Technical Memorandum Number 798

Component Procurement Strategies in Decentralized Assembled-to-Order Systems with Leadtime-Dependent Product Pricing

by

Xiang Fang
Yunzeng Wang
Rick So

July 2005

Department of Operations
Weatherhead School of Management
Case Western Reserve University
330 Peter B Lewis Building
Cleveland, Ohio 44106

Component Procurement Strategies in Decentralized Assemble-to-Order Systems with Leadtime-Dependent Product Pricing

Xiang Fang
Weatherhead School of Management, Case Western Reserve University

Kut C. So
The Paul Merage School of Business, University of California, Irvine, and School of Business, National University of Singapore

and

Yunzeng Wang
Weatherhead School of Management, Case Western Reserve University

Abstract

We consider a manufacturer who procures components from multiple suppliers to produce an assemble-to-order customized product for a client. The product pricing depends on the manufacturer's delivery lead times. We explore how the manufacturer can use a revenue-sharing scheme to manage the underlying risk and to coordinate independent suppliers' decisions on the production quantity of their components when facing demand uncertainty and long procurement lead times. We formulate the problem as a Stackelberg game played by the manufacturer against her multiple suppliers to determine her optimal revenue-sharing scheme. This Stackelberg game consists of a sub-game played by the component suppliers against each other to choose their individual production quantities. We show that while this sub-game may have numerous equilibria, there exists a unique one that is Pareto-optimal, and develop an efficient algorithm for finding this Pareto-optimal equilibrium. From this result, we derive the optimal revenue-sharing scheme that maximizes the expected profit of the manufacturer. Our results provide useful insights into key managerial issues for managing components in these types of assemble-to-order environments and for understanding how demand uncertainty and component procurement lead times affect individual firms' performance in decentralized assembly channels.

Keywords: Assemble-To-Order Manufacturing; Optimal Inventory Policies; Lead Time Management; Vendor Managed Inventory; Pricing Strategies

June 21, 2005

1 Introduction

Global outsourcing is becoming a common business practice. Many firms, especially in the computers and consumer electronic products industries, are increasingly relying on contract manufacturers in low-cost economies to manufacture their products to meet demands in a global market. At the same time, the business of contract manufacturing is becoming increasingly competitive. Contract manufacturers need to deliver their products in the most cost-effective and timely manner to compete and achieve profitability. Effective management of component inventory is an important success factor for these contract manufacturers, as cost of components constitutes a significant portion of their total product cost and component availability determines the delivery time performance. This paper studies component procurement strategies for contract manufacturers in an assemble-to-order environment.

In rapidly changing high technology industries where competition is fierce, firms must introduce new products in a faster pace to stay ahead. Contract manufacturers for such industries must be able to react to a dynamic market and deliver products in a timely manner to meet their clients' needs. Stocking components ahead of an actual order commitment is a commonly used strategy to achieve timely delivery where the procurement lead times for these components are long. However, this strategy can pose significant risk to contract manufacturers as component obsolescence is common for high technology products. Excessive components could be subject to severe price erosion when their clients' order falls short of expectations. Faced with demand uncertainty, stringent delivery performance and long procurement lead times, contract manufacturers often have to think strategically and rely on contractual arrangements for managing their component inventory.

In this paper we develop a model for analyzing the optimal procurement strategies for a contract manufacturer who produces an assemble-to-order customized product for her client. The objective of the contract manufacturer is to devise a revenue-sharing scheme with the component suppliers for managing the required components to fulfill an anticipated future order from a major client, where the specific order quantity is uncertain and can only be confirmed by the client at some future time point. Timely delivery is essential, and any delay of a shipment will reduce the unit price of the assembled product that the contract manufacturer will receive, i.e., the longer the delay, the lower the unit product price. This assemble-to-order product requires multiple components, each produced by an independent outside supplier with a different lead time. The contract manufacturer can stock the components before the final order quantity is confirmed; however, any excess components are subject to obsolescence.

A recent paper by Hsu, et al. (2005a) analyzes a similar manufacturing situation and determines the optimal component stocking quantities for the contract manufacturer. In this paper we consider a different component procurement strategy where the contract manufacturer manages the underlying risk of overstocking the components using a vendor-managed-inventory (VMI) contract with the different component suppliers. Specifically, the manufacturer will specify a revenue-sharing scheme with all her component suppliers, and each supplier then makes his own decision in stocking the required component before the actual order quantity is confirmed. Thus, under this contractual arrangement, each supplier needs to manage his own risk of stocking excessive components, taken into account the revenue-sharing scheme and the stocking quantity of other component suppliers.

We formulate the above situation as a two-stage optimization problem. Using a game-theoretical framework, we first analyze the equilibrium component stocking quantities of the suppliers for any revenue-sharing scheme chosen by the contract manufacturer (referred as the assembler in our subsequent discussions). We call this the suppliers problem. We derive sufficient conditions for the Nash equilibrium solutions and show that there could exist multiple Nash equilibria for the suppliers problem. We show that among all these Nash equilibria, however, there always exists a unique one that is strictly Pareto-optimal. We further devise an efficient algorithm for finding this Pareto-optimal equilibrium. From the result of the suppliers problem, we then analyze the optimal revenue-sharing scheme that maximizes the expected profit of the assembler. We call this the assembler problem. We show, under a mild condition on the demand distribution function, that there exists a unique optimal solution to the assembler problem. We develop a simple algorithm to find this unique optimal revenue-sharing scheme. Finally, we derive some sensitivity results that illustrate how different cost parameters of the model can affect the optimal solution of the assembler problem and the corresponding equilibrium solution of the suppliers problem.

