} dF_l dF_c + 4 \iint_{\Delta_6} dF_l dF_c \\
\left| \frac{\partial^2 \Pi_2}{\partial K_1 \partial K_2} \right| &= \frac{2}{3} \iint_{\Delta_3} dF_l dF_c + 2 \iint_{\Delta_6} dF_l dF_c
\end{aligned}
$$

So (30) holds. ∎

Proof of Proposition 4.

Proof. Under Assumptions 1 and 2, area 3 takes the whole space in Figure 4, so $q_l = (2M_l - M_c + 3K^h)/9$, $q_c = (M_c - 2M_l + 6K^h)/9$, and (6) becomes

$$c_h = 2(\mu_l + \mu_c - 3K^h)/3 \tag{31}$$

So $K^h = (\mu_l + \mu_c)/3 - c_h/2$, $p_c^h = M_c - 2q_c$, $p_l^h = M_l - 2q_l$ and

$$
\begin{aligned}
\Pi^h &= 2p_l^h q_l + p_c^h q_c - 2c_h K^h \\
&= \frac{1}{81} \iint_{\Delta} \Big[(7x_c + 4x_l - 12K^h)(x_c - 2x_l + 6K^h) \\
&\quad + 2(5x_l + 2x_c - 6K^h)(2x_l - x_c + 3K^h) \Big] dF_l dF_c - 2c_h K^h
\end{aligned}
\tag{32}
$$

The result follows after some algebraic manipulations. ∎

Appendix B   Point-to-Point Networks

Airline one's problem is

$$\max_{q_{l1}, q_{c1}} \pi_1 = 2[M_l - (q_{l1} + q_{l2}]q_{l1} + [M_c - (q_{c1} + q_{c2})]q_{c1} \tag{33}$$

$$s.t. \quad 0 \leq q_{c1} \leq K_{c1} \qquad 0 \leq q_{l1} \leq K_{l1}$$

The KKT conditions of (33) are

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$$2M_l - 4q_{l1} - 2q_{l2} - u_{l1} + v_{l1} = 0$$

$$q_{l1}v_{l1} = 0$$

$$M_c - 2q_{c1} - q_{c2} - u_{c1} + v_{c1} = 0$$

$$v_{c1}q_{c1} = 0$$

$$u_{l1}(q_{l1} - K_{l1}) = 0 \qquad (34)$$

$$u_{c1}(q_{c1} - K_{c1}) = 0$$

$$0 \leq q_{c1} \leq K_{c1}$$

$$0 \leq q_{l1} \leq K_{l1}$$

$$u_{c1}, u_{l1}, v_{c1}, v_{l1} \geq 0$$

The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and the slack variables from (26) and its analogues for airline two.

Neither airline is capacity-constrained (Δ_1)

$$q_{yi} = A_y/3$$

Airline one is capacity-constrained in the local markets (Δ_4)

$$q_{l1} = K_{l1} \qquad q_{c1} = q_{c2} = M_c/3 \quad q_{l2} = (M_l - K_{l1})/2 \quad u_{l1} = M_l - 3K_{l1} > 0$$

Airline one is capacity-constrained in the connecting market (Δ_2)

$$q_{c1} = K_{c1} \quad q_{c2} = (M_c - K_{c1})/2 \quad q_{l1} = q_{l2} = M_l/3 \quad u_{c1} = (M_c - 3K_{c1})/2 > 0$$

Airline one is capacity-constrained in all markets (Δ_3)

$$q_{y1} = K_{y1} \quad q_{y2} = (A_y - K_{y1})/2 \quad u_{y1} = (A_y - 3K_{y1})/2 > 0$$

