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Information: Valuation Issues in Supply Chains**

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## **Abstract**

We investigate the role of forward commitments and option contracts between a seller (supplier) and a buyer (retailer) in the presence of asymmetric information. In our case both parties face price and demand uncertainty but the retailer, being closer to the market, has additional information about the true demand. The supplier, aware of this asymmetry, and acting as a Stackelberg leader, designs contracting arrangements that best meet his interest. We contrast the role of forward and option contracts in this environment and identify cases where combinations of the two are dominant. We examine how profits are affected by the contracting arrangements and by the degree of asymmetric information. Finally, we investigate how alternative contracting arrangements alters the expected value of obtaining information that eliminates asymmetric information.

In many industries retailers are faced with long lead times, short selling seasons, and high demand uncertainties. Very frequently, the retailer and supplier may have substantially different information sets. For example, it may be the case that the retailer, who is close to the market, has a very good sense of the future demand distribution, whereas the supplier is less confident. For such problems one avenue of research is concerned with the design of contracting relationships between the informed and uninformed that minimizes the impact of asymmetric information, and yet maximizes the interest of the less informed party.

This paper examines such contracting arrangements with asymmetric information, in a supply chain setting consisting of a supplier whose access to the product market is via a single retailer. To manage the risk of inventories associated with uncertain demand, it is fairly common for the manufacturer to make available to the retailer an array of purchasing choices ranging from *forward contracts*, where the price and quantity are predetermined and delivery takes place prior to the selling season, to *option contracts*, where for a fee, the retailer can obtain contracts that provide the right to order up to a predetermined maximum quantity of goods at predetermined per unit costs once demand has been realized. By providing both forward and option contracts, the supplier gives the retailer more choices that may allow the latter to select a strategy that maximizes her profits. The supplier's goal is to design the terms of the forward and option contracts so as to induce the retailer to take optimal actions that best serve the supplier's interests.

Our primary objective is to evaluate the role of forward and option contracts in the presence of asymmetric information. As part of our analysis, we investigate how option contracts can be valued in the presence of asymmetric information. Ex ante, the supplier's unconditional view of uncertainty, as measured by standard deviation, is much larger than the more informed retailer's assessment. With effort, the supplier can perform market research to obtain the same knowledge about demand as that possessed by the retailer. So, viewed in this light, total uncertainty can be broken down into two components. The first is *natural uncertainty*, that would still be present even if the supplier had equal information as the retailer. The second is *information uncertainty*, created by the fact that there is information that the supplier can learn, perhaps at a cost. In investigating the option program, it will be necessary to value the option in terms of both natural and informational uncertainty. The value of the option as a function of natural and informational uncertainty provides an interesting step towards better understanding the value of obtaining perfect information. Indeed, our framework where asymmetric information exists provides a very concrete platform to establish the *value of information* and its impact on option pricing. Conversely, we are also interested in how various contracting relationships affect the supplier's assessment of the expected value of perfect information.

To focus exclusively on the contracting arrangements we assume both supplier and retailer are risk neutral. We provide several interesting results about contracting relationships in this

setting. First, we show that given the choice between forward and option contracts, the supplier will generally prefer option contracts. However, policies involving forwards and options will be weakly preferred over options or forward policies alone. Only when there is no natural demand uncertainty will the supplier be indifferent to the optimal use of options and forwards.

Second, we establish the marginal value of option contracts to the supplier, over and above forwards, in the presence of asymmetric information. We show that the greater the asymmetry of information, the greater the value of the option program, over and above forwards. Further, as demand uncertainty increases, the marginal value of option contracts, relative to forwards, increases for the supplier. In contrast, as retail price uncertainty increases, the role of forward commitments for the supplier takes on a bigger role.

Third, we investigate the value of obtaining perfect information. With forward contracts, the expected value of perfect information typically exceeds the value when options are used alone. In general, when both options and forwards are used, the expected value of perfect information typically declines. However, this is not always the situation, and cases can be established where the value of perfect information actually increases when the supplier moves from forward policies to option policies to mixtures of both.

Finally, we investigate the implications of these policies on the retailer's profits. As the supplier moves from forwards only, to options only, there may, or may not be an increase in profits for the retailer. Similarly, if the supplier moves to a policy involving both options and forwards, the retailer may find expected profits increase, or decrease. Unlike traded derivative contracts, option and forward contracts in supply chains are not zero sum games, and there are cases where both parties benefit. Our results are interesting in that they demonstrate sets of conditions under which contracting relationships might be improved by the simultaneous use of options and forwards.

The paper proceeds as follows. In Section 1 we review related literature in supply chain management. In Section 2 we list the basic assumptions and establish the decision and pricing problems. In Section 3 we consider the Bayesian Nash equilibrium that results if the supplier uses forward contracts alone, option contracts alone, and both contracts. We carefully compare the optimal profits for the supplier and the retailer for these three policies and identify the impact of asymmetric information. In Section 4 we examine the value of the supplier using options (forwards), over and above forwards (options). We identify conditions on demand and price uncertainty when the use of options (forwards) adds significantly to profits, and highlight how asymmetric information alters the contribution of the two contract types to profit. We also investigate the role of options and forwards in hedging against natural price and demand uncertainties as well as informational uncertainty. In section 5 we examine the value of perfect information and explore how contracting arrangements alter this value. Section 6 concludes.

# 1 Literature Review

We are interested in valuing option and forward contracts in a supply chain setting. The usual approach in pricing real options follows the Black-Scholes (1973) and Merton (1973) paradigm, in which the underlying price is unaltered by the introduction of the option. This approach is not meaningful in our setting because the option contract is not a redundant asset that can be replicated by dynamic self financing trading schemes in the underlying asset and in riskless bonds, and hence should not affect the underlying price. Moreover, in our model we explicitly acknowledge asymmetric information, a feature absent in traditional option models in finance.<sup>1</sup> In our model, however, the underlying is a specific good, and the specific contracts offered by the supplier induce a chain of reactions that alter profits for both parties as well as for the entire channel. We shall assume that the supplier chooses terms of the contracting vehicle (options and/or forwards) so as to maximize his expected profits. Since, at the outset the supplier can only assign a probability distribution to the possible demand distributions that could occur, while the retailer knows the exact distribution type that will occur, the contracting relationships that we consider are self revealing, and the equilibria that result are Bayesian Nash equilibria.<sup>2</sup>

Like our study, Cachon and Lariviere (2001) consider contracting arrangements using forwards and options. However, in our setting the less-informed supplier offers the contract while in their setting the more informed buyer offers the contract. The two settings are fundamentally different from each other in terms of model formulation of the underlying gaming problems and solution approaches. Specifically, our model relies on the revelation principle to derive Bayes-equilibria that induce truth-telling of the more informed buyer, while theirs falls within a class of gaming problems known as signaling models where the concern is about information credibility. Another related paper is by Ozer and Wei (2004), who consider option-only and forward-only contracts in a capacity reservation setting which is quite different from our two-production mode setting.

Barnes-Schuster et al (2002) study option contracts in a two-period setting where a supplier offers the contract to allow a down-stream buyer to commit firm orders, at the beginning of the first period, for both periods and buy options to order later for the second period. Like our model, they assume that the supplier has the choice of producing early at a low cost or later at a higher cost. Their model, however, does not consider issues related to information asymmetry.

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<sup>1</sup>For an example of an exception to this see Beck (1993) who investigates asymmetric information and the influence to the dynamics of the underlying asset when an option is listed.

<sup>2</sup>Asymmetric information is also considered in a real option setting by Granadier (1999), and Nordlund (2004), who adopt an information revelation mechanism to study the formation of an equilibrium in a market where potential entrants have asymmetric information, for example on the size of the market, and their actions affect competitors decisions to enter. For an excellent survey of real options and strategic competition see Boyer, Gravel and Lassere (2004).

Without considering information asymmetry, researchers have examined other type of option contracts or option-like agreements to provide flexibility to the buyer in a supply chain. For example, Eppen and Iyer (1997) study backup agreements where a buyer commits to a total order quantity for a selling season. Pre-specified fraction of the total quantity is delivered initially, and the buyer may purchase additional units up to the remaining commitment at a later date. Tsay and Lovejoy (1999) consider quantity flexibility contracts, where the buyer commits to a range within which she can choose to place future orders. Brown and Lee (1997) examine a pay-to-delay agreement, where the buyer reserves a capacity with the supplier at a fixed fee, and she could then place orders at a later date up to the capacity reservation. Many researchers study buy-back contracts under which a buyer commits to and pays for an initial quantity but has the option to return unused units later at a credit. Examples include Pasternack (1985), Emmons and Gilbert (1998), Burnetas and Ritchken (2005), and Donohue (2000). Finally, the literature on forward contracts (or contracts other than options) with asymmetric information is fairly abundant. Since the focus of this paper is on option contract, we will omit a detailed review here. Interested readers are referred to recent works by Corbett (2001), Corbett et al. (2004), Ha (2001), Taylor (2003), Cachon and Zhang (2003), Burnetas et al (2004), and the references therein.

## 2 General Model

At date 0 the supplier provides the retailer with a choice of contracts. The retailer then decides which, if any, contract to choose. Based on the selection, the supplier establishes production quantities and build an initial inventory at date 0, at the per unit cost of  $c_0$ . At date 1, the uncertainties in both demand and retail price are resolved, and, depending on the contract negotiated at date 0, committed deliveries of units to the retailer are made and/or the retailer places limited additional orders at predetermined per unit prices. The supplier can deliver the units from inventory, or, if necessary, produce units at an expedited cost of  $c_1$  per unit, where  $c_1 \geq c_0$ .

At date 0, demand, viewed through the eyes of the supplier, is uncertain with two possible distributions. Let  $F_L(x)$  be the distribution function with low demand and  $F_H(x)$  be the distribution function of high demand. We assume that the domains of both distributions are finite, i.e., for low demand  $0 \leq x \leq \bar{D}_L$  and for high demand  $0 \leq x \leq \bar{D}_H$ , where  $\bar{D}_L < \bar{D}_H$ . Assume that  $F_L(x) > F_H(x)$ ,  $\forall 0 \leq x \leq \bar{D}_L$ , i.e. the high demand distribution stochastically dominates the low demand distribution. The retailer knows which of the two demand distributions is relevant, whereas the supplier, who has less information than the retailer, only has a prior probability over the two possible demand distributions. Let  $p$  be the probability that the supplier thinks the demand will be a drawing from the low demand distribution.

Let  $m$  be the quantity of product the supplier procures at cost  $c_0$ , and  $m'$  be the quantity of product the supplier produces at the expedited cost  $c_1$ . The amount produced in period 1,  $m'$ , by the supplier is fully determined by  $m$ , the realization of demand  $D$ , and the nature of the contract, and therefore is not an explicit control variable. The price that the retailer sells one unit of product is  $r$ . We assume that  $r$  is a nonnegative random variable with density function  $g(x)$  and distribution function  $G(x)$  and it is independent of demand. Let  $\mu_r$  and  $\sigma_r$  denote the mean and standard deviation of price  $r$ , respectively. The assumption of independence of price and demand is made for analytical convenience and is not essential for the model. What is important here is that there is common knowledge (in our case represented by the distribution of price and the two possible distributions of demand) and asymmetric information, here represented by the relevance of the two demand distributions.<sup>3</sup>

We consider three types of contracting arrangements. The first is a standard *forward* contract between the supplier and the retailer. At date 0 the retailer (supplier) commits to buying (selling) a certain quantity of goods at a fixed price to be paid upon delivery. The second type of contract is an *option* contract. At date 0 the supplier offers option contracts to the retailer. Each option allows the retailer to purchase one unit at date 1 for a predetermined strike price of  $X$ . The retailer responds by buying a package of these contracts. Then, once demand is realized, the retailer exercises as many options as is appropriate, up to the amount of contracts purchased only if  $r > X$ . The third contract we consider involves a portfolio of forwards and options.

Since our focus is on comparing contract designs in the presence of asymmetric information, we assume away risk aversion issues and address the problem when both supplier and retailer are risk neutral.<sup>4</sup> Furthermore, for simplicity we assume the interest rate is 0.

Due to asymmetric information, the supplier will offer two alternative “bundles” to the retailer, and request that the retailer choose one. Each bundle specifies the quantity of forward contracts, and/or the quantity of options, together with the total cost. Let  $Q$  be the quantity of commitments,  $q$  the quantity of options and  $T$  the total payment. One bundle corresponds to a selection that the retailer will find optimal to choose if the demand distribution is high; the other bundle corresponds to the optimal response by the retailer if the true demand scenario corresponds to the lower demand distribution. We restrict attention to direct mechanisms in which it is a Bayesian Nash equilibrium for the retailer to reveal the truth about the distribution function.<sup>5</sup>

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<sup>3</sup>If price and demand are correlated, then there cannot be a common source of knowledge regarding uncertainties. In this case we could project price onto demand and an orthogonal uncorrelated variable, and assume common knowledge over the uncorrelated variable. Alternatively, we could just assume there is asymmetry of information involving the joint distribution of prices and demand.