Our results provide several important insights on how demand uncertainty and component procurement lead times affect individual firms' performance in decentralized assembly channels and how to improve their performance. Specifically, we observe that a lead time reduction of any individual supplier, while beneficial to the assembler and the overall system, does not increase the expected profit of this individual supplier. Similarly, a cost reduction of the component of any individual supplier, again while beneficial to the assembler and the system, does not necessarily increase the expected profit of this individual supplier. These two results imply that there is no incentive for any individual supplier to unilaterally reduce his lead time and component cost. In addition, we observe that increased demand uncertainty will benefit the component suppliers while hurting the assembler and the overall system. Also, the higher the demand uncertainty, the more the assembler prefers a centralized system. These two observations suggest that while the objective of supplier-managed inventory is to allow the assembler to share the underlying risk of the system due to demand uncertainty, it is most beneficial to the assembler to reduce the demand uncertainty so as to maximize her expected profit under a supplier-managed inventory system.

The paper is organized as follows. In Section 2 we provide a literature review of related research. In Section 3 we describe the basic model setting and the revenue-sharing scheme used by the assembler to manage component inventory from different suppliers. In Section 4 we formulate and analyze the suppliers problem. Specifically, we show that there always exists a unique Pareto-optimal Nash equilibrium for the suppliers problem and provide an efficient algorithm for finding this equilibrium. In Section 5 we formulate and analyze the assembler problem. We characterize the optimal revenue-sharing scheme and devise a simple algorithm for finding the optimal solution. In Section 6 we derive some sensitivity results that illustrate how the component cost and product price can affect the optimal solution. We further provide some numerical results to illustrate how demand variability and lead time can affect the system and individual firms' performance. In Section 7 we give some comparative results between our supplier managed inventory (decentralized) system with that of a centralized system where the assembler also determines the component stocking quantities. We conclude our results in Section 8. All mathematical proofs are given in the Appendix.

2 Literature Review

There exists a substantial body of recent research on assemble-to-order (ATO) systems, which have been widely adopted in practice to achieve mass customization. Key basic research issues on ATO systems include the optimal component inventory policy and optimal component allocation policy among various final products. Song and Zipkin (2003) present a general formulation of ATO systems and provide a comprehensive survey on recent research on ATO systems.

Our research addresses the component stocking policy in a single-order assembly system with product pricing dependent on delivery times. A number of research papers have studied the optimal component stocking policies for single-order assembly systems, including Chu, et al. (1993), Fu, et al. (2005), Gurnani, et al. (1996), Hopp and Spearman (1993), Kumar (1989), Shore (1995), Song, et al. (2000), Gerchak et al. (1994), and Yano (1987). Most recently, Hsu, et al. (2005a) analyzes a single-order assembly system with a delivery time-dependent pricing structure. They characterize the structure of the optimal component procurement policy and provide an efficient algorithm for finding the optimal solution. In that paper, the assembler is allowed to deliver the total order quantity in multiple partial shipments. Hsu, et. al (2005b) later extend their earlier model to the case where only one single full shipment is allowed.

All papers referenced above can be considered as a centralized decision making by the assembler. More closely related to this paper, however, is a recent body of literature studying decentralized assembly systems, where the focus is on the contractual arrangements between the assembler and her multiple component suppliers. Similar to our model here, Gerchak and Wang (2004), Bernstein and DeCroix (2004a), Granot and Yin (2004), and Wang (2004) all consider a setting where an assembler uses revenue-sharing contracts to induce independent component suppliers to choose their individual production quantities. However, all of the above papers assume that there is only one chance to order the components and fill the demand under which all suppliers will produce the same quantity in equilibrium – a fact that dramatically simplifies the decision of the underlying problems. In contrast, our model allows for multiple deliveries of the final product such that different suppliers, with different component production lead times, may choose to produce different quantities. Other recent papers studying decentralized assembly systems include Wang and Gerchak (2003), Bernstein and DeCroix (2004b), Tomlin (2003), Gurnani and Gerchak (1998), Carr and Karmarkar (2005), and Zhang (2004).

3 The Basic Model

Consider a contract assembler who is anticipating a future order for an assemble-to-order customized product. The specific order size, denoted by D, is uncertain and can only be confirmed by the client at some future time point, denoted by t^0. Let $f(.)$ and $F(.)$ be the probability density function and cumulative distribution function of D, respectively. After the order size is revealed at time t^0, the assembler can deliver the total order quantity in multiple shipments. However, the assembler will receive a different unit price for the products in each shipment depending on the delay of a shipment; the longer the delay, the lower the unit price.

The final product consists of n different components, each produced by an independent supplier at a constant cost of c_i, $i = 1, 2, .., n$. The assembler uses a vendor-managed-inventory (VMI) type of contract with revenue-sharing to coordinate the component production of the n suppliers. Under the contract, the assembler first offers a scheme for sharing the sales revenue of the product, and the suppliers then choose their individual production quantities of their components. In the following, we first delineate how total sales revenue can depend on the suppliers' production decisions in the system, and then specify the structure of the revenue-sharing scheme.

Facing uncertain demand before time t^0, supplier i, $i = 1, 2..., n$, needs to choose an initial production quantity Q_i. Assume that all suppliers have enough time and capacity to produce this initial quantity before time t^0. At time t^0, demand D for the final product is realized. Based on the initial production quantity of each component Q_i and the realized demand D, the assembler will assemble and deliver $\min\{\min_i(Q_i), D\}$ units of the product to her client. Without loss of generality, assume that the assembly time is negligible, so this initial shipment will be delivered to the client at time t^0.