Airline one is capacity-constrained in all markets and airline two is capacity-constrained in local markets (Δ_6)

$$q_{l1} = K_{l1} \quad q_{c1} = K_{c1} \quad q_{l2} = K_{l2} \quad u_{l1} = M_l - 2K_{l1} - K_{l2} > 0$$

$$u_{c1} = (M_c - 3K_{c1})/2 > 0 \quad u_{l2} = M_l - 2K_{l2} - K_{l1} > 0$$

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Airline one is capacity-constrained in all markets and airline two is capacity-constrained in the connecting market (Δ_8)

$$u_{c2} = (M_c - 3K_{c1})/2 > 0 \quad u_{l1} = (M_l - 3K_{l1})/2 > 0$$

$$u_{c1} = M_c - 2K_{c1} - K_{c2} \quad q_{l2} = (M_l - K_{l1})/2$$

Both airlines are capacity-constrained in all markets (Δ_9)

$$q_{yi} = K_{yi}^p \quad u_{yi} = A_y - 2K_{yi} - K_{yj} > 0 \quad i,j = 1,2; i \neq j; y = l,c$$

Both airlines are capacity-constrained in the local markets (Δ_5)

$$q_{l1} = K_{l1}^p \quad q_{l2} = K_{l2} \quad q_{c1} = q_{c2} = M_c/3 \quad u_{li} = M_l - 2K_{li} - K_{lj} > 0$$

Both airlines are capacity-constrained in the connecting market (Δ_7)

$$q_{c1} = K_{c1} \quad q_{c2} = K_{c2} \quad q_{l1} = q_{l2} = M_c/3 \quad u_{ci} = A_y - 2K_{ci} - K_{cj} > 0$$

Proof of Proposition 2

Proof. Following the same reasoning as in the proof of Proposition 1, for airline one, $\partial \Pi_1/\partial K_{c1} = \partial E\pi_1/\partial K_{c1} - c_p = 0$ which translates into

$$c_p = \frac{1}{2} \iint_{\Delta_{2,3,6}} (x_c - 3K_{c1})dF_l dF_c + \iint_{\Delta_{7,8,9}} (x_c - 2K_{c1} - K_{c2})dF_c dF_l \tag{35}$$

Similarly, $\partial \Pi_1/\partial K_{l1} = \partial E\pi_1/\partial K_{l1} - 2c_p = 0$ which translates into

$$c_p = \iint_{\Delta_4} (x_l - K_{l1})dF_l dF_c + \frac{1}{2} \iint_{\Delta_{3,8}} (x_l - 3K_{l1})dF_l dF_c$$
$$+ \iint_{\Delta_{5,6,9}} (x_l - 2K_{l1} - K_{l2})dF_l dF_c \tag{36}$$

Similarly for airline two, the FOC of Π_2 w.r.t K_{c2} and K_{l2} yields, respectively.

$$\frac{1}{2} \iint_{\Delta_7} (x_c - 3K_{c1})dF_L dF_c + \iint_{\Delta_{8,9}} (x_c - 2K_{c2} - K_{c1})dF_c dF_l = c_p \tag{37}$$

$$\iint_{\Delta_{5,6,9}} (x_l - 2K_{l2} - K_{l1})dF_l dF_c = c_p \tag{38}$$

When the airlines have the same capacities, areas 2, 3, 4, 6 and 8 disappear, Figure 5(b) shows the situation. As a result, (37) and (38) are simplified to (7) and (8), respectively. Similar to the proof of Proposition 1, a sufficient condition for uniqueness is

$$\left|\frac{\partial^2 \Pi_i}{\partial (K_{yi})^2}\right| > \left|\frac{\partial^2 \Pi_i}{\partial K_{yi} \partial K_{yj}}\right| \quad (y = c, l; i, j = 1, 2; i \neq j) \tag{39}$$

The second-order derivatives are

$$\left|\frac{\partial^2 \Pi_1}{\partial (K_{l1})^2}\right| = 2 \iint_{\Delta_{5,9}} dF_l dF_c \tag{40}$$

$$\left|\frac{\partial^2 \Pi_1}{\partial K_{l1} \partial K_{l2}}\right| = \iint_{\Delta_{5,9}} dF_l dF_c \tag{41}$$