<sup>4</sup>This assumption may be reasonable if the two firms are publically traded. If the supplier and/or retailer are privately held firms then issues of risk aversion could become more important.

<sup>5</sup>From the Revelation Principle, any Bayesian Nash equilibrium of any Bayesian game can be represented by



Let  $\Pi_H^s$  be the profit of the supplier if the demand is high and  $\Pi_L^s$  be the profit of the supplier if the demand is low. Let  $\Pi_\theta^r(B_\tau)$  be the profit of the retailer given she chooses the bundle  $B_\tau$ ,  $\tau = H, L$ , when the demand distribution is of type  $\theta$ . The general model of the problem at time 0 before the retailer makes a decision is:

$$\begin{aligned} \max_{B_H, B_L} \quad & p\Pi_L^s(B_L) + (1-p)\Pi_H^s(B_H) && \text{[Problem I]} \\ \text{s.t.} \quad & \Pi_L^r(B_L) \geq 0 && (1) \end{aligned}$$

$$\Pi_H^r(B_H) \geq 0 \quad (2)$$

$$\Pi_L^r(B_L) \geq \Pi_L^r(B_H) \quad (3)$$

$$\Pi_H^r(B_H) \geq \Pi_H^r(B_L) \quad (4)$$

The first two constraints ensure that the retailer does not expect to lose money in each state. The last two are the incentive compatible constraints that ensure truth telling by the retailer is optimal. Through the design of the two contracts, once the retailer has chosen a bundle, the supplier will know the true demand distribution. So the problem to the supplier after the retailer makes a choice is:

$$\max_{m_\theta} \quad \Pi_\theta^s(B_\theta) \quad (\theta = H, L) \quad \text{[Problem II]}$$

We use backward induction to solve this problem, i.e. solve Problem II first then substitute the optimal solution into Problem I and solve it. We now consider the equilibrium implications for the three contract designs.

### 3 Bayesian Nash Equilibria

#### 3.1 Forward Contracts Only

In this model, the supplier issues two contract choices to the retailer. Each choice contains a commitment quantity and a total payment. Let  $B_\theta = \{(Q_\theta, T_\theta) | \theta = H, L\}$  represent the two contracts. The profit function of the supplier if the demand distribution is  $\theta$  is:

$$\Pi_\theta^s = T_\theta - c_0 m_\theta - c_1 m'_\theta \quad (5)$$

where  $m_\theta + m'_\theta = Q_\theta$ .

The expected profit function for the retailer in the demand state  $\theta$ , given that contract of type  $\tau$  is chosen, is:

$$\Pi_\theta^r(B_\tau) = E[r \min(Q_\tau, D_\theta)] - T_\tau$$

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a truth telling or incentive compatible direct mechanism.

$$\begin{aligned}
&= \mu_r [Q_\tau - E[(Q_\tau - D_\theta)^+] - T_\tau] \\
&= \mu_r [Q_\tau - \int_0^{Q_\tau} F_\theta(x) dx] - T_\tau \\
&= \mu_r \int_0^{Q_\tau} \bar{F}_\theta(x) dx - T_\tau
\end{aligned} \tag{6}$$

Now, Problem II of the general model becomes:

$$\begin{aligned}
\max_{m_\theta} \quad & \Pi_\theta^s = T_\theta - c_0 m_\theta - c_1 (Q_\theta - m_\theta) \\
\text{s.t.} \quad & 0 \leq m_\theta \leq Q_\theta
\end{aligned}$$

Since  $\frac{\partial \Pi_\theta^s}{\partial m_\theta} = c_1 - c_0 > 0$ ,  $\Pi_\theta^s$  is increasing in  $m_\theta$ . Hence,  $\Pi_\theta^s$  is maximized when  $m_\theta$  is at the upper boundary, i.e.,  $m_\theta^* = Q_\theta$ .

So, for this case, Problem I becomes:

$$\begin{aligned}
\max_{Q_L, T_L, Q_H, T_H} \quad & p(T_L - c_0 Q_L) + (1-p)(T_H - c_0 Q_H) \\
\text{s.t.} \quad & \mu_r \int_0^{Q_L} \bar{F}_L(x) dx - T_L \geq 0 \\
& \mu_r \int_0^{Q_H} \bar{F}_H(x) dx - T_H \geq 0 \\
& \mu_r \int_0^{Q_L} \bar{F}_L(x) dx - T_L \geq \mu_r \int_0^{Q_H} \bar{F}_L(x) dx - T_H \\
& \mu_r \int_0^{Q_H} \bar{F}_H(x) dx - T_H \geq \mu_r \int_0^{Q_L} \bar{F}_H(x) dx - T_L
\end{aligned} \tag{7}$$

We now characterize the optimal policy with forward contracts.

### Proposition 1

(i) If the supplier uses forward contracts, the Bayesian Nash equilibrium strategy is for the supplier to offer the retailer two contracts,  $B_L = (Q_L^*, T_L^*)$  and  $B_H = (Q_H^*, T_H^*)$ , where  $Q_L^*$  and  $Q_H^*$  satisfy:

$$p(\mu_r \bar{F}_L(Q_L^*) - c_0) = (1-p)\mu_r(\bar{F}_H(Q_L^*) - \bar{F}_L(Q_L^*)) \tag{8}$$

$$\mu_r \bar{F}_H(Q_H^*) = c_0 \tag{9}$$

and

$$\begin{aligned}
T_L^* &= \mu_r \int_0^{Q_L^*} \bar{F}_L(x) dx \\
T_H^* &= \mu_r \left[ \int_0^{Q_H^*} \bar{F}_H(x) dx - \int_0^{Q_L^*} [\bar{F}_H(x) - \bar{F}_L(x)] dx \right]
\end{aligned}$$

(ii) The optimal production plan is for the supplier to produce  $Q_\theta^*$  at date 0, where  $\theta$  is the retailer's optimal response.

*Proof:* See Appendix.

The expected profit of a low demand type retailer is always 0, whereas the expected profit of a high demand type retailer is given by

$$\Pi_H^r = \mu_r \int_0^{Q_L^*} [\bar{F}_H(x) - \bar{F}_L(x)] dx \quad (10)$$

which is positive and is an increasing function of  $Q_L^*$ . Since the difference of the two demand distributions, namely,  $[\bar{F}_H(x) - \bar{F}_L(x)]$ , represents the degree of information asymmetry, from (10), we see that the expected profit of the high demand type retailer increases as asymmetry of information increases.

Interestingly, using forward contracts, the supplier's profit does not depend on the volatility of the retail price.

### 3.2 Option Contracts Only

When the contract only consists of options, the two bundles offered to the retailer are  $B_\theta = \{(q_\theta, T_\theta) | \theta = H, L\}$ . Here  $q_\theta$  is the number of options purchased by the retailer given contract type  $\theta$  is chosen. For the moment we assume the strike price,  $X$ , is exogenous, rather than a decision variable. That is, each option contract allows the retailer to buy one unit at price  $X$  at date 1, once the true demand is realized. The profit function of the supplier, given the retailer chooses  $B_\theta$ , is:

$$\Pi_\theta^s = T_\theta - c_0 m_\theta + \bar{G}(X) [X E_D[\min(q_\theta, D_\theta)] - c_1 m'_\theta]$$

where

$$\begin{aligned} m'_\theta &= E_D[\min(q_\theta, D_\theta)] - E_D[\min(m_\theta, D_\theta)] \\ m_\theta &\leq q_\theta. \end{aligned}$$

Hence:

$$\begin{aligned} \Pi_\theta^s &= T_\theta - c_0 m_\theta + \bar{G}(X) \left[ X \int_0^{q_\theta} \bar{F}_\theta(x) dx - c_1 \int_0^{q_\theta} \bar{F}_\theta(x) dx + c_1 \int_0^{m_\theta} \bar{F}_\theta(x) dx \right] \\ &= T_\theta - c_0 m_\theta + \bar{G}(X) \left[ (X - c_1) \int_0^{q_\theta} \bar{F}_\theta(x) dx + c_1 \int_0^{m_\theta} \bar{F}_\theta(x) dx \right]. \end{aligned} \quad (11)$$

The expected profit function for the retailer, given that contract  $B_\tau$  is chosen is:

$$\begin{aligned} \Pi_\theta^r(B_\tau) &= E_r[r - X | r > X] E_D[\min(q_\tau, D_\theta)] - T_\tau \\ &= \int_X^\infty g(t)(t - X) dt \int_0^{q_\tau} \bar{F}_\theta(x) dx - T_\tau \\ &= \left[ \mu_r - \int_0^X \bar{G}(t) dt \right] \int_0^{q_\tau} \bar{F}_\theta(x) dx - T_\tau. \end{aligned}$$

Let

$$A = \mu_r - \int_0^X \overline{G}(t) dt. \quad (12)$$

Note that  $A$  is positive, decreasing in  $X$  and  $\lim_{X \rightarrow \infty} A = 0$ .

For this case, Problem II becomes:

$$\begin{aligned} \max_{m_\theta} \quad & \Pi_\theta^s = T_\theta + \overline{G}(X)(X - c_1) \int_0^{q_\theta} \overline{F}_\theta(x) dx - c_0 m_\theta + \overline{G}(X) c_1 \int_0^{m_\theta} \overline{F}_\theta(x) dx \\ \text{s.t.} \quad & 0 \leq m_\theta \leq q_\theta \end{aligned} \quad (13)$$

Moreover, since  $\frac{\partial \Pi_\theta^s}{\partial m_\theta} = \overline{G}(X) c_1 \overline{F}(m_\theta) - c_0$ , the solution to (13) is:

$$m_\theta^* = \min(n_\theta, q_\theta) \quad (14)$$

where

$$n_\theta = \overline{F}_\theta^{-1} \left( \frac{c_0}{\max(\overline{G}(X) c_1, c_0)} \right) \quad \theta = H, L. \quad (15)$$

Problem I reduces to:

$$\begin{aligned} \max_{q_L, T_L, q_H, T_H} \quad & \Pi^s = p \left[ T_L + \overline{G}(X)(X - c_1) \int_0^{q_L} \overline{F}_L(x) dx - c_0 m_L^* + \overline{G}(X) c_1 \int_0^{m_L^*} \overline{F}_L(x) dx \right] \\ & + (1 - p) \left[ T_H + \overline{G}(X)(X - c_1) \int_0^{q_H} \overline{F}_H(x) dx - c_0 m_H^* + \overline{G}(X) c_1 \int_0^{m_H^*} \overline{F}_H(x) dx \right] \\ \text{s.t.} \quad & A \int_0^{q_L} \overline{F}_L(x) dx - T_L \geq 0 \\ & A \int_0^{q_H} \overline{F}_H(x) dx - T_H \geq 0 \\ & A \int_0^{q_L} \overline{F}_L(x) dx - T_L \geq A \int_0^{q_H} \overline{F}_L(x) dx - T_H \\ & A \int_0^{q_H} \overline{F}_H(x) dx - T_H \geq A \int_0^{q_L} \overline{F}_H(x) dx - T_L \end{aligned}$$

Under this formulation we obtain the following:

## Proposition 2

(i) If option contracts are used, the Bayesian Nash Equilibrium strategy is for the supplier to offer the retailer the following two bundles,  $B_L = \{q_L^*, T_L^*\}$  and  $B_H = \{q_H^*, T_H^*\}$ , where  $q_L^*$  satisfies the equation:

$$\begin{aligned} q_L^* &= \begin{cases} q_1 & \text{if } q_1 \geq n_L \\ q_2 & \text{if } q_1 < n_L \end{cases} \\ q_H^* &= \overline{D}_H \end{aligned}$$

where  $q_1$  and  $q_2$  satisfy:

$$p \left[ A + \overline{G}(X)(X - c_1) \right] \overline{F}_L(q_1) = (1 - p)A[\overline{F}_H(q_1) - \overline{F}_L(q_1)] \quad (16)$$

$$p \left[ A + \overline{G}(X)X \right] \overline{F}_L(q_2) - c_0 = (1 - p)A[\overline{F}_H(q_2) - \overline{F}_L(q_2)] \quad (17)$$

(ii) In addition:

$$\begin{aligned} T_L^* &= A \int_0^{q_L^*} \overline{F}_L(x) dx \\ T_H^* &= A \left[ \int_0^{q_H^*} \overline{F}_H(x) dx - \int_0^{q_L^*} [\overline{F}_H(x) - \overline{F}_L(x)] dx \right] \end{aligned}$$

(iii) Further, the optimal production schedule for the supplier is:

$$\begin{aligned} m_L^* &= \begin{cases} n_L & \text{if } q_1 \geq n_L \\ q_L^* & \text{if } q_1 < n_L \end{cases} \\ m_H^* &= n_H^* \end{aligned}$$

*Proof:* See Appendix.

It can be verified that at equilibrium the expected profits for the retailer are given by:

$$\begin{aligned} \Pi_L^r &= 0 \\ \Pi_H^r &= A \int_0^{q_L^*} [\overline{F}_H(x) - \overline{F}_L(x)] dx. \end{aligned}$$

These results are similar to the results for forward contracts, in that the profit for a low demand type retailer is zero. In addition, the expected profit of a high demand type retailer depends on  $q_L^*$ , the gap between the two demand distributions, and in this case, the strike,  $X$ . However, unlike the case for forwards, the profit for the supplier will depend on the volatility of retail prices, as well as the volatility of demand.