If the initial shipment is not enough to fully satisfy the realized demand D due to shortages of one or more of the components, then those suppliers with shortage of components will initiate a second batch of production at time t^0 to make up the difference. In particular, supplier i will produce $(D - Q_i)^+$ units of component i, where $(x)^+$ denotes $\max(x, 0)$. An important feature of our problem is that the second batch production of components experiences a lead time that is different across the multiple component suppliers. Let L_i be the delivery lead time for supplier i, which includes the transportation time for shipping the components to the assembler. Number the suppliers such that $L_1 \leq L_2 \leq ... \leq L_n$. Define $L_0 = 0$.

Upon receiving subsequent batches of components from the suppliers, the assembler will immediately assemble them into final products and make deliveries to the client, so as to capture the highest unit price possible. For example, suppose that supplier 1 chose an initial production quantity of Q_1 units with $Q_1 < \min_{i>1}(Q_i)$ and $Q_1 < D$. Then, the assembler is able to assemble and deliver Q_1 units of the products to the client at time t^0. Also, the assembler will receive a second batch of $(D-Q_1)$ units of components from supplier 1 at time $t^0 + L_1$, and immediately assemble and deliver an additional $\min\{D - Q_1, \min_{i>1}(Q_i) - Q_1\}$ units of the product to the client. Therefore, there are at most $(n+1)$ possible delivery epochs for the products, namely, $t^0, t^0 + L_1, ..., t^0 + L_n$. Let $P^0, P^1, ..., P^n$ be the corresponding unit product prices received by the assembler. Since $0 \leq L_1 \leq L_2 \leq ... \leq L_n$, we have $P^0 \geq P^1 \geq ... \geq P^n$, indicating that a longer delivery time results in a lower unit price for the product.

The assembler uses the following revenue-sharing scheme to manage component production with her n component suppliers: For each unit of the product delivered at time $t^0 + L_t$ capturing a unit price of P^t, $t = 0, 1, ..., n$, the assembler will allocate P_i^t to supplier i and keep the remaining amount of $(P^t - \sum_{i=1}^n P_i^t)$ for herself. Thus, a revenue-sharing scheme is fully specified by the revenue-sharing matrix $\{P_i^t\}_{i=1,2,...,n}^{t=0,1,...,n}$.

To simplify our exposition, we next make several simplifying assumptions in the basic model. We point out, however, that all these assumptions can be relaxed without affecting the structure of the solution. First, assume that the unit production cost c_i is constant and independent of the production timing. This implies that when a unit of a component is to be produced, producing it early is most beneficial to the assembler as the unit product price is decreasing in delivery times. To induce each supplier to produce early rather than late, the assembler would therefore set P_i^t to be decreasing in t for all $i = 1, 2, ..., n$. Furthermore, each supplier i needs to at least recoup his unit production cost c_i. Thus, any feasible revenue-sharing contract needs to satisfy

$$P_i^0 \geq P_i^1 \geq ... \geq P_i^n \geq c_i. \tag{1}$$

Also, we assume that the product price of the last possible delivery at time $t^0 + L_n$ is higher than the total cost of components, i.e., $P^n > \sum_{i=1}^n c_i$, so that the assembler will always choose to deliver the full order quantity D. Furthermore, assume that the assembler incurs no cost for assembling and delivering the final product to the client, and there is no salvage value for excess components produced.

The sequence of the overall decision-making process is modeled as a Stackelberg leader-followers' game: The assembler, acting as the leader, moves first to unilaterally choose and announce the revenue-sharing scheme, i.e., the values of $\{P_i^t\}_{i=1,2,...,n}^{t=0,1,...,n}$. The n component suppliers, as the followers, then simultaneously choose their initial production quantities $\{Q_i\}_{i=1,2,...,n}$. We assume that all information about demand distribution, product prices and costs is common knowledge to all parties.

For a given revenue-sharing scheme chosen by the assembler, the profit of any one supplier depends on his own production quantity *as well as* the production quantities of all other suppliers. Thus, for a given revenue-sharing scheme, the decisions of the n suppliers constitute a gaming problem as well, which we refer to as the suppliers problem. Following the backward induction, we first analyze the suppliers problem in Section 4.

4 The Suppliers Problem

For a given revenue-sharing scheme $\{P_i^t\}_{i=1,2,...,n}^{t=0,1,...,n}$, the n suppliers simultaneously choose the initial production quantities $(Q_1, ..., Q_n)$ of their individual components. Our first goal is to characterize the structure of the equilibrium production quantities of the n suppliers. For notational convenience, define $Q_0 = 0$ and $Q_{-i} = (Q_1, ..., Q_n)$ with the element Q_i omitted. For any given production quantities Q_{-i} chosen by all other suppliers, supplier i chooses his production quantity Q_i so as to maximize his expected profit $\Pi_i(Q_i|Q_{-i})$, which can be written as

$$\Pi_i(Q_i|Q_{-i}) = \sum_{k=0}^{n-1} P_i^k E[\min(Q_{k+1}, ..., Q_n, D) - Q_k]^+ + P_i^n E[D - Q_n]^+ - c_i E[\max(Q_i, D)]. \tag{2}$$

We first establish the following result:

Proposition 1: *Given any production quantities Q_{-i} chosen by all other suppliers, the optimal production quantity of supplier i, denoted by Q_i^*, satisfies the condition that* $Q_i^* \leq \min\{Q_{i+1}, ..., Q_n\}$.

Recall that the suppliers are indexed according to their production lead times such that $L_1 \leq L_2 \leq ... \leq L_n$. Proposition 1 states that a supplier will never choose to produce more than any other supplier who has a longer production lead time than his own. This can be intuitively explained by considering two suppliers with $L_1 < L_2$. Suppose that supplier 2 has chosen a specific production quantity Q_2. Then, for supplier 1, choosing a higher production quantity $Q_1 > Q_2$ will obviously hurt himself *if* the demand D is less than Q_2. Even if the demand D is higher than Q_2, choosing $Q_1 > Q_2$ is dominated by choosing $Q_1 = Q_2$ for supplier 1, as both decisions generate the same revenue of $P_1^0 Q_2 + P_1^2(D - Q_2)$ due to the fact that supplier 2 has a longer lead time, and yet the decision of $Q_1 > Q_2$ could lead to overstocking when $Q_1 > D > Q_2$.