So (39) holds. ∎

Proof of Proposition 5

Proof. Under Assumptions 1 and 2, area 9 takes the whole space in Figure 5, so equations (7) and (8) translate into $\mu_c - 3K_c^p = c_p$ and $\mu_l - 3K_l^p = c_p$ respectively. Thus $K_y^p = (\mu_y - c_p)/3$ for $y = l, c$. So $p_y^p = (A_y + 2c_p)/3$

$$
\begin{aligned}
\Pi^p &= 2p_l^p K_l^p + p_c^p K_c^p - c_p(2K_l^p + K_c^p) \\
&= 2(\mu_l - 2K_l^p)K_l^p + (\mu_c - 2K_c^p)K_c^p - c_p(2K_l^p + K_c^p) \\
&= (2\mu_l^2 + \mu_c^2)/9 - 2c_p(2\mu_l + \mu_c)/9 + c_p^2/3
\end{aligned}
\tag{42}
$$

∎

Appendix C   Different Networks

Let airline one use a hub-and-spoke network and the other airline a point-to-point network. Airline one's problem is defined by (25) and the corresponding KKT conditions are defined by (26). Airline two's problem is the analogues of (33) and the corresponding KKT conditions are the analogues of (34). The quantity game is solved for each area by taking appropriate values of the Lagrange multipliers and slack variables.

Neither airline is capacity-constrained (Δ_1)

$$q_{l1} = q_{l2} = M_l/3 \quad q_{c1} = q_{c2} = M_c/3$$

Airline two is capacity-constrained in the local markets (Δ_2)

$$q_{l1} = (M_l - K_{l2})/2 \quad q_{l2} = K_{l2} \quad q_{c1} = q_{c2} = M_c/3 \quad \mu_{l2} = M_l - 3K_{l2} > 0$$

Airline two is capacity-constrained in the connecting market (Δ_3)

$$q_{c1} = (M_c - K_{c2})/2 \quad q_{c2} = K_{c2} \quad q_{l1} = q_{l2} = M_c/3 \quad u_{c2} = (M_c - 3K_{c2})/2 > 0$$

Only airline one is capacity-constrained (Δ_4)

$$q_{c1} = (M_c - 2M_l + 6K_1)/9 \quad q_{c2} = (4M_c + M_l - 3K_1)/9$$

$$q_{l1} = (2M_l - M_c + 3K_1)/9 \quad q_{l2} = (M_c + 7M_l - 3K_1)/18 \quad u_1 = (M_c + M_l - 3K_1)/3 > 0$$

Airline one is capacity-constrained; airline two is capacity-constrained in the local markets (Δ_5)

$$q_{l1} = (4M_l - M_c + 3K_1 - 4K_{l2})/11 \quad q_{l2}^m = K_{l2}$$

$$q_{c1}^m = (M_c - 4M_l + 8K_1 + 4K_{l2})/11 \quad q_{c2} = (5M_c + 2M_l + 4K_1 + 2K_{l2})/11$$

$$u_1 = 2(2M_c + 3M_l - 6K_1 - 3K_{l2})/11 > 0 \quad u_{l2} = 2(M_c + 7M_l - 3K_1 - 18K_{l2})/11 > 0$$

Airline one is capacity-constrained; airline two is capacity-constrained in the connecting market (Δ_6)

$$q_{c1} = (M_c - M_l + 3K_1 - K_{c2})/5 \quad q_{c2} = K_{c2}$$

$$q_{l1} = (M_l - M_c + 2K_1 + K_{c2})/5 \quad q_{l2} = (M_c + 4M_l - 2K_1 - K_{c2})/10$$

$$u_1 = (3M_c + 2M_l - 6K_1 - 3K_{c2})/5 > 0 \quad u_{c2} = (4M_c + M_l - 3K_1 - 9K_{c2})/5 > 0$$

Airline one is capacity-constrained and serves only the local market where airline two is capacity-constrained (Δ_7)

$$q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{l2} = K_{c2} \quad q_{lc2} = M_c/2 \quad u_{l2} = 2M_l - 4K_{l2} - 2K_1 > 0$$

$$u_1 = 2M_l - 4K_1 - 2K_{l2} > 0 \quad v_{c1} = (4M_l - M_c - 8K_1 - 4K_{l2})/2 > 0$$