From Proposition 2, we see that since  $q_H^* = \overline{D}_H$ , the optimal policy is for the supplier to guarantee that all the retailer's customers will be satisfied if the demand type is high, provided the retail price is above the strike. In contrast, if the demand type is low, not all the demand is guaranteed when the retail price is above the strike, since the number of options granted is strictly less than  $\overline{D}_L$ .

With options, the supplier bears significant quantity risk. In particular, after the retailer accepts one of the two contracts, the amount of goods that will actually be purchased remains unclear. As a result, the solution of the supplier's contingent production schedule is of interest. If the true distribution is of type  $L$ , then the optimal production schedule will depend on whether  $q_L^*$  is above or below  $n_L$ . If  $q_L^* < n_L$ , the supplier makes up the maximum inventory that the

retailer could possible order, and there is no possibility of having expedited orders produced at a per unit cost of  $c_1$ . However, if  $q_L^* \geq n_L$  then the supplier does not produce up to the maximum order size that the retailer can request at date 1. That is a “just-in-case” inventory policy is no longer optimal. If demand in period 1 is sufficiently high then the supplier may have to produce additional units to satisfy the requests from options that are exercised.

### 3.3 Comparison of Forward Commitments with Option Contracts

To illustrate the results obtained so far, assume that retail price is lognormal with  $\mu_r = 50$ , and  $\sigma_r = 10$ . The two possible demand distributions are also lognormal. The means of the two distributions are  $\mu_L = 1000$ , and  $\mu_H = 2000$ . They have a common standard deviation of  $\sigma_d = 200$ . We assume  $c_0 = 20$ . In the analysis that follows we will vary  $p$ ,  $\sigma_r$  and  $\sigma_d$  to see how asymmetry and volatility effects values of the solutions.

The top panel of Figure 1 provides a plot of the expected profit for the supplier against strike prices, first for the case when  $c_1 = c_0$ , and then for the case when  $c_1 = \infty$ . These two values represent an upper and lower bound on profits for  $c_0 \leq c_1 < \infty$ . The dashed line shows the corresponding expected profit when forward contracts are used.

Figure 1: Here

Notice that when  $X = 0$ , the lower bound on expected profits using options equals the expected profit using forwards. Notice too, that the slope of the option profits at  $X = 0$  is positive at first, but for sufficiently large strikes eventually goes negative. This indicates that there is an  $X^c$  such that for  $0 \leq X < X^c$ , option contracts are preferred to forwards and thereafter forward contracts are preferred.

The advantage of options over forwards depends on the strike price  $X$  and on the cost  $c_1$ . The bottom panel of Figure 1 shows the regions in  $(c_1, X)$  space where options are preferred to forwards and vice versa. Notice that for sufficiently low strikes options are preferred regardless of  $c_1$ . Then there is an interval of strikes where forwards dominate provided the expedited cost  $c_1$  is sufficiently high. Finally, there is a threshold value of the strike  $X$  above which forwards are always dominant, regardless of  $c_1$ . Note that in this example, this critical strike price is below the expected price of 50.

The above illustrations are true in general and the results are summarized in the Proposition below.

### Proposition 3

(i) For a given cost of expediting,  $c_1$ , there exists a critical strike price,  $X^c > 0$ , such that for  $0 \leq X < X^c$ , the supplier prefers option contracts to forward contracts, and for  $X > X^c$ , the supplier prefers forwards.

(ii) There exists a region of strike prices,  $X < X_L^c$ , where options are preferred to forwards regardless of  $c_1$ , and a region of strike prices,  $X > X_U^c$ , where Forwards dominate options. In the region  $X_L^c < X < X_H^c$ , there is a threshold value of expedited costs,  $c_1^*(X)$  say, below which options are preferred, and above which forwards are preferred.

*Proof:* See Appendix.

So far we have assumed the strike price is predetermined. When we permit the strike price to be a decision variable, option contracts always dominate forward contracts for the supplier, as shown formally in Proposition 4 below. From Figure 1, we know there exists an  $X^*$  which maximize expected profit of supplier. Since all contract parameters are functions of strike  $X$ , so does the expected profit of supplier which is written as  $\Pi^s(X)$ . Unfortunately,  $\Pi^s(X)$  has no closed form solution, and is therefore obtained using numerical methods.

The top panel of Figure 2 shows a plot of the optimal strike price,  $X^*$ , as price volatility  $\sigma_r$  increases. The parameters used are from the previous example, with the asymmetric information parameter,  $p$ , set at  $p = 0.5$ . The bottom panel shows a plot of the optimal strike price as demand volatility,  $\sigma_d$  increases.

Figure 2: Here

First, consider the case when  $\sigma_d = 0$ . In this case, it is easy to show that the optimal strike price is  $X^* = 0$ . As a result, the retailer will exercise all options. The supplier, anticipating this, will make up all the required goods in period 0. As a result, the supplier will be indifferent to using forwards or options.

When  $\sigma_r = 0$ , then it can easily be shown that  $X^* = \mu_r$ . As a result, the high demand type retailer does not profit, and the supplier is able to extract all the surplus.

From Figure 2, we observe that  $X^*$  is decreasing in  $\sigma_r$ . Since, in this example,  $X^* < \mu_r$  when the variance of retail price increases, the probability that the retailer will not exercise the options increases. So, as variance increases, the supplier must set the strike price lower so that the retailer has an incentive to exercise. As demand volatility increases, the cost to the supplier of providing these options with “low” strikes increases, and therefore the supplier’s

natural response to increasing demand uncertainty, is to increase the strike. This behavior is confirmed in the lower panel of Figure 2.

To better understand the role of uncertainties, Figure 3 plots the supplier and retailer's profits against  $\sigma_d$  and  $\sigma_r$ , when the strike  $X$  is chosen optimally by the supplier.

Figure 3: Here

As discussed, when  $\sigma_d = 0$ , the two contracts are equivalent, in the sense that the supplier's profits are the same. As  $\sigma_d$  increases, the supplier's profit using either contract decreases, but profits from the options contract exceed profits from the forwards contract. The right figure in the top panel of Figure 3 shows that the profits for the supplier using options always exceeds the profit using forwards for all price volatility values.

Figure 3 shows that the retailer's profit under either contract is not monotone in demand volatility, and neither contract type is consistently preferred.

We summarize and prove formally some of the key properties illustrated above, together with some additional ones, in the following proposition:

**Proposition 4**

*(i) Regardless of the degree of asymmetry of information, for the supplier, option contracting always weakly dominates forward contracting, and the two contracts are equivalent only when the demand volatility is zero, i.e., when  $\sigma_d = 0$ .*

*(ii) When there is no asymmetry of information, the retailer is unable to profit with either forwards or options. That is, both contracts deliver a zero profit to the retailer.*

*(iii) With asymmetric information, the retailer may be able to profit, but it is not clear which contract will be preferred. However, if demand volatility approaches zero, i.e., when  $\sigma_d = 0$ , the retailer cannot profit. Further, if price volatility approaches zero, i.e., when  $\sigma_r = 0$ , the retailer can only profit if the supplier uses forward contracts.*

*Proof:* See Appendix.

The following table summarizes the preferences among the contracts for the supplier and the retailer.



---

$\sigma_d > 0, \sigma_r > 0$ $\sigma_d = 0, \sigma_r \geq 0$ $\sigma_d > 0, \sigma_r = 0$				
Symmetric	Supplier	$F \prec O$	$F \sim O$	$F \prec O$
	Retailer	$F \sim O$	$F \sim O$	$F \sim O$
Asymmetric	Supplier	$F \prec O$	$F \sim O$	$F \prec O$
	Retailer	ambiguous	$F \sim O$	$F \succ O$

---

### 3.4 Forward and Option Contracts

We now consider the general case where the supplier considers using both forward and option contracts. Let  $B_\theta = \{(Q_\theta, q_\theta, T_\theta) | \theta = H, L\}$  represent the two contracts that the supplier provides the retailer. Again, for the moment let the strike price be exogenous. The profit functions of the supplier is:

$$\begin{aligned}\Pi_\theta^s &= T_\theta - c_0 m_\theta + \overline{G}(X)[X E_D[\min(Q_\theta + q_\theta, D_\theta) - \min(Q_\theta, D_\theta)] - c_1 m'_\theta] \\ m'_\theta &= E_D[\min(Q_\theta + q_\theta, D_\theta) - \min(\max(Q_\theta, m_\theta), D_\theta)] + \max(Q_\theta, m_\theta) - m_\theta\end{aligned}$$

and the profit for the retailer is:

$$\begin{aligned}\Pi_\theta^r(B_\tau) &= -T_\tau + E_r[r E_D[\min(Q_\tau, D_\theta)] | r < X] \\ &\quad + E_r[(r - X) E_D[\min(Y_\tau, D_\theta)] + X E_D[\min(Q_\tau, D_\theta)] | r > X] \\ &= A \int_0^{Y_\tau} \overline{F}_\theta(x) dx + B \int_0^{Q_\tau} \overline{F}_\theta(x) dx - T_\tau\end{aligned}\tag{18}$$

where  $Y_\tau = Q_\tau + q_\tau$ ,  $A$  is defined earlier, and

$$B = \int_0^X \overline{G}(t) dt = \mu_r - A.\tag{19}$$

Clearly,  $m_\theta < Q_\theta$  is suboptimal. Thus  $\max(Q_\theta, m_\theta) = m_\theta$ . So,

$$\Pi_\theta^s = T_\theta + \overline{G}(X) \left[ (X - c_1) \int_0^{Y_\theta} \overline{F}_\theta(x) dx - X \int_0^{Q_\theta} \overline{F}_\theta(x) dx + c_1 \int_0^{m_\theta} \overline{F}_\theta(x) dx \right] - c_0 m_\theta$$

Problem II of the general model reduces to:

$$\begin{aligned}\max_{m_\theta} \Pi^s &= T_\theta + \overline{G}(X) \left[ (X - c_1) \int_0^{Y_\theta} \overline{F}_\theta(x) dx - X \int_0^{Q_\theta} \overline{F}_\theta(x) dx + c_1 \int_0^{m_\theta} \overline{F}_\theta(x) dx \right] - c_0 m_\theta \\ \text{s.t.} \quad & m_\theta \leq Y_\theta\end{aligned}$$

Since  $\frac{\partial \Pi_\theta^s}{\partial m_\theta} = \overline{G}(X) c_1 \overline{F}(m_\theta) - c_0$ , the solution to Problem II is:

$$m_\theta^* = \min(n_\theta, Y_\theta)\tag{20}$$

where

$$n_\theta = \overline{F}_\theta^{-1} \left( \frac{c_0}{\max(\overline{G}(X)c_1, c_0)} \right).$$

Problem I of the general model now is:

$$\begin{aligned} \max_{\overline{B}_L, \overline{B}_H} & \quad p \left[ T_L - c_0 m_L^* + \overline{G}(X) [(X - c_1) \int_0^{Y_L} \overline{F}_L(x) dx - X \int_0^{Q_L} \overline{F}_L(x) dx + c_1 \int_0^{m_L^*} \overline{F}_L(x) dx] \right] \\ & + (1-p) \left[ T_H - c_0 m_H^* + \overline{G}(X) [(X - c_1) \int_0^{Y_H} \overline{F}_H(x) dx - X \int_0^{Q_H} \overline{F}_H(x) dx + c_1 \int_0^{m_H^*} \overline{F}_H(x) dx] \right] \\ \text{s.t.} & \quad A \int_0^{Y_L} \overline{F}_L(x) dx + B \int_0^{Q_L} \overline{F}_L(x) dx - T_L \geq 0 \\ & \quad A \int_0^{Y_H} \overline{F}_H(x) dx + B \int_0^{Q_H} \overline{F}_H(x) dx - T_H \geq 0 \\ & \quad A \int_0^{Y_L} \overline{F}_L(x) dx + B \int_0^{Q_L} \overline{F}_L(x) dx - T_L \geq A \int_0^{Y_H} \overline{F}_L(x) dx + B \int_0^{Q_H} \overline{F}_L(x) dx - T_H \\ & \quad A \int_0^{Y_H} \overline{F}_H(x) dx + B \int_0^{Q_H} \overline{F}_H(x) dx - T_H \geq A \int_0^{Y_L} \overline{F}_H(x) dx + B \int_0^{Q_L} \overline{F}_H(x) dx - T_L \end{aligned}$$

We now have:

### Proposition 5

(i) If forward and option contracts are used, the Bayesian Nash equilibrium strategy is for the supplier to offer the retailer the following two bundles,  $B_L = (Q_L^*, q_L^*, T_L^*)$  and  $B_H = (Q_H^*, q_H^*, T_H^*)$ . If we define  $Y_\theta = Q_\theta + q_\theta$ , then  $Y_L^*$  satisfies the equation:

$$\begin{aligned} Y_L^* &= \begin{cases} Y_1 & \text{if } Y_1 \geq n_L \\ Y_2 & \text{if } Y_1 < n_L \end{cases} \\ Y_H^* &= \overline{D}_H \end{aligned}$$

where  $Y_1$  and  $Y_2$  satisfy:

$$p \left[ A + \overline{G}(X)(X - c_1) \right] \overline{F}_L(Y_1) = (1-p)A[\overline{F}_H(Y_1) - \overline{F}_L(Y_1)] \quad (21)$$

$$p \left[ [A + \overline{G}(X)X] \overline{F}_L(Y_2) - c_0 \right] = (1-p)A[\overline{F}_H(Y_2) - \overline{F}_L(Y_2)] \quad (22)$$

with

$$n_L = \overline{F}_L^{-1} \left( \frac{c_0}{\max(\overline{G}(X)c_1, c_0)} \right).$$

(ii) The optimal commitments,  $Q_L^*$  and  $Q_H^*$ , satisfy:

$$p[B - \overline{G}(X)X] \overline{F}_L(Q_L^*) = (1-p)B[\overline{F}_H(Q_L^*) - \overline{F}_L(Q_L^*)] \quad (23)$$

$$Q_H^* = n_H \quad (24)$$

In addition:

$$\begin{aligned}
T_L^* &= A \int_0^{Y_L^*} \bar{F}_L(x) dx + B \int_0^{Q_L^*} \bar{F}_L(x) dx \\
T_H^* &= A \left[ \int_0^{Y_H^*} \bar{F}_H(x) dx - \int_0^{Y_L^*} [\bar{F}_H(x) - \bar{F}_L(x)] dx \right] + B \left[ \int_0^{Q_H^*} \bar{F}_H(x) dx - \int_0^{Q_L^*} [\bar{F}_H(x) - \bar{F}_L(x)] dx \right]
\end{aligned}$$

(iii) Further, the optimal production schedule for the supplier is:

$$\begin{aligned}
m_L^* &= \begin{cases} n_L & \text{if } q_1 \geq n_L \\ q_L^* & \text{if } q_1 < n_L \end{cases} \\
m_H^* &= n_H.
\end{aligned}$$

*Proof:* See Appendix

From Proposition 5, we know that  $Q_L^* > 0$  and  $Q_H^* > 0$ . Since forwards do enter the optimal solution, combinations of option and forwards must dominate option strategies alone.

### 3.5 Comparison of Different Contracts

The following proposition establishes relationships between Forward-Option and Option-only contracts.

#### Proposition 6

(i) Regardless of the degree of asymmetric information, for the supplier Forward-Option contracts always weakly dominate Option contracts alone, with the two contracts being equivalent when the demand volatility and/or the price volatility become zero, i.e., when  $\sigma_d = 0$  and/or  $\sigma_r = 0$ .

(ii) With symmetric information, the retailer will find that Forward-Option contracts and option contracts are equivalent, each delivering a zero profit.

(iii) With asymmetric information, however, the retailer finds that there is no dominant contract in general. However, if  $\sigma_d = 0$  and/or  $\sigma_r = 0$ , the retailer will be indifferent between the contracts, with both delivering zero profits.

*Proof:* See Appendix

Together with Proposition 4, we now can fully establish the relationships among the three contracts: forwards, options and forward-options. For ease of reference, we summarize the results as:

		$\sigma_d > 0, \sigma_r > 0$	$\sigma_d = 0, \sigma_r \geq 0$	$\sigma_d > 0, \sigma_r = 0$
Symmetric	Supplier	$F \prec O \prec F\&O$	$F \sim O \sim F\&O$	$F \prec O \sim F\&O$
	Retailer	$F \sim O \sim F\&O = 0$	$F \sim O \sim F\&O = 0$	$F \sim O \sim F\&O = 0$
Asymmetric	Supplier	$F \prec O \prec F\&O$	$F \sim O \sim F\&O$	$F \prec O \sim F\&O$
	Retailer	ambiguous	$F \sim O \sim F\&O = 0$	$F \succ O \sim F\&O = 0$

Using the same benchmark parameters as in the earlier examples, Table 1 provides a numerical illustration that highlights important results. First, we notice from Table 1 that the supplier's expected profit from using forward & option contracts does dominate profits from using option contracts which in turn dominates forward contracts, for all values corresponding to asymmetric information.

Table 1: Here

The Table shows that when  $p = 0.5$  the retailer would prefer that the supplier use forward & option contracts rather than option contracts alone. However, when  $p = 0.2$ , the retailer would prefer option contracts. As discussed, for the retailer, none of the three contracts is always dominating or being dominated.

Everything else being the same, the optimal strike under the forward & option contract should be greater than the optimal strike for option contracts alone, since using both forwards & options the supplier can reduce revenue uncertainty by using forwards. In particular, using options alone, the delivery of goods will be zero if  $r < X^*$ ; however, using forwards & options, the delivery of goods is at least the number of forwards even if  $r < X^*$ . Thus the supplier can raise the strike to extract more profit.

Table 1 shows the optimal strike prices and the production policy. Consistent with our discussion, the optimal strikes under the forward & option policy are always higher than the optimal strikes under the pure option policy. Notice that the quantities of forward contracts under the forward policy are always higher than the quantities of forwards under the forward & option policy. This is to be expected, since given that additional demand can be potentially filled through options, the number of forwards offered can be reduced in a forward & option policy. Similarly, the number of options offered under the option only policy is always higher than the number of options issued under the forward & option policy. Finally, due to the higher number of forward commitments, the initial production quantities under the forward only policy is always higher than that under the forward & option policy or under the option only policy.

## 4 Valuation of Option and Forward Programs

We have seen from Propositions 4 and 6 that when combining forwards and options, instead of using either options or forwards individually, the supplier is always weakly better off, while the retailer can be either better or worse off. In this section, we more carefully explore the marginal values or loss of adding options (forwards) into the contracting relationship over and above forward (option) contracts.

We define the value of the option program to the supplier, over and above the forward contract as,  $V^s(O|F)$ , where:

$$V^s(O|F) = \Pi^s(O\&F) - \Pi^s(F) \quad (25)$$

where  $O$  denotes the option contract,  $F$ , the forward program alone, and  $O\&F$  denotes the combined option and forward contracting program. Similarly, we define the marginal value of the forward program to the supplier as

$$V^s(F|O) = \Pi^s(O\&F) - \Pi^s(O) \quad (26)$$

Both  $V^s(O|F)$  and  $V^s(F|O)$  are clearly non-negative.

Since the expected profit of the low demand type retailer is always zero in each of the three contracts, we only consider the high demand type retailer. Define the value of the option program to the retailer as:

$$V^r(O|F) = \Pi_H^r(O\&F) - \Pi_H^r(F) \quad (27)$$

and the value of forward program to retailer as:

$$V^r(F|O) = \Pi_H^r(O\&F) - \Pi_H^r(O) \quad (28)$$

For the high type demand retailer,  $V^r(O|F)$  and  $V^r(F|O)$  can be either positive or negative.

We numerically examine how price and demand uncertainties affect the relative valuations of options and forwards as defined above. Uncertainties can be classified into two categories: natural uncertainties and information uncertainties. Natural uncertainties are those that are known to and faced by both the supplier and the retailer, while information uncertainties are those related to asymmetric information between the two parties. In our model setting, natural uncertainties include retail price volatility  $\sigma_r$  and demand volatility  $\sigma_d$ , and information uncertainties are represented through the supplier's prior probability  $p$  about two possible demand distributions and their dispersion  $\mu_H - \mu_L$ .

### 4.1 Marginal Valuation of the Option Program

We first investigate how  $V^s(O|F)$  and  $V^r(O|F)$  vary against  $\sigma_d$  and  $\sigma_r$ . The top panel of Figure 4 plot  $V^s(O|F)$  against  $\sigma_d$  fixing  $\sigma_r = 10$  and then against  $\sigma_r$  fixing  $\sigma_d = 200$ , where  $p$  is fixed

at 0.5.

Figure 4: Here

First, we notice that for the supplier, the valuation of the option program,  $V^s(O|F)$ , increases as the volatility of demand,  $\sigma_d$ , increases, and decreases as the volatility of retail price,  $\sigma_r$ , increases.

As uncertainty in demand increases, the use of options becomes more important to the retailer. Providing this flexibility to the retailer, could increase overall purchases by the retailer and hence add additional profit for the supplier, but also comes at the cost of the supplier having to possibly build contingent inventories or manufacture goods at higher expedited costs. Overall, however, as demand uncertainty for the retailer increases, the value of being able to build or expedite orders increases, and the value of the option program to the supplier increases.

On the other hand, as the volatility of the retail price increases, the ability of the retailer to exercise options at a low strike relative to a high retail price increases. Moreover, as we have already seen, the optimal strike price generally *decreases* as volatility increases. Hence, the value of the option program, while positive for the supplier, will generally decrease as retail price uncertainty increases. That is, when retail price uncertainty is very high, the marginal contribution of option contracts above and beyond forward contracts is reduced.

To more clearly see this, the middle panel of Figure 4 shows the marginal contribution of the option program to total profits as  $\sigma_d$  increases and as  $\sigma_r$  increases. The figure also shows the marginal contribution of the option program to total profits for the case where there is no asymmetric information. The figures clearly show that for the supplier, the contribution to profits expands with demand uncertainty and shrinks with regard to price uncertainty.

The bottom panel of Figure 4 shows plots of the value of the option program over and above the forward program for the retailer. Since the retailer's profits depend on the contract chosen to maximize the supplier's profits, there is no guarantee that the value of the option program to the retailer will be positive. Furthermore, the way in which volatilities ( $\sigma_d$  or  $\sigma_r$ ) affect the retailer's value of the option program could be more complex and not monotone. In particular, Figure 4 shows the cost of the option program to the retailer increasing as demand volatility increases, and then decreasing as demand volatility expands further. In contrast, as retail price uncertainty increases, the cost of the option program decreases. In these figures, the retailer is worse off with the introduction of options when the volatility of demand is low, but at very high volatilities, the retailer's profit increases. Of course, in real product markets options are not zero sum games, so the increase in costs for the retailer are not equal to the increase in profits for the supplier. Indeed, illustrative cases can be constructed under which option contracts would

actually benefit both parties.

## 4.2 Marginal Valuation of the Forward Program

The top panel of Figure 5 shows the marginal value of forwards over and above options for the supplier as uncertainties in demand and price increase. Unlike options, the marginal value of the forward program above and beyond options to the supplier,  $V^s(F|O)$ , increases both in the volatility of demand,  $\sigma_d$ , and in the volatility of prices,  $\sigma_r$ .

Figure 5: Here

As price uncertainty increases, the value of the forward program over and above the option component increases. The middle panel of Figure 5 shows the contribution to profits of the supplier by forward over and above the profits from using options.

The bottom panel of Figure 5 plot the values of forwards for the retailer. Again, since the retailer's profits depend on the contract chosen to maximize supplier's profits, there is no guarantee that the value of the forward program to the retailer will be positive. As can be seen, the way in which volatilities ( $\sigma_d$  or  $\sigma_r$ ) affects the retailer's value of the forward program is complex. As demand volatility expands, the value of the forward commitment to the retailer increases. However, the value of forwards is not monotone with respect to retail price uncertainty.

Expected profits for the supplier using forwards are unchanged by increasing retail price volatility. When  $\sigma_r = 0$  the contribution of options is the greatest, and the marginal value of forwards is zero. As volatility increases, supplier profits of either using forwards and options or options only, fall, and the contribution of the option component above forwards declines.

Expected profits for the supplier using forwards are affected by increasing demand volatility. When  $\sigma_d = 0$  the contribution of options to profits is the smallest, and the marginal value of options above forwards is zero. As demand volatility increases, supplier profits of using any contract fall, but the contribution of the option component above forwards increases.

Our specific results are based on independence between demand and price uncertainties. This assumption may be valid in some product markets but in general demand and prices might be correlated. Our main conclusion, however, is that in the presence of asymmetric information, the role of forwards and option contracting in a supply chain target in on different aspects of uncertainty. In general, the dual role of options and forwards may depend critically on the joint determinants of price and quantity uncertainties.

### 4.3 Impact of Asymmetric Information on Valuations

We now turn to investigating the effects of information asymmetry as represented through the supplier's prior probability  $p$  about the two possible demand distributions.

The left panels of Figure 6 show the values of the option program for different  $p$  values for the supplier, and for the high demand type retailer. The right hand panels show the same figures but under a higher volatility,  $\sigma_d$ .

Figure 6: Here

Generally speaking, for the low volatility of demand, the value of the option program for the supplier increases with asymmetry of information. That is, the value of option is smaller when  $p$  is near 0 or 1 than when  $p$  is the middle range, and is maximized near  $p = 0.5$ . For the high-demand type retailer the value of the option program becomes negative as  $p$  increases, then, when  $p$  is near 1, it sharply increases, before jumping to zero at  $p = 1$ .