A direct implication of Proposition 1 is that in any equilibrium, suppliers' production quantities are ordered according to their lead times. We describe this property formally as the following Corollary:

Corollary 1: *Let $(Q_1^*, Q_2^*, ..., Q_n^*)$ denote any Nash equilibrium solution of the n suppliers' production quantities. Then,*

$$Q_1^* \leq Q_2^* \leq ... \leq Q_n^*. \tag{3}$$

From Corollary 1, we can restrict our search for the equilibrium production quantities to those satisfying (3). Given any production quantities Q_{-i} chosen by all other suppliers such that $Q_1 \leq ... \leq Q_{i-1} \leq Q_{i+1} \leq Q_n$, it follows from Proposition 1 that the expected profit of supplier i given by (2) for $Q_i \leq Q_{i+1}$ reduces to

$$\begin{aligned}\Pi_i(Q_i|Q_{-i}) \;\; = \;\; & \sum_{k=0}^{i-2} P_i^k E[\min(Q_{k+1}, Q_i, D) - Q_k]^+ + P_i^{i-1} E[\min(Q_i, D) - Q_{i-1}]^+ \\ & + P_i^i E[\min(Q_{i+1}, D) - Q_i]^+ + \sum_{k=i+1}^{n-1} P_i^k E[\min(Q_{k+1}, D) - Q_k]^+ \\ & + P_i^n E[D - Q_n]^+ - c_i E[\max(Q_i, D)],\end{aligned}$$

which can be expressed as

$$\Pi_i(Q_i|Q_{-i}) = \begin{cases} G_i^j(Q_i) + K_i^j(Q_{-i}), & Q_j \leq Q_i \leq Q_{j+1}, \; j = 0, 1, ..., i-2 \\ G_i^{i-1}(Q_i) + K_i^{i-1}(Q_{-i}), & Q_{i-1} \leq Q_i \leq Q_{i+1} \end{cases} \tag{4}$$

where $Q_0 \equiv 0$,

$$G_i^j(Q_i) = (P_i^j - P_i^i)\int_0^{Q_i} \bar{F}(x)dx - c_i \int_0^{Q_i} F(x)dx, \tag{5}$$

and

$$\begin{aligned} K_i^j(Q_{-i}) &= \sum_{k=0}^{j-1} P_i^k E[\min(Q_{k+1}, D) - Q_k]^+ + \sum_{k=i+1}^{n-1} P_i^k E[\min(Q_{k+1}, D) - Q_k]^+ \\ &\quad + P_i^n E(D - Q_n)^+ - P_i^j \int_0^{Q_j} \bar{F}(x)dx + P_i^i \int_0^{Q_{i+1}} \bar{F}(x)dx - c_i E(D), \end{aligned}$$

for all $j = 0, 1, 2, ..., i-1$. It is important to note that $G_i^j(Q_i)$ is a function of Q_i only, and $K_i^j(Q_{-i})$ is a function of Q_{-i} only and is independent of Q_i.

We next provide some properties of the profit function of supplier i:

Lemma 1: *(a) The functions $G_i^j(Q_i)$ given in (5) is strictly concave in Q_i and reaches its maximum at*

$$q_i^j = \bar{F}^{-1}\Big(\frac{c_i}{P_i^j - P_i^i + c_i}\Big). \tag{6}$$

Furthermore, q_i^j is decreasing in j (j=0,1,...,i-1) for each $i = 1, 2, ..., n$.
(b) The profit function $\Pi_i(Q_i|Q_{-i})$ given in (4) is continuous and strictly concave in Q_i for $0 \le Q_i \le Q_{i+1}$.

The shape of $\Pi_i(Q_i|Q_{-i})$, as depicted by Lemma 1, is illustrated in Figure 1. The results of Lemma 1 imply that the optimal Q_i^* must be equal to either one of the q_i^k that maximizes $G_i^k(Q_i)+K_i^k(Q_{-i})$, $k = 0, 1, 2, ..., i-1$, or one of the break points $Q_i = Q_j$ for $j = 1, 2, \ldots, i-1$ and Q_{i+1}. Specifically, we can characterize Q_i^* in the following proposition:

Proposition 2: *Given the production quantities Q_{-i} chosen by all other suppliers with $Q_1 \le \ldots \le Q_{i-1} \le Q_{i+1} \le \ldots \le Q_n$, the optimal production quantity of supplier i is given by*

$$Q_i^* = \min_{0\le j \le i-1} \Big\{\max(Q_j, q_i^j), Q_{i+1}\Big\} \tag{7}$$

where $Q_0 \equiv 0$ and $Q_{n+1} \equiv \infty$.

To summarize, equations (3) and (7) provide the necessary and sufficient conditions for which any Nash equilibrium solution of the n-suppliers' production quantities must satisfy. We next analyze the equilibrium solutions of the n-supplier problem by solving (3) and (7).

4.1 Equilibrium Solutions of the n-Supplier Problem

We first provide some specific conditions under which a unique Nash equilibrium solution of the n-supplier problem exists.

Theorem 1: *Suppose that* $q_i^{i-1} < q_{i+1}^i$ *for all* $i = 1, \ldots, n-1$*. Then, the production quantities* $Q_i = q_i^{i-1}$*,* $i = 1, \ldots, n$*, constitute the unique Nash equilibrium solution of the* n*-supplier problem.*

Theorem 1 provides the sufficient condition that guarantees the existence of a unique equilibrium where each supplier i chooses the production quantity of q_i^{i-1}. In general, the n-supplier problem is much more complex to analyze when this sufficient condition is violated. In particular, the game may have numerous equilibria. We shall construct a procedure for finding a unique equilibrium that is Pareto-optimal for the general case. To that end, we first present the following result, which characterizes the structure of any equilibrium solution when the condition in Theorem 1 fails to hold.