Airline one is capacity-constrained and serves only the local markets; airline two is capacity-constrained in all markets (Δ_8)

$$q_{l1} = K_1 \quad q_{c1} = 0 \quad q_{c2} = K_{c2} \quad q_{l2} = K_{l2}$$

$$u_{c2} = M_c - 2K_{c2} > 0 \quad u_{l2} = 2M_l - 4K_{l2} - 2K_1 > 0$$

$$u_1 = 2M_l - 4K_1 - 2K_{l2} > 0 \quad v_{c1} = 2M_l - M_c - 4K_1 - 2K_{l2} + K_{c2} > 0$$

Both airlines are capacity-constrained in all markets (Δ_9)

$$q_{l2} = K_{l2} \quad q_{l1} = (2M_l - M_c + 2K_1 + K_{c2} - 2K_{l2})/6$$

$$q_{c2} = K_{c2} \quad q_{c1} = (M_c - 2M_l + 4K_1 - K_{c2} + 2K_{l2})/6$$

$$u_1 = 2(M_c + M_l - 2k_1 - K_{c2} - K_{l2})/3 > 0$$

$$u_{c2} = (5M_c + 2M_l - 4K_1 - 11K_{c2} - 2K_{l2})/6 > 0$$

$$u_{l2} = (4M_l + M_c - 2K_1 - K_{c2} - 10K_{l2})/3 > 0$$

Proof of Proposition 3.

Proof. For airline one, $\partial\Pi_1/\partial K_1 = E(\partial\pi_1/\partial K_1) - 2c_h = 0$, and $E(\partial\pi_1/\partial K_1)$ can be derived by summing the product of the first-order derivative of profit w.r.t K_1 and the probability of each area in Figure 6 using the Leibnitz rule. So the FOC of Π_1 w.r.t K_1 is $E\partial\pi_1/\partial K_1 = 2c_h$, which translates into

$$c_h = \frac{1}{3}\iint_{\Delta_4}(x_l + x_c - 3K_1)dF_ldF_c + \frac{2}{11}\iint_{\Delta_5}(2x_c + 3x_l - 6K_1 - 3K_{l2})dF_ldF_c$$
$$+ \frac{1}{5}\iint_{\Delta_6}(3x_c + 2x_l - 6K_1 - 3K_{l2})dF_ldF_c + 2\iint_{\Delta_{7,8}}(x_l - 2K_1 - K_{l2})dF_ldF_c$$
$$+ \frac{2}{3}\iint_{\Delta_9}(x_c + x_l - 2K_1 - K_{c2} - K_{l2})dF_ldF_c$$

For airline two, $\partial\Pi_2/\partial K_{l2} = \partial E\pi_2/\partial K_{l2} - 2c_p = 0$, which can be changed into

$$c_p = \iint_{\Delta_2}(x_l - 3K_{l2})dF_ldF_c + \frac{2}{11}\iint_{\Delta_5}(x_c + 7x_l - 3K_1 - 18K_{l2})dF_ldF_c$$
$$+ 2\iint_{\Delta_{7,8}}((x_l - 2K_{l2} - K_1)dF_ldF_c + \frac{1}{3}\iint_{\Delta_9}(4x_l + x_c - 2K_1 - K_{c2} - 10K_{l2})dF_ldF_c$$

similarly, $\partial\Pi_2/\partial K_{c2} = \partial E\pi_2/\partial K_{c2} - c_p = 0$, which is equivalent to

$$c_h = \frac{1}{5}\iint_{\Delta_6}(4x_c + x_l - 3K_1 - 9K_{c2})dFdG + \iint_{\Delta_8}(x_c - 2K_{c2})dF_ldF_c$$
$$+ \frac{1}{6}\iint_{\Delta_9}(5x_c + 2x_l - 4K_1 - 11K_{c2} - 2K_{l2})dF_ldF_c$$

∎

Proof of Proposition 6.