When  $\sigma_d$  is high, the value of the option as a function of  $p$  becomes more complex. We notice that the value of the option program for the high demand type retailer is not continuous because  $Y_L^*$  and  $Q_L^*$  are not continuous in  $p$ , which makes the expected profit of the high demand type retailer discontinuous. The value of the option program over and above the forward program is continuous for the supplier, since the expected profit of supplier is the same at the jump point. The figure clearly reveals that the value of the option program to the supplier over and above forward contracts is *not* monotone in  $p$  even over a range from 0 to 0.5.

Figure 7: Here

Figure 7 repeats the same analysis for forward contracts. In general, the value of forwards increases with asymmetric information, but the results are not clean, in the sense that the maximum value is not necessarily accomplished when information is most asymmetric. ( $p$  near 0.5) The bottom row shows the complex value of the forward contracts to the retailer as information asymmetry varies.

## 5 Expected Value of Perfect Information

As discussed earlier, the supplier is faced with two sources of uncertainty, namely the natural uncertainty from both retail prices and demand which cannot be learnt, and asymmetric infor-



mation, that could possibly be learnt. Ex ante, the demand distribution faced by the supplier is a mixture of two lognormals, with unconditional variance,  $\sigma^2 = (\mu_H - \mu_L)^2 p(1-p) + \sigma_d^2$ . The retailer knows which of these two distributions demand will come from. We now investigate the fair price the supplier would be prepared to pay for perfect information. We define the expected value of perfect information, given the supplier's ex-ante probability that  $\theta = L$  is  $p$ ,  $EVPI(p)$ , say, as:

$$EVPI_i(p) = [p\Pi_i^s(1) + (1-p)\Pi_i^s(0)] - \Pi_i^s(p) \quad (29)$$

Here  $\Pi_i^s(p)$  is the expected profit for the supplier where  $i = O, F$  or  $O\&F$  denotes option, forward, and both respectively, when the ex-ante probability that  $\theta = L$  is  $p$ .

The top panel of Figure 8 shows the expected value of perfect information  $EVPI_i(p)$  for the supplier for different values of  $p$  for each of the three contract types.

Figure 8: Here

Clearly, when  $p = 0$  and 1, there is no asymmetric information, and the value of perfect information for all three types is zero. Depending on the parameter values the maximum expected value of perfect information occurs somewhere between 0.4 and 0.6 for each of the three cases. To better see the relative differences in the expected value of perfect information across contracts, the lower panel shows the differences. For relatively small values of  $p$ ,  $EVPI_F < EVPI_O < EVPI_{O\&F}$ . However, this relationship is not maintained as  $p$  increases. Generally, the expected value of perfect information, varies in complex ways with the type of contracting mechanism. Further, when options are used rather than forwards, it is not necessarily the case that the expected value of perfect information decreases.

## 6 Conclusion

This paper has studied the role of forward commitments and option contracts in a supply chain with asymmetric information. In our case, the Stackelberg leader was the supplier, who was less informed about the demand distribution than the retailer, but equally informed about the possible retail price uncertainty. We showed that in such a market there is a role for both forwards and options. Forwards are particularly important in the presence of retail price uncertainty, while option contracts assist greatly in the presence of demand uncertainty.

Of interest is the marginal value of options to the supplier, over and above forwards, in the presence of asymmetric information. We have seen that the greater the asymmetry of information, the greater the value of the option program, over and above forwards. Further, as

demand uncertainty increases, the marginal value of options, relative to forwards increases for the supplier. In contrast, as retail price uncertainty increases, the role of forward commitments for the supplier takes on a bigger role. Finally, the value of obtaining perfect information depends on the policy adopted by the supplier. With forward contracts, the expected value of perfect information, typically exceeds the value when options are used alone. In general, when both options and forwards are used, the expected value of perfect information typically declines. However, this is not always the case, and cases can be established where the value of perfect information actually increases, when the supplier moves from forward policies to option policies to mixtures of both.

Our results have several interesting applications that relate to the introduction of new technologies. For example, assume the supplier has a new technology where the cost of expediting orders is dramatically reduced from  $c_1$  to  $c'_1$ . This has enormous consequences on the optimal contracting relationship, since it moves the new equilibrium further away from using forwards to options. The implication of this for the retailer could be good or bad, depending on the nature of the asymmetric information, and the magnitudes of the uncertainties. It also alters the value of perfect information. It remains for future research to more closely investigate how the existence of a new technology alters the equilibrium, and, in a dynamic model, how it could affect the timing of the introduction of the new technology.

It also remains for future research to consider additional policies, other than options and forwards as contracting mechanisms. Since we have implicitly assumed that these contracts are non-transferable, there is no reason for pricing to be linear, or the strike prices to be independent of quantity. As the complexity of the contracting relationship increases, the supplier is provided with additional degrees of freedom that allow further extraction of rents away from the high demand state retailer. Of course, in our analysis we have assumed that the supplier has full bargaining power. It remains for further research to examine alternative models of equilibrium, where more power is granted to the retailer. There is also the possibility of extending the analysis to allow for a continuum of retailer demand types, rather than two, as in Baron and Myerson (1982). While such extensions are possible, it is unclear what additional insights will be obtained.

# Appendix

## Proof of Proposition 1

(i) First, we claim that if the first and last constraints hold, then the second constraint holds automatically. To see this, notice that from the first constraint, we have:

$$-T_L \geq -\mu_r \int_0^{Q_L} \bar{F}_L(x) dx.$$

Substituting this into the last constraint, we get

$$r \int_0^{Q_H} \bar{F}_H(x) dx - T_H \geq \mu_r \int_0^{Q_L} [\bar{F}_H(x) - \bar{F}_L(x)] dx.$$

The right hand side is strictly positive as long as  $F_H$  stochastically dominates  $F_L$ , which is the assumption we have made.

Now, we claim that at the optimal solution the first and the last constraints must be binding. Recall that the second constraint is not binding and the profit of  $\theta = H$  retailer is strictly positive. If the first constraint is not binding, we can increase  $T_L$  and  $T_H$  by the same amount  $\delta$  such that all constraints hold and the object function increases which violates the optimality condition. Thus, the first constraint must be binding. Similarly, if at the optimal solution, the last constraint is not binding, we can increase  $T_H$  by  $\delta$  such that the last constraint is binding. In this case, all other constraints will hold and the objective function will increase, which violates the optimality condition. So, the last constraint is also binding.

Since the first and the last constraints are binding, we can solve for  $T_L$  and  $T_H$  through these two equations:

$$\begin{aligned} T_L &= \mu_r \int_0^{Q_L} \bar{F}_L(x) dx \\ T_H &= \mu_r \left[ \int_0^{Q_H} \bar{F}_H(x) dx - \int_0^{Q_L} [\bar{F}_H(x) - \bar{F}_L(x)] dx \right]. \end{aligned}$$

Substituting  $T_L$  and  $T_H$  into the third constraint, we can get the following inequality:

$$\mu_r \int_{Q_L}^{Q_H} [\bar{F}_H(x) - \bar{F}_L(x)] dx \geq 0$$

Hence, as long as  $Q_L < Q_H$ , the third constraint will hold.

So, relaxing the third constraint, we can convert the original constrained optimization problem into an unconstrained optimization problem. Using the first order condition, we obtain a necessary condition for optimal  $Q_L^*$  and  $Q_H^*$ :

$$\begin{aligned} p(\mu_r \bar{F}_L(Q_L^*) - c_0) &= (1-p)\mu_r (\bar{F}_H(Q_L^*) - \bar{F}_L(Q_L^*)) \\ \mu_r \bar{F}_H(Q_L^*) &= c_0 \end{aligned}$$

Now, we verify that  $Q_L^* < Q_H^*$ . Since  $\bar{F}_H(Q_L^*) > \bar{F}_L(Q_L^*)$ ,  $pc_0 + (1-p)\mu_r\bar{F}_H(Q_L^*) = \mu_r\bar{F}_L(Q_L^*) < \mu_r\bar{F}_H(Q_L^*)$ , or  $\mu_r\bar{F}_H(Q_L^*) > c_0 = \mu_r\bar{F}_H(Q_H^*)$ . So,  $Q_L^* < Q_H^*$ .

(ii) The results here follow from the optimal solution to supplier's production problem, as discussed in the paper.

## Proof of Proposition 2

(i) Using the same method in the proof of Proposition 1, we can show that the first and the last constraints are binding. The second constraint holds as long as high demand stochastically dominates low demand and the third constraint holds as long as  $q_L < q_H$ . Then we are facing a similar unconstrained optimization problem.

But here,  $m_\theta^* = \min(n_\theta, q_\theta)$ . We consider two cases:

$$\frac{\partial m_\theta^*}{\partial q_\theta} = \begin{cases} 0 & \text{if } n_\theta \leq q_\theta \\ 1 & \text{if } n_\theta > q_\theta. \end{cases}$$

For  $\theta = L$ , if  $n_L \leq q_L$ , the first order condition is,

$$p \left[ A + \bar{G}(X)(X - c_1) \right] \bar{F}_L(q_1) = (1-p)A[\bar{F}_H(q_1) - \bar{F}_L(q_1)].$$

If  $n_L > q_L$ , the first order condition is,

$$p \left[ A + \bar{G}(X)X \right] \bar{F}_L(q_2) - c_0 = (1-p)A[\bar{F}_H(q_2) - \bar{F}_L(q_2)].$$

$$q_L^* = \begin{cases} q_1 & \text{if } q_1 \geq n_L \\ q_2 & \text{if } q_2 < n_L \end{cases}$$

For  $\theta = H$ , only one case ( $n_H \leq q_H$ ) can happen. In this case,  $q_H^* = \bar{D}_H$ .

If  $n_H > q_H$ , then we know  $q_H^*$  satisfies the first order condition:

$$\left[ \mu_r - \int_0^X \bar{G}(t)dt + \bar{G}(X)X \right] \bar{F}_H(q_H^*) = c_0,$$

and

$$n_H = \bar{F}_H^{-1} \left( \frac{c_0}{\max(\bar{G}(X)c_1, c_0)} \right).$$

Since  $\mu_r - \int_0^X \bar{G}(t)dt + \bar{G}(X)X > \bar{G}(X)c_1$  if  $\mu_r > c_1$ , then we know  $q_H^* > n_H$  must be true, which is a contradiction with  $n_H > q_H^*$ .

Obviously,  $q_L^* < q_H^*$ , so the third constraint holds.

(ii) The results follow directly from optimal solution.

(iii) The results here follow from the optimal solution to supplier's production problem, as discussed in the paper.

### Proof of Proposition 3

(i) It is clear that

$$\begin{aligned}\frac{\partial \Pi^s}{\partial X} &= (1-p)\overline{G}(X) \int_0^{q_L^*} [\overline{F}_H(x) - \overline{F}_L(x)] dx \\ &\quad - pg(X) \left[ (X - c_1) \int_0^{q_L^*} \overline{F}_L(x) dx + c_1 \int_0^{m_L^*} \overline{F}_L(x) dx \right] \\ &\quad - (1-p)g(X) \left[ (X - c_1) \int_0^{q_H^*} \overline{F}_H(x) dx + c_1 \int_0^{m_H^*} \overline{F}_H(x) dx \right]\end{aligned}$$

So that  $\left. \frac{\partial \Pi^s}{\partial X} \right|_{X=0} = (1-p) \int_0^{q_L^*} [\overline{F}_H(x) - \overline{F}_L(x)] dx > 0$  (recall that  $\overline{G}(0) = 1$  and  $g(0) = 0$ ). When strike price is infinity, definitely the supplier will not profit. Therefore, we proved that  $\Pi^s$  is increasing then decreasing in  $X$ , and also  $\Pi^s \rightarrow 0$  when  $X \rightarrow \infty$ .

When  $c_1 = \infty$  and  $X = 0$ , then  $A = \mu_r$ . Thus  $q_L^*$  comes from equation (17), which becomes

$$p(\mu_r \overline{F}_L(q_2) - c_0) = (1-p)\mu_r(\overline{F}_H(q_2) - \overline{F}_L(q_2)) \quad (\text{A.1})$$

And  $n_L = \overline{D}_L$ . Comparing (A.2) and (8), we can get  $q_L^* = Q_L^*$  and  $m_L^* = q_L^*$ . However, when  $c_1 > \mu_r$ ,  $q_H^* = \overline{D}_H$  does not hold. When  $X = 0$ ,

$$\frac{\partial \Pi^s}{\partial q_H} = (1-p) \left[ (\mu_r - c_1) \overline{F}_H(q_H) + \frac{\partial m_H^*}{\partial q_H} (c_1 \overline{F}_H(m_H^*) - c_0) \right]$$

If  $m_H^* = n_H$ ,  $\frac{\partial m_H^*}{\partial q_H} = 0$ .

$$\frac{\partial \Pi^s}{\partial q_H} = (1-p)(\mu_r - c_1) \overline{F}_H(q_H) \quad (\text{A.2})$$

If  $m_H^* = q_H$ ,  $\frac{\partial m_H^*}{\partial q_H} = 1$ .

$$\frac{\partial \Pi^s}{\partial q_H} = (1-p)(\mu_r \overline{F}_H(q_H) - c_0) \quad (\text{A.3})$$

If  $c_1 > \mu_r$ ,  $q_H^*$  comes from (A.3). Comparing (A.3) and (9), we can find that  $q_H^* = Q_H^*$ . Thus we have proved that when  $X = 0$  and  $c_1 = \infty$ , option contract and forward contract are equivalent.

We have

$$\frac{\partial \Pi^s}{\partial c_1} = p \left[ \frac{\partial q_L^*}{\partial c_1} (A + \overline{G}(X)(X - c_1) \overline{F}_L(q_L^*)) + \frac{\partial m_L^*}{\partial c_1} (\overline{G}(X)c_1 \overline{F}_L(m_L^*) - c_0) - \overline{G}(X) \int_{m_L^*}^{q_L^*} \overline{F}_L(x) dx \right]$$

$$\begin{aligned}
& (1-p) \left[ \frac{\partial q_H^*}{\partial c_1} (A + \overline{G}(X)(X - c_1)) \overline{F}_H(q_H^*) + \frac{\partial m_H^*}{\partial c_1} (\overline{G}(X)c_1 \overline{F}_H(m_H^*) - c_0) \right. \\
& \quad \left. - \frac{\partial q_L^*}{\partial c_1} A (\overline{F}_H(q_L^*) - \overline{F}_L(q_L^*)) - \overline{G}(X) \int_{m_H^*}^{q_H^*} \overline{F}_H(x) dx \right] \\
& = -p \overline{G}(X) \int_{m_L^*}^{q_L^*} \overline{F}_L(x) dx - (1-p) \overline{G}(X) \int_{m_H^*}^{q_H^*} \overline{F}_H(x) dx \\
& \leq 0
\end{aligned}$$

The second equality holds since (16), (17) and (15) hold and  $\frac{\partial q_H^*}{\partial c_1} = 0$ . The inequality holds since  $m_L^* \leq q_L^*$  and  $m_H^* \leq q_H^*$ .

Since  $\Pi^s$  is decreasing in  $c_1$ , we have shown that for any  $c_1 \geq c_0$ , the profit of option contract at  $X = 0$  is no less than the profit of the forward contract. Recall that the profit of the forward contract is independent of  $X$  and strict positive. So for any  $c_1 \geq c_0$ , the two curves cross each other, that is, there exists an  $X^c > 0$ , such that for  $0 \leq X \leq X^c$ , the value of option contract is greater than the value of forward contract, and for  $X > X^c$ , the value of forward contract is greater than option contract.

(ii) If we let  $c_1 = \infty$ , from (i) we can get a critical value  $X_L^c$  such that for  $X < X_L^c$  the value of forward contract is less than the option contract at  $c_1 = \infty$ , which is no greater than the value of option contract at any  $c_1$ . Similarly, if we let  $c_1 = c_0$ , from (i) we can get a critical  $X_U^c$  such that for  $X > X_U^c$ , the value of forward contract is greater than the value of option contract at  $c_1 = c_0$ , which is greater than the value of option contract at any  $c_1$ . For  $X_L^c < X < X_U^c$ , the relationship depends on  $c_1$ . If  $c_1$  is small enough, then option contract is preferred, otherwise, the forward contract is better. Thus there is a critical  $c_1^*(X)$ , below which option is preferred, and above which forward is preferred.

#### Proof of Proposition 4

(i) If  $\sigma_d > 0, \sigma_r > 0$ , from Proposition 3, it is clear that  $O \succ F$ .

If  $\sigma_d = 0, \sigma_r \geq 0$ , for the forward contract, the optimal solution is:

$$\begin{aligned}
Q_\theta^* &= \overline{D}_\theta \\
T_\theta^* &= \overline{r} \overline{D}_\theta.
\end{aligned}$$

Thus,  $\Pi^s = (\overline{r} - c_0)[p \overline{D}_L + (1-p) \overline{D}_H]$  and  $\Pi_L^r = \Pi_H^r = 0$ .

For the option contract, the optimal solution is:

$$\begin{aligned}
X^* &= 0 \\
q_\theta^* &= \overline{D}_\theta \\
T_\theta^* &= \overline{r} \overline{D}_\theta.
\end{aligned}$$

Thus,  $\Pi^s = (\bar{r} - c_0)[p\bar{D}_L + (1 - p)\bar{D}_H]$  and  $\Pi_L^r = \Pi_H^r = 0$ .

In this case,  $O \sim F$ .

If  $\sigma_d > 0, \sigma_r = 0$ , the optimal solution for forward contract is same as in case (1). However, for option contracts, the optimal strike price  $X^* = \bar{r}$ . The supplier's profit decreases when  $\sigma_r$  increases. So, for this case, the profit from using options, exceeds that obtained from forwards. That is  $\Pi_O^S(\sigma_r = 0) > \Pi_O^S(\sigma_r > 0) > S_F$  ( $\Pi_F^S$  is independent of  $\sigma_r$ ). Under symmetric information, we know that for the retailer,  $F \sim O$ . Under asymmetric information,  $\Pi_O^R = 0$  since  $X^* = \mu_r$  and  $\Pi_F^R > 0$  since  $\Pi_F^R$  is independent of  $\sigma_r$ .

(ii) This result is obvious since the supplier always charges retailer her expected revenue under full information.

(iii) This result is true from (i).

### Proof of Proposition 5

After transforming the original constrained optimization program to the unconstrained optimization program, we get the following problem:

$$\begin{aligned} \max_{B_L, B_H} \quad & \Pi^s = p\Pi_L^s + (1 - p)\Pi_H^s \\ \text{where} \quad & \\ \Pi_L^s = \quad & [A + \bar{G}(X)(X - c_1)] \int_0^{Y_L} \bar{F}_L(x) dx + [B - \bar{G}(X)X] \int_0^{Q_L} \bar{F}_L(x) dx \\ & + \bar{G}(X)c_1 \int_0^{m_L^*} \bar{F}_L(x) dx - c_0 m_L^* \\ \Pi_H^s = \quad & [A + \bar{G}(X)(X - c_1)] \int_0^{Y_H} \bar{F}_H(x) dx + [B - \bar{G}(X)X] \int_0^{Q_H} \bar{F}_H(x) dx \\ & + \bar{G}(X)c_1 \int_0^{m_H^*} \bar{F}_H(x) dx - c_0 m_H^* - A \int_0^{Y_L} [\bar{F}_H(x) - \bar{F}_L(x)] dx \\ & - B \int_0^{Q_L} [\bar{F}_H(x) - \bar{F}_L(x)] dx \end{aligned}$$

Using the same method as in the proof of Proposition 2, leads to the results.

### Proof of Proposition 6

(i) If  $\sigma_d > 0, \sigma_r > 0$ , since  $Q_L^* > 0, Q_H^* > 0$ , we can say that forward & option contract strictly dominates option contract. Otherwise, if two contracts are equivalent, then it is no need to use forwards, then  $Q_L^* = Q_H^* = 0$ , which is a contradiction. Thus, for the supplier,  $F \& O \succ O$ . Under symmetric information, it is clear that for the retailer,  $\Pi_{F \& O}^R = \Pi_O^R = 0$ . However, under asymmetric information, Table 1 gives two cases in which either  $\Pi_{F \& O}^R > \Pi_O^R$  or  $\Pi_O^R > \Pi_{F \& O}^R$  can happen.

If  $\sigma_d = 0, \sigma_r \geq 0$ , for option contract, the optimal solution is

$$\begin{aligned} X^* &= 0 \\ q_\theta^* &= \bar{D}_\theta \\ T_\theta^* &= \bar{r}\bar{D}_\theta \end{aligned}$$

Thus  $\Pi^s = (\bar{r} - c_0)[p\bar{D}_L + (1 - p)\bar{D}_H]$  and  $\Pi_L^r = \Pi_H^r = 0$ .

For forward & option contract, the optimal solution is

$$\begin{aligned} X^* &= 0 \\ Y_\theta^* &= \bar{D}_\theta \\ Q_\theta^* &= 0 \\ T_\theta^* &= \bar{r}\bar{D}_\theta \end{aligned}$$

Thus  $\Pi^s = (\bar{r} - c_0)[p\bar{D}_L + (1 - p)\bar{D}_H]$  and  $\Pi_L^r = \Pi_H^r = 0$ .

From above, it is clear  $S_{F\&O} = S_O$  and  $R_{F\&O} = R_O$ . In this case,  $F\&O \sim O$ .

If  $\sigma_d > 0, \sigma_r = 0$ , for the forward & option contract, it can be shown that  $Q_\theta^* = 0$ ; thus forward & option contract is equivalent with the option-only contract. Therefore  $S_{F\&O} = S_O$ .

The optimal strikes for these two contracts are  $\mu_r$ , so  $\Pi_{F\&O}^R = \Pi_O^R = 0$ .

(ii) This result is obvious since the supplier always charges retailer her expected revenue under full information.

(iii) This result is true from (i).



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**Table 1:**  
**Optimal Policies with Forward, Options, and Both Contracts**

The Table provides details of solutions to the supplier's problem for 3 different values of the asymmetric information parameter,  $p$ . The case parameters are described in the text. For each problem the optimal bundles under the three policies, forwards-only, options-only and forwards& options, are provided. The strike prices of options are also provided, together with the production quantities in period 0. Finally, the expected profit of the supplier is noted, as well as the profit for the high type retailer.

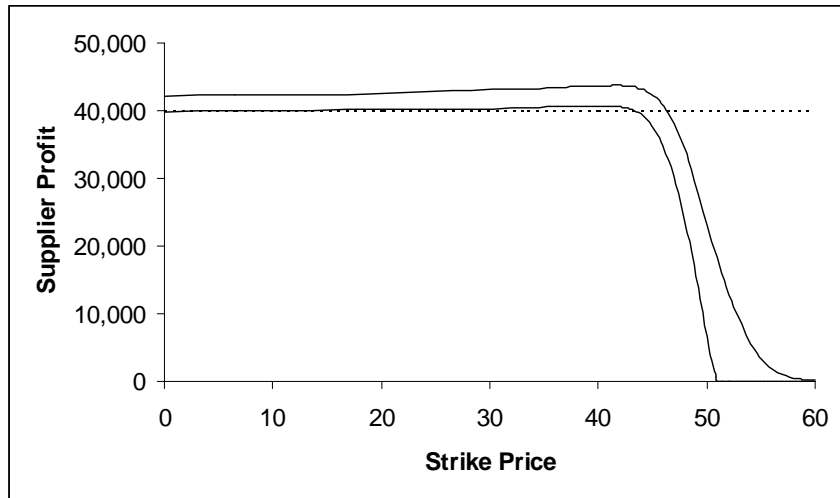
	Forwards	Options	Forward & Options	
p=0	Forwards, $(Q_L, Q_H)$	(-, 2047)	-	(-, 1857)
	Options, $(q_L, q_H)$	-	(-, 8000)	(-, 6143)
	Strike, $X$	-	30.9	40.0
	Charge, $(T_L, T_H)$	(-, 85696)	(-, 38187)	(-, 6143)
	Production, $(m_L, m_H)$	(-, 2047)	(-, 1857)	(-, 1857)
	Supplier's Profit, $\pi^s$	44751	47996	47996
	Retailer's Profit, $\pi_H^R$	0	0	0
p=0.2	Forwards, $(Q_L, Q_H)$	(341, 2047)	-	(118, 1844)
	Options, $(q_L, q_H)$	-	(590, 8000)	(523, 6159)
	Strike, $X$	-	40.8	43.48
	Charge, $(T_L, T_H)$	(16531, 85155)	(4805, 17793)	(8727, 82986)
	Production, $(m_L, m_H)$	(341, 2047)	(590, 1856)	(641, 1844)
	Supplier's Profit, $\pi^s$	37308	40708	40846
	Retailer's Profit, $\pi_H^R$	541	630	572
p=0.5	Forwards, $(Q_L, Q_H)$	(540, 2047)	-	(155, 1847)
	Options, $(q_L, q_H)$	-	(847, 8000)	(794, 6153)
	Strike, $X$	-	41.4	43.18
	Charge, $(T_L, T_H)$	(24375, 83085)	(5699, 15626)	(11483, 82100)
	Production, $(m_L, m_H)$	(540, 2047)	(778, 1855)	(772, 1847)
	Supplier's Profit, $\pi^s$	27860	31268	31400
	Retailer's Profit, $\pi_H^R$	2611	1569	1640

**Figure 1:  
Comparing Option Contracts with Forward Contracts**

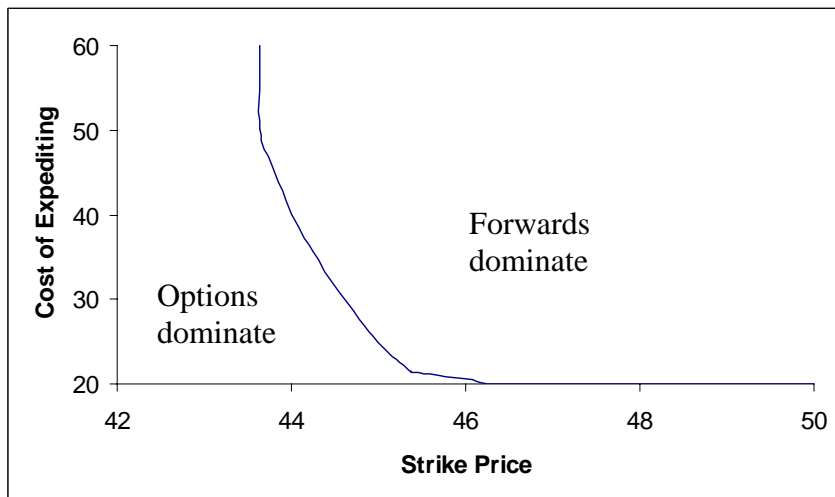
The top panel shows the bounds on supplier profits using Options and using Forwards. The solid lines represent the bounds on supplier profits using options. The upper bound represents the case where the cost of expediting is at its lower bound, while the lower bound represents the case where no expediting can take place. The dashed line represents profits from using forward contracts, and is independent of the strike price and expediting cost.