Lemma 2: *Suppose that* $q_i^{i-1} \geq q_{i+1}^i$ *for some* $i = 1, \ldots, n-1$*. If* $(\tilde{Q}_1, \ldots, \tilde{Q}_n)$ *is a Nash equilibrium solution, then* $\tilde{Q}_i = \tilde{Q}_{i+1}$*.*

Lemma 2 shows that when the condition in Theorem 1 fails to hold, some immediate neighbor suppliers will form "clusters" in equilibria, with all members in the same cluster having the same production quantity. We can describe such a form of an equilibrium as an m-cluster solution, where the n suppliers are partitioned into m clusters, $m \leq n$. Let l_i and r_i denote respectively the indexes of the first and last suppliers in cluster i, such that $l_1 = 1$, $r_m = n$ and $r_{i-1} = l_i - 1$. Thus, the original n suppliers can now be represented by m clusters as $\{(l_1, r_1), (l_2, r_2), \ldots, (l_m, r_m)\}$. Also, we use subscript $[i]$ to indicate a quantity that is common for all members in cluster i. For example, we use $Q_{[i]}$ to denote the production quantity of all suppliers in cluster i, i.e., $Q_j = Q_{[i]}$ for $l_i \leq j \leq r_i$.

We next present a simple algorithm (Algorithm 1) for finding a *unique* m-cluster solution to the n-supplier problem. The algorithm starts by treating each of the n suppliers as a separate cluster. At each iteration, it merges two neighboring clusters into one single cluster until a unique solution is reached. Later in Theorem 2, we provide a constructive proof to show that the unique solution found through this algorithm corresponds to an *equilibrium point* of the n-supplier game problem. Furthermore, this equilibrium point is Pareto-optimal among all possible equilibria of the n-supplier problem. The basic idea for the proof of Theorem 2 is as follows. We first show that every equilibrium point of the m-cluster system formed at each iteration of Algorithm 1 corresponds to an equilibrium point of the original n-supplier problem. We then show that every equilibrium point of the original system not contained in the equilibrium set of the m-cluster system is dominated by an equilibrium point of the m-cluster system in terms of the expected profit of each individual supplier. Since the algorithm always ends up with an m-cluster system having a unique equilibrium solution, this unique equilibrium point must dominate all other equilibria of the original system.

Algorithm 1 (Finding the unique Pareto-optimal Nash equilibrium of the n-supplier problem)
Step 0: (Initialization) Compute $q_i^j = \bar{F}^{-1}\left(\frac{c_i}{P_i^j - P_i^i + c_i}\right)$ *for all* $i = 1, \ldots, n$ *and* $j = 0, \ldots, i-1$. *Let* $m = n$ *and* $l_i = i, r_i = i$ *for all* $i = 1, ..., n$. *Set* $r_0 = 0$.

Step 1: For all $i = 1, \ldots, m$, *compute*

$$q_{[i]}^{i-1} = \min_{l_i \le a \le r_i} \{q_a^{r_{i-1}}\} \tag{8}$$

If $q_{[i]}^{i-1} < q_{[i+1]}^{i}$ *for all* $i = 1, \ldots, m-1$, *then* $Q_{[i]} = q_{[i]}^{i-1}$ *is the* m*-cluster equilibrium solution of the* n*-supplier problem and stop. Otherwise, go to step 2.*

Step 2: Let k *be the smallest index* i, *such that* $q_{[i]}^{i-1} \ge q_{[i+1]}^{i}$ *and merge the corresponding clusters* k *and* $(k+1)$ *into a single cluster, i.e., set*

$$\begin{aligned} &r_k \leftarrow r_{k+1} \\ &l_j \leftarrow l_{j+1}, r_j \leftarrow r_{j+1}, \quad j = k+1, \ldots, m-1 \\ &m \leftarrow m-1. \end{aligned}$$

Go back to step 1.

Theorem 2: *If* $q_i^{i-1} < q_{i+1}^{i}$ *for all* $i = 1, \ldots, n-1$, *then Algorithm 1 finds the unique Nash equilibrium solution as described in Theorem 1. Otherwise, Algorithm 1 finds a unique Pareto-optimal Nash equilibrium for the* n*-supplier problem.*

The unique equilibrium found through Algorithm 1 is Pareto-optimal, i.e., this equilibrium maximizes the expected profit of *every* supplier among all possible equilibria. Consequently, we shall consider this equilibrium point as the outcome of the suppliers problem for the remainder of our analysis in this paper.

5 The Assembler Problem

The assembler problem is to choose the optimal revenue-sharing scheme that maximizes her own expected profit, assuming that the n suppliers will respond with their production quantities as given by the equilibrium solution of Algorithm 1. Let $\bar{\mathbf{P}} = \{P_i^t\}_{i=1,\ldots,n}^{t=0,\ldots,n}$ be any assembler's revenue-sharing scheme satisfying (1) and $Q_i(\bar{\mathbf{P}})$ be the corresponding equilibrium production quantity of supplier i as found by Algorithm 1 in response to the revenue-sharing scheme $\bar{\mathbf{P}}$. Note that $Q_1(\bar{\mathbf{P}}) \leq \ldots \leq Q_n(\bar{\mathbf{P}})$. Define $Q_0(\bar{\mathbf{P}}) \equiv 0$ and $Q_{n+1}(\bar{\mathbf{P}}) \equiv \infty$. Then, the assembler problem is to choose the optimal values of P_i^t, for $i = 1, \ldots, n$ and $t = 0, \ldots, n$, to maximize her expected profit $\Pi_0(\bar{\mathbf{P}})$ given by

$$\Pi_0(\bar{\mathbf{P}}) = \sum_{j=0}^{n}(P^j - \sum_{i=1}^{n} P_i^j)E[\min(Q_{j+1}(\bar{\mathbf{P}}), D) - Q_j(\bar{\mathbf{P}}))]^+. \tag{9}$$

As shown in Algorithm 1, the suppliers' equilibrium production quantities $(Q_1, ..., Q_n)$ form m clusters denoted by $\{(l_1, r_1), \ldots, (l_m, r_m)\}$ where for any $l_i \leq j \leq r_i$,

$$Q_j = Q_{[i]} = q_{[i]}^{i-1} = \min_{l_i \leq a \leq r_i} (q_a^{r_i - 1}), \tag{10}$$

with $q_{[i]}^{i-1} < q_{[i+1]}^{i}$ for all $i = 1, 2, ..., m-1$. There may exist numerous revenue-sharing schemes $\bar{\mathbf{P}}$ that can result in the same equilibrium solution $(Q_{[1]}, \ldots, Q_{[m]})$ with m clusters $\{(l_1, r_1), \ldots, (l_m, r_m)\}$. The following Proposition shows that among all such feasible schemes, however, there exists a unique one that maximizes the expected profit of the assembler:

Proposition 3: *To induce a given equilibrium solution $(Q_{[1]}, \ldots, Q_{[m]})$ with m clusters $\{(l_1, r_1), \ldots, (l_m, r_m)\}$ and $0 \leq Q_{[1]} < Q_{[2]} < \ldots < Q_{[m]}$, the revenue-sharing scheme $\{P_i^t\}$ maximizing the expected profit of the assembler is given by*

$$P_k^j = \begin{cases} \frac{c_k}{\bar{F}(Q_{[i]})}, & 0 \leq j \leq l_i - 1 \\ c_k & l_i \leq j \leq n, \end{cases} \tag{11}$$

for $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$.

Proposition 3 allows us to establish a *one-to-one correspondence* between any equilibrium solution $(Q_{[1]}, \ldots, Q_{[m]})$ of the suppliers and the assembler's corresponding revenue-sharing scheme $\bar{\mathbf{P}}$, as given by (11). Therefore, determining the optimal revenue-sharing scheme that maximizes the expected profit of the assembler is equivalent to determining the optimal cluster formation $\{(l_1, r_1), ..., (l_m, r_m)\}$ and the optimal production quantities $\mathbf{Q} = (Q_{[1]}, ..., Q_{[m]})$. Using this one-to-one correspondence, we can substitute (11) into (9) and express the expected profit of the assembler as

$$\Pi_0(\mathbf{Q}) = \sum_{i=1}^{m} M_{(l_i, r_i)}(Q_{[i]}) + (P^n - \sum_{i=1}^{n} c_i)E(D) \tag{12}$$

where

$$M_{(a,b)}(Q) \equiv \left[P^{a-1} - P^b + \sum_{k=a}^{b} c_k \left(1 - \frac{1}{\bar{F}(Q)}\right)\right] \int_0^Q \bar{F}(x)dx \tag{13}$$

for any integers $1 \le a \le b \le n$. The next result provides some structural properties regarding the function $M_{(a,b)}(Q)$. We first define

$$R(Q) = \frac{f(Q)}{\bar{F}(Q)^2} \int_0^Q \bar{F}(x)dx. \tag{14}$$

Lemma 3: *Assume that $R(Q)$ is increasing in Q.*
(a) $M_{(a,b)}(Q)$ is concave and reaches its maximum at $\hat{q}_{(a,b)}$ which solves the equation

$$m_{(a,b)} + 1 = \frac{1}{\bar{F}(\hat{q}_{(a,b)})} + R(\hat{q}_{(a,b)}), \tag{15}$$

where

$$m_{(a,b)} \equiv \frac{P^{a-1} - P^b}{\sum_{k=a}^b c_k}; \tag{16}$$

(b) For any $1 \le a \le b \le n$ and $1 \le a' \le b' \le n$, $m_{(a,b)} < m_{(a',b')}$ if and only if $\hat{q}_{(a,b)} < \hat{q}_{(a',b')}$.

The requirement in Lemma 3 that $R(Q)$ is increasing in Q is a very weak condition, as first discussed by Wang and Gerchak (2003). For example, this condition is satisfied by all distributions with increasing failure rate (IFR), which include many commonly used distributions such as the Normal and Uniform distributions, as well as the Gamma and Weibull families subject to parameter restrictions (Barlow and Proschan 1965). We shall assume that $R(Q)$ is increasing in Q in our subsequent discussions, which enables us to develop a simple algorithm for finding the optimal revenue-sharing scheme by the assembler.

It is clear from the definition of $M_{(a,b)}(Q)$ that for any $1 \le i \le j < k \le n$,

$$M_{(i,j)}(Q) + M_{(j+1,k)}(Q) = M_{(i,k)}(Q).$$

Consider any cluster formation $\{(l_1, r_1), (l_2, r_2), \ldots, (l_m, r_m)\}$ and production quantities $(Q^m_{[1]}, ..., Q^m_{[m]})$ with $Q^m_{[1]} < Q^m_{[2]} < \ldots < Q^m_{[m]}$. For all $1 \le i \le m$ and $l_i \le j \le r_i$, let $Q_j = Q^m_{[i]}$. Then,

$$\sum_{i=1}^m M_{(l_i,r_i)}(Q^m_{[i]}) = \sum_{i=1}^n M_{(i,i)}(Q_i).$$

Therefore, determining the optimal cluster formation and optimal production quantity $Q^m_{[i]}$ for each cluster i to maximize $\Pi_0(\mathbf{Q})$ in (12) is equivalent to determining the optimal production quantity of each supplier. Since the second term on the right hand side of (12) is independent of the production quantities $(Q_1, ..., Q_n)$, we can write the assembler problem as

$$\max_{0 \le Q_1 \le Q_2 \le \cdots \le Q_n} \sum_{i=1}^n M_{(i,i)}(Q_i). \tag{17}$$

Assume that $R(Q)$ is increasing in Q. We can use the results in Lemma 3 to construct the following algorithm to find the assembler's optimal cluster formation and production quantities of suppliers and the corresponding optimal revenue-sharing scheme:

Algorithm 2 (Finding the assembler's optimal policy)
Step 0: (Initialization) Set $m = n$, $r_0 = 0$, and $l_i = i$, $r_i = i$ for all $i = 1, \ldots, m$.
Step 1: For all $i = 1, \ldots, m$, compute $m_{(l_i,r_i)}$ as given in (16). If $m_{(l_i,r_i)} < m_{(l_{i+1},r_{i+1})}$ for all $i = 1, \ldots, m-1$, then for all $i = 1, \ldots, m$, and $l_i \leq j \leq r_i$, set $Q_j^* = Q_{[i]}^* = \hat{q}_{(l_i,r_i)}$ as given in (15), and find the corresponding optimal revenue-sharing scheme $\{P_i^t\}$ using (11). Otherwise, go to step 2.
Step 2: Let k be the smallest index i, such that $m_{(l_i,r_i)} \geq m_{(l_{i+1},r_{i+1})}$. Merge clusters k and $(k+1)$ into a single cluster, i.e., set

$$\begin{aligned} & r_k \leftarrow r_{k+1} \\ & l_j \leftarrow l_{j+1}, r_j \leftarrow r_{j+1}, \quad j = k+1, \ldots, m-1 \\ & m \leftarrow m - 1. \end{aligned}$$

Go back to step 1.

Algorithm 2 begins by treating each of the n suppliers as a separate cluster. At each iteration, it merges two neighboring clusters into one and reduces an m-cluster system to an $(m-1)$-cluster system. The algorithm continues until the values of $m_{(l_i,r_i)}$ are strictly decreasing in the newly formed m-cluster system. Observe from (16) that the values of $m_{(l_i,r_i)}$ are given by the ratio of the price difference and the summation of the component costs in the same cluster, and are independent of the demand distribution. Hence, while the optimal production quantity for each cluster depends on the demand distribution, the optimal cluster formation is independent of the demand distribution. The next result shows that Algorithm 2 indeed finds the optimal revenue-sharing scheme for the assembler.

Theorem 3: *Algorithm 2 finds the unique optimal solution for the assembler problem (17).*

6 Sensitivity Analysis

In this section we provide some results to illustrate how the different model parameters affect the optimal solution and the profitability of the assembler and the suppliers. We first derive two analytical results that characterize the impact of the unit component production cost and the leadtime-sensitive product pricing structure. We then provide some results from our numerical experiments to illustrate the impact of demand variability on the system and to discuss their managerial implications.

Consider any equilibrium solution $\mathbf{Q} = (Q_1, Q_2, ..., Q_n)$ with the corresponding optimal revenue-sharing scheme $\{P_i^t\}$ as stipulated in Proposition 3. Substituting (11) into (2) and after some algebraic simplifications, the expected profit of supplier i, $i = 1, ..., n$, can be expressed as

$$\Pi_i(\mathbf{Q}) = \frac{c_i}{\bar{F}(Q_i)} \int_0^{Q_i} \bar{F}(x)dx - c_i Q_i. \tag{18}$$

Also, the expected profit of the assembler as given in (12) can be written as

$$\Pi_0(\mathbf{Q}) = \sum_{j=1}^{n} \left[P^{j-1} - P^j + c_j \left(1 - \frac{1}{\bar{F}(Q_j)} \right) \right] \int_0^{Q_j} \bar{F}(x)dx + (P^n - \sum_{j=1}^{n} c_j)E(D). \tag{19}$$

Hence, the total expected profit of the suppliers and assembler is given by

$$\Pi_s(\mathbf{Q}) = \sum_{j=1}^{n} \left[(P^{j-1} - P^j + c_j) \int_0^{Q_j} \bar{F}(x)dx - c_j Q_j \right] + (P^n - \sum_{j=1}^{n} c_j)E(D). \tag{20}$$

Now consider two n-component systems with product price and component cost parameters denoted by $\{\mathbf{P}, \mathbf{c}\}$ and $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$, respectively. Let $\mathbf{Q}^* = (Q_1^*, Q_2^*, \ldots, Q_n^*)$ and $\tilde{\mathbf{Q}}^* = (\tilde{Q}_1^*, \tilde{Q}_2^*, \ldots, \tilde{Q}_n^*)$ denote the optimal equilibrium production quantities of the suppliers. Similarly, Π_i^*, Π_0^* and Π_s^* denote, respectively, the optimal expected profit of each supplier i, the assembler and the channel for the system with parameters $\{\mathbf{P}, \mathbf{c}\}$, and $\tilde{\Pi}_i^*$, $\tilde{\Pi}_0^*$ and $\tilde{\Pi}_s^*$ denote the corresponding values for the system with parameters $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$. We first obtain the following results:

Proposition 4: *Suppose that $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$ is the same as $\{\mathbf{P}, \mathbf{c}\}$ except $\tilde{c}_k < c_k$ for some $1 \leq k \leq n$. Then,*
(a) $\tilde{Q}_i^* = Q_i^*$ for all i where $Q_i^* < Q_k^*$, and $\tilde{Q}_i^* \geq Q_i^*$ for all i where $Q_i^* \geq Q_k^*$;
(b) $\tilde{\Pi}_i^* = \Pi_i^*$ for all i where $Q_i^* < Q_k^*$, and $\tilde{\Pi}_i^* \geq \Pi_i^*$ for all $i \neq k$ where $Q_i^* \geq Q_k^*$;
(c) $\tilde{\Pi}_0^* > \Pi_0^*$; and
(d) $\tilde{\Pi}_s^* > \Pi_s^*$.