Proof. Under Assumptions 1 and 2, area 9 takes the whole space in Figure 6. So (9), (10), and (11), respectively, become

$$c_h = 2(\mu_l + \mu_c - 2K_h^m - K_c^m - K_l^m)/3$$

$$c_p = (4\mu_l + \mu_c - 2K_h^m - K_c^m - 10K_l^m)/3$$

$$c_p = (2\mu_l + 5\mu_c - 4K_h^m - 11K_c^m - 2K_l^m)$$

Solving above equations yields expected equilibrium capacities. The expected equilibrium quantities for area 9 is

$$q_{l2} = K_l^m \quad q_{l1} = (2x_l - x_c + 2K_h^m + K_c^m - 2K_l^m)/6$$

$$q_{c2} = K_c^m \quad q_{c1} = (x_c - 2x_l + 4K_h^m - K_c^m + 2K_l^m)/6$$

$$Ep_l^m = E[x_l - (q_{l1} + q_{l2})] = (2\mu_l + c_p + c_h)/6$$

$$Ep_c^m = E[x_c - (q_{c1} + q_{c2})] = (\mu_c + c_p + c_h)/3$$

The airlines' expected profits are

$$\Pi_h^m = 2p_l^m q_{l1} + p_c^m q_{c1} - 2c_h K_h^m = (p_c^m - 2p_l^m)q_{c1}^m + (2p_l^m - 2c_h)K_h^m$$

$$= \frac{1}{18}\iint_\Delta (x_c - 2x_l)(x_c - 2x_l + 4K_h^m - K_c^m + 2K_l^m)dF_l dF_c$$

$$+ K_l^m(2\mu_l + c_p - 5c_h)/3$$

$$\Pi_p^m = 2(p_l^m - c_p)K_l^m + (p_c^m - c_p)K_c^m$$

$$= [K_l^m(2\mu_l - 5c_p + c_h) + K_c^m(\mu_c + c_h - 2c_p)]/3 \tag{43}$$

The results follow after a few algebraic manipulations. ∎

Appendix D   The Proof of Proposition 11

Proof. By definition, at equilibrium each player's strategy is a best response to its rival's strategy. In case a, from Propositions 8 and 9, a hub-and-spoke network is the best response regardless of its rival's network structure. The same is true for case b(i). In case b(ii), from

Airline Two

	H	P
H	(3 ,3)	(3.5, 1.5)
P	(1.5, 3.5)	(4, 4)

Airline One

Figure 7: Case b(ii)

$$\underbrace{\frac{H\to H}{P\to H}}_{\underline{c_2}} + \underbrace{\frac{H\to H}{P\to P}}_{\underline{c_1}} + \frac{H\to P}{P\to P} \quad \underbrace{\frac{1}{c_1}}_{} \quad \underbrace{\frac{H\to H}{P\to P}}_{\underline{c_2}} \quad \frac{H\to H}{P\to H}$$
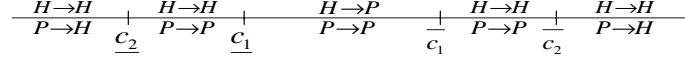
Figure 8: Case c

Propositions 8 and 9, each airline follows a tit-for-tat strategy, i.e., using the same network structure as its rival. It can be verified that $c < \bar{c}_2$ is equivalent to $6c^2 - 8\mu c + 5\sigma^2 < 0$. So

$$\Pi^h - \Pi^p = (6c^2 - 8\mu c + 5\sigma^2 + 3c^2 - 4\mu_c)/27 < 0 \tag{44}$$

$$\Pi^p - \Pi_h^m = -(6c^2 - 8\mu c + 5\sigma^2)/18 > 0 \tag{45}$$

Therefore, (44), (45) and Proposition 10 yield $\Pi_p^m < \Pi^h < \Pi_h^m < \Pi^p$ in case b(ii). This confirms that (P,P) and (H,H) are the pure strategy NEPs. Figure 7 is an numerical example of the network game for case b(ii).

It can be verified that $\bar{c}_1 < \bar{c}_2$ and $\underline{c_1} > \underline{c_2}$. Case c can be described by Figure 8 where $\to$ denotes responses, e.g., $P \to H$ indicates that a hub-and-spoke network is the best response to a point-to-point network. The results for part c are established following the same reasoning for part b.

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