The lower panel shows the curve that separates out the regions where Option Policies and Forward Policies are Optimal. Below the curve is the region where option contracts dominate forwards. Above the curve is the region where forwards dominate options.

**Bounds on Supplier Profits Using Options and Profits Using Forwards**

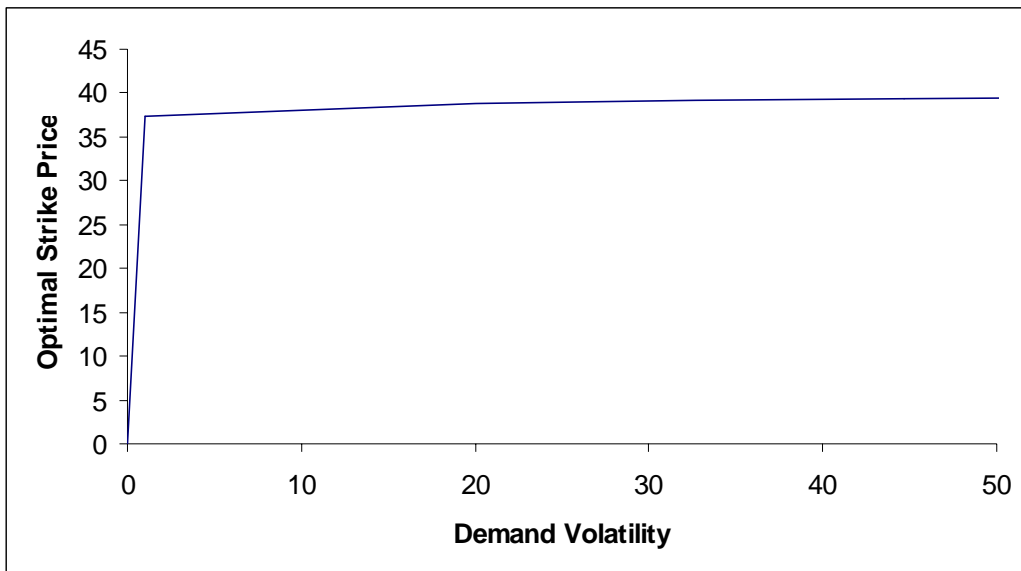
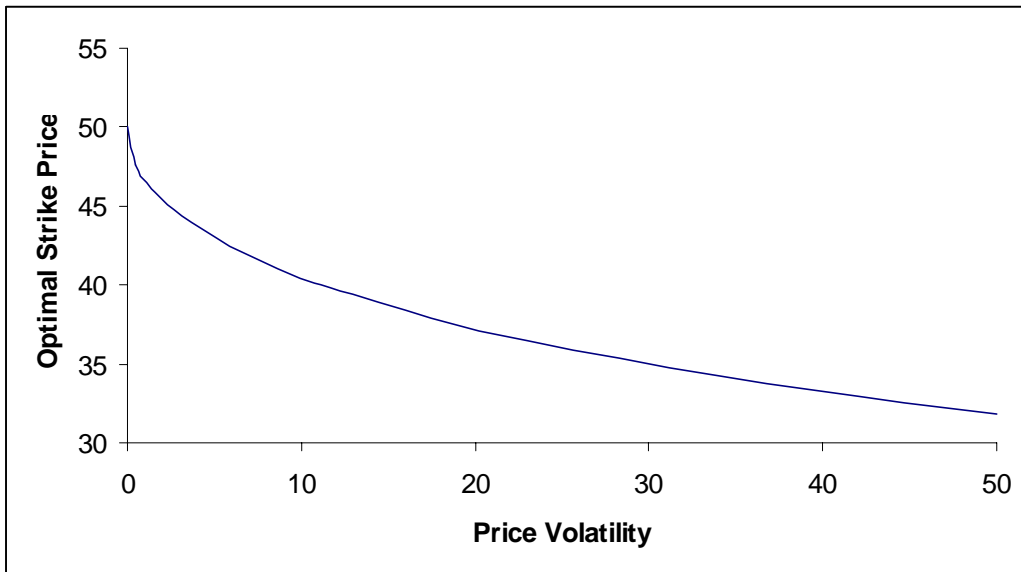


**Optimal Regions for Option and Forward Policies**



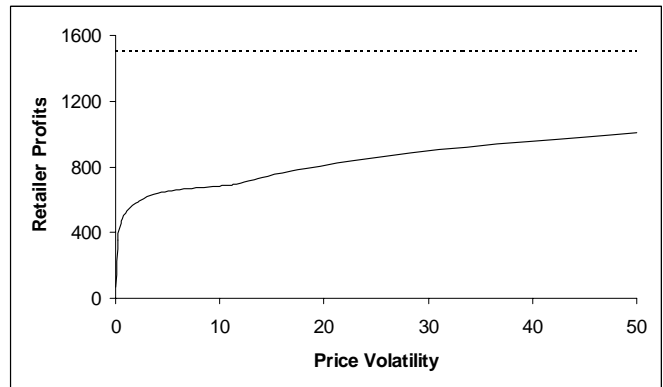
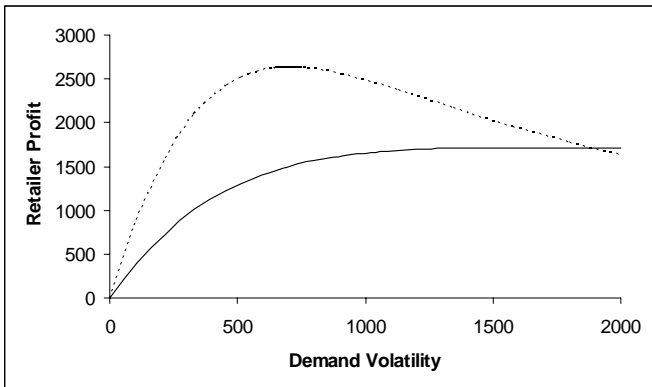
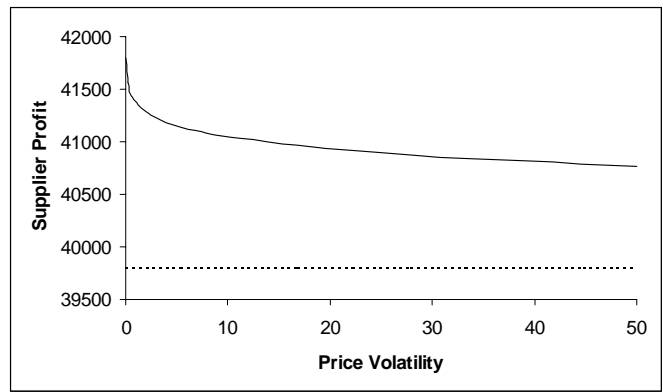
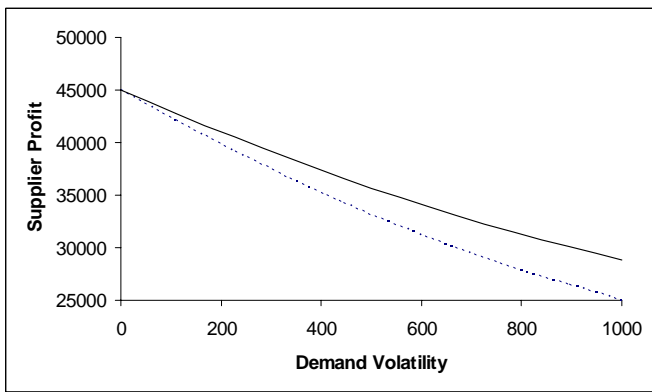
**Figure 2**  
**Behavior of Optimal Strike Price with Respect to Price Volatility**

The top panel shows the optimal strike for various levels of price volatility. The bottom panel shows the optimal strike for various levels of demand volatility. The case parameters are discussed in the text.



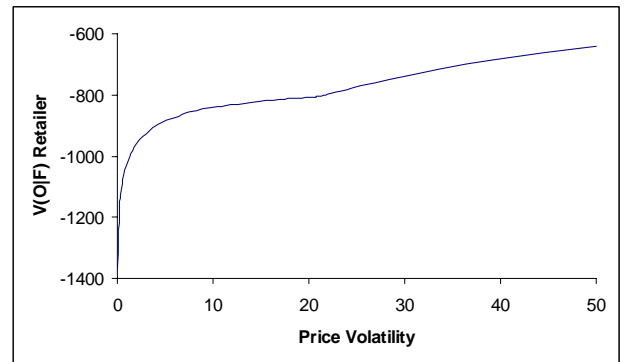
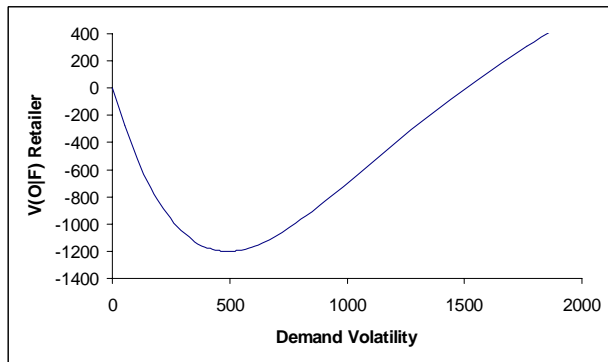
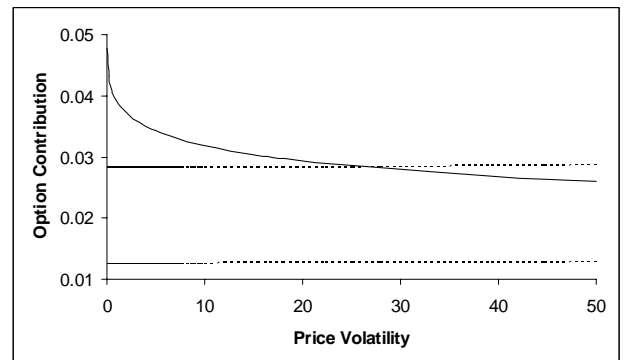
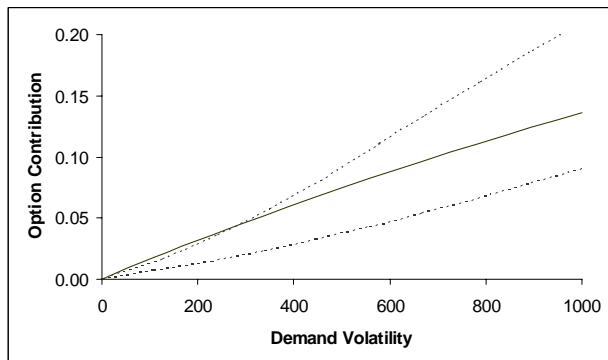
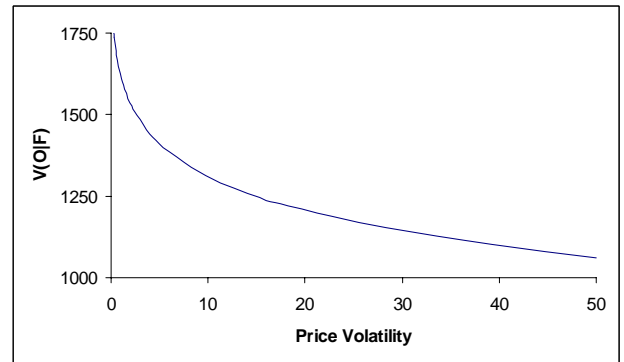
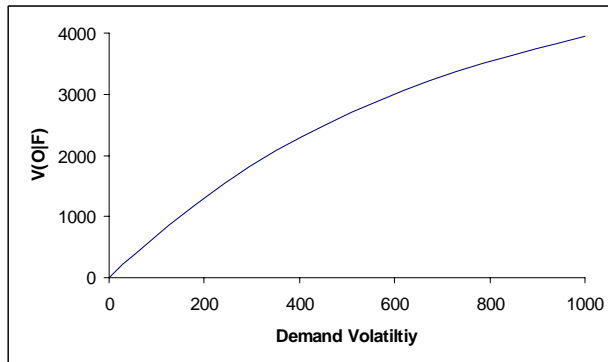
**Figure 3:  
Profit for Supplier and Retailer using Options and using Forwards**