The results of Proposition 4 show the impact of a reduction in the component cost of supplier k. Specifically, a reduction of the component cost c_k have no impact on the equilibrium production quantity of the components or the profitability of the suppliers in the lower-indexed clusters than the one containing k. On the other hand, the reduction of the component cost c_k do not decrease, but could possibly increase the equilibrium production quantity of the components in the same cluster containing k or higher-indexed clusters as well as the expected profits of all other suppliers in these clusters. Also, a reduction of the component cost c_k increases the expected profits of the assembler as well as the overall system.

What remains unspecified in Proposition 4 is the impact of a reduction of cost c_k on the expected profit of this specific supplier k. One might expect that any supplier should benefit from the reduction of his own component cost. Surprisingly, it is not necessarily the case as illustrated by the following simple numerical example. Suppose that $n = 1$, $P^0 = 100$, $P^1 = 50$ and demand D is uniformly distributed between 0 and 1000. When $c_1 = 15$, the supplier's profit $\Pi_1^* = 5305$. When c_1 is decreased to 10, $\Pi_1^* = 5532$. However, as c_1 is further decreased to 4, $\Pi_1^* = 5227$. This result seems to be rather surprising, as it shows that while a reduction in component cost of one supplier would benefit the system as a whole, the assembler and other suppliers with longer lead times, it might not necessarily benefit this specific supplier. We can explain this observation as follows. Everything else stays the same, a lower production cost would increase his profit margin and prompt a supplier to produce a larger quantity of his components. However, the assembler, acting as the Stackelberg leader and realizing the potential increase of profit margin of the supplier, would accordingly lower the revenue share of this supplier and increase the revenue share of other suppliers with longer lead times so as to induce these suppliers to produce a higher quantity of their components to go along with the increase of the production quantity of this specific supplier. As a result, this specific supplier might not necessarily benefit, while the assembler and other suppliers with longer lead times would benefit.

The next result shows the impact of unit product price on the production quantities of the suppliers and the profitability of all the players in the system.

Proposition 5: *Suppose that* $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$ *is the same as* $\{\mathbf{P}, \mathbf{c}\}$ *except* $P^{k-1} \geq \tilde{P}^k > P^k$ *for some* $1 \leq k \leq n$*. Then,*
(a) $\tilde{Q}_i^* \leq Q_i^*$ *for all* $1 \leq i \leq k$*, and* $\tilde{Q}_i^* \geq Q_i^*$ *for all* $k+1 \leq i \leq n$*;*
(b) $\tilde{\Pi}_i^* \leq \Pi_i^*$ *for all* $1 \leq i \leq k$*, and* $\tilde{\Pi}_i^* \geq \Pi_i^*$ *for all* $k+1 \leq i \leq n$*; and*
(c) $\tilde{\Pi}_0^* \geq \Pi_0^*$*.*

Proposition 5 shows the impact of changing the unit product price P^k, i.e., when the product is delivered at time $(t^0 + L_k)$. Specifically, an increase of the price P^k would reduce the optimal equilibrium production quantities and the expected profits of all component suppliers whose lead times are less than or equal to L_k. On the other hand, the increase of the price P^k would increase the optimal equilibrium production quantities and the expected profit of all component suppliers whose lead times are greater than L_k. The expected profit of the assembler would also increase as unit product P^k increases. However, it is unclear as to the impact of the increase of P^k on the total expected profit of the system Π_s. It is plausible that the total expected profit of the system should increase due to a higher unit product price. Indeed, we have observed in all our numerical results that if $\tilde{P}^k > P^k$, then $\tilde{\Pi}_s \geq \tilde{\Pi}_s$. However, we are unable to establish an analytical proof for this observation, even for the simplest case with $n = 1$.

The results of Proposition 5 have some interesting implications on component procurement lead times. Specifically, we can interpret the increase in unit product price P^k as a result of a reduction in L_k, the procurement lead time for supplier k. Under this interpretation, Proposition 5 shows that a reduction in a supplier's lead time would reduce the optimal equilibrium production quantities and the expected profit of all component suppliers with lower lead times (including this particular supplier), but would increase the optimal equilibrium production quantities and the expected profit of all component suppliers with longer lead times as well as the expected profit of the assembler. Thus, this result suggests that there is no incentive for any supplier to unilaterally reduce his lead time under this supplier managed inventory system.

To delve further into the impact of procurement lead time on the expected profit of individual suppliers, we consider the special case where all suppliers incur the same unit component cost, i.e., $c_1 = c_2 = ... = c_n$. In this case, it follows directly from equation (18) and $Q_1 \leq Q_2 \leq ... \leq Q_n$ that $\Pi_1 \leq \Pi_2 \leq ...\Pi_n$. In other words, a supplier with a longer lead time has a higher expected profit than that of a supplier with a shorter lead time. This again illustrates the observation that there is no incentive for any supplier to unilaterally reduce his lead time under the supplier managed inventory system. We can explain this observation as follows. Since it is beneficial for suppliers with longer lead times to produce a larger quantity of components than other suppliers with shorter lead times, the assembler would need to provide a larger share of her revenue for the supplier with a longer lead time to motivate this supplier to produce more, resulting in a higher expected profit