The top panel compares the profits for the supplier when options are used alone versus forwards alone. The bottom panel compares the profits for the retailer when the supplier offers options alone and forwards alone. The profits using options (forwards) are indicated by solid (dashed) lines. The left panel shows the behavior with respect to demand volatility, while the right panel shows the behavior with respect to price volatility



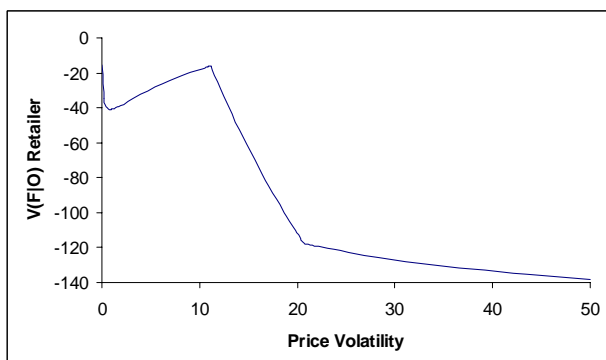
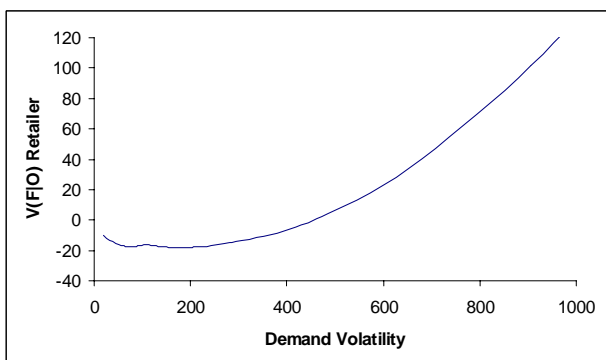
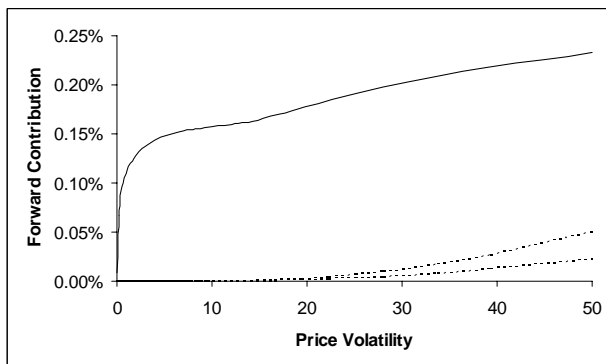
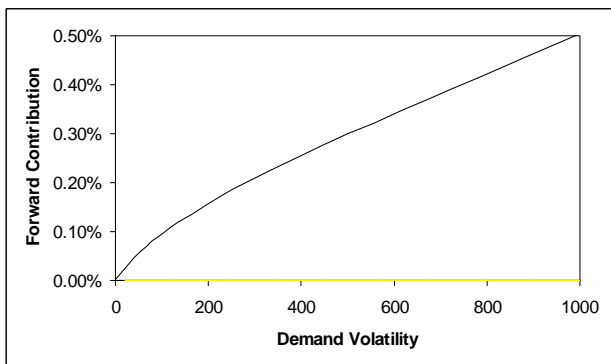
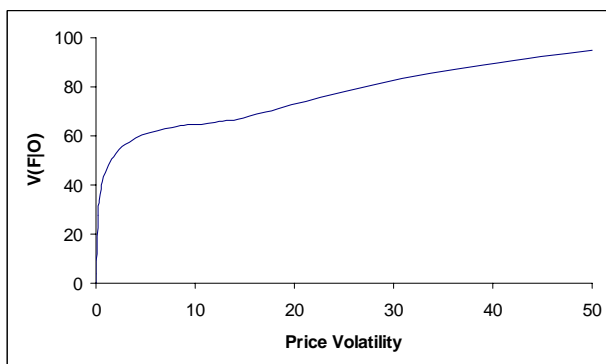
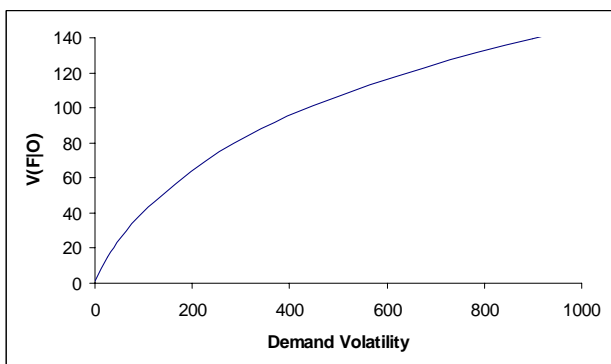
**Figure 4:  
Valuation of the Option Program**

The top panel shows how the value of the option program to the supplier, over and above the forward program, behaves as volatility of demand and prices change. The second panel shows how the contribution of options to total supplier profits changes. The dashed lines in the second panel show the relative contributions of the option program when there is no asymmetric information. The higher (lower) dashed lines correspond to the case where  $p=1$  ( $p=0$ ). The bottom panel shows the impact of the option program to the retailer.



**Figure 5:  
Valuation of the Forward Program**

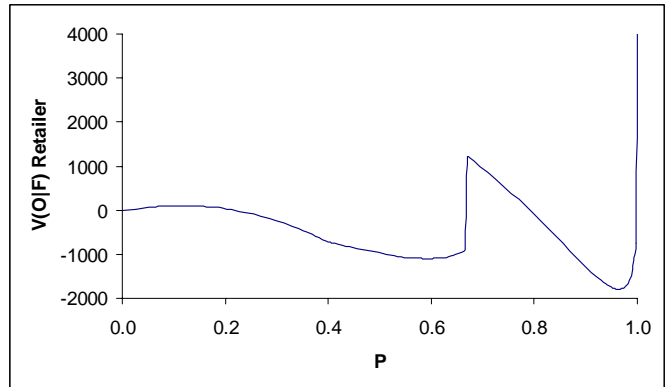
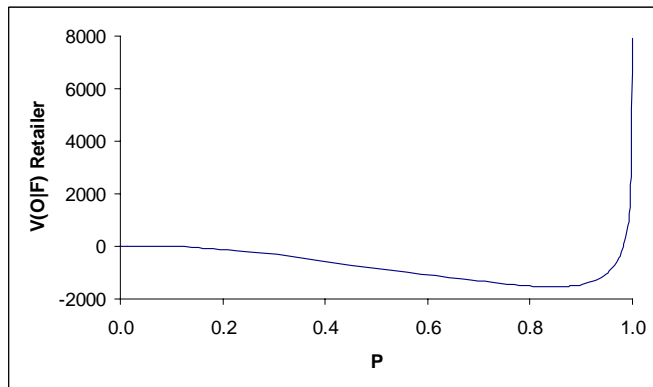
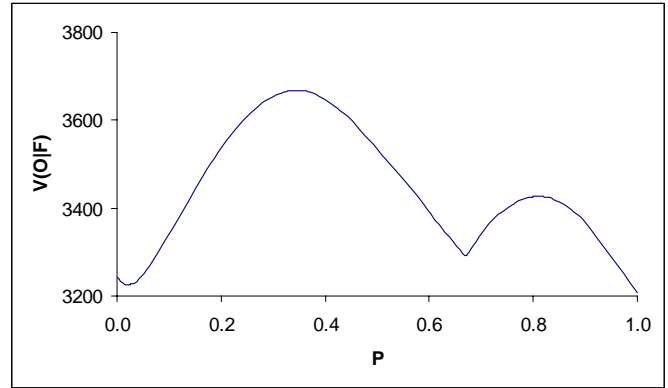
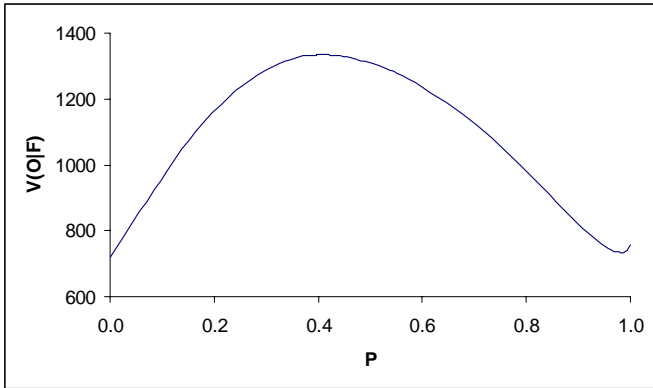
The top panel shows how the value of the forward program to the supplier, over and above the option program, behaves as volatility of demand and prices change. The second panel shows how the contribution of forwards to total supplier profits changes. The dashed lines in the second panel show the relative contributions of the option program when there is no asymmetric information. The higher (lower) dashed lines correspond to the case where  $p=1$  ( $p=0$ ). For the plot against demand volatility, the contributions of forward contracts with perfect information were negligible, and are not shown. The bottom panel shows the impact of the forward program to the retailer.





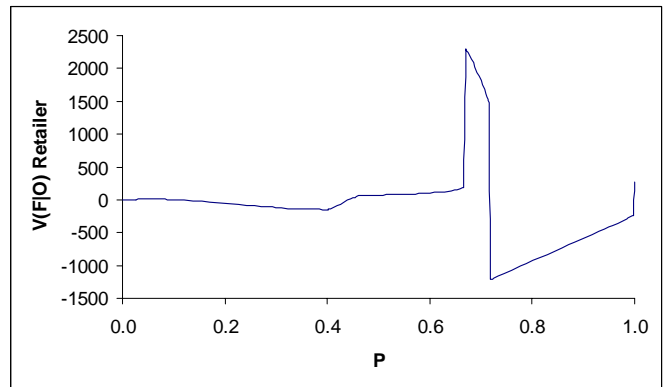
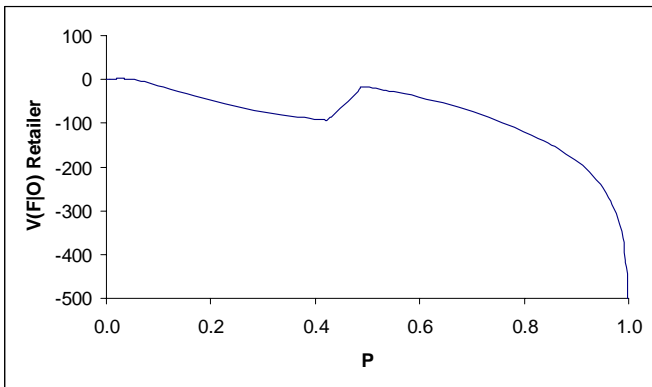
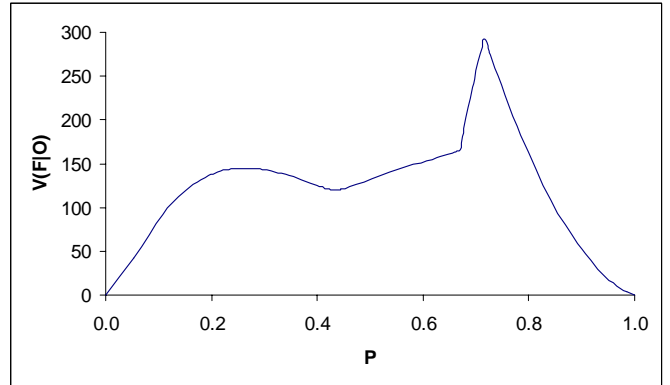
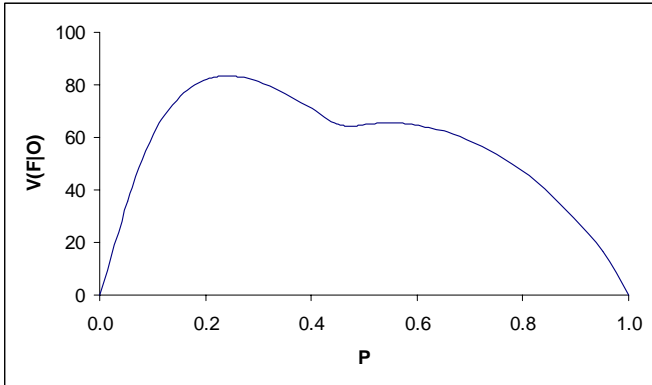
**Figure 6:**  
**Value of Option Programs and Asymmetric Information**

The top panel shows the value of the option program, over and above that of forwards, for the supplier and the bottom shows the values for the retailer. The left panel shows the results when demand volatility is low, while the right panel repeats the plots when demand volatility is high. The case parameters are discussed in the text.



**Figure 7:**  
**Value of Forward Programs and Asymmetric Information**

The top panel shows the value of the forward program, over and above that of options, for the supplier and the bottom panel shows the value for the retailer. The left panel shows the results when demand volatility is low, while the right panel repeats the plots when demand volatility is high. The case parameters are discussed in the text.



**Figure 8:  
Expected Value of Perfect Information**

The top figure plots out the Expected value of Perfect Information for the Supplier as a function of  $p$ . The EVPI is shown for three curves, namely for options only, forwards only and for both. Since the curves intersect at several places, the bottom panel shows the differences between the expected values of perfect information for the three contracts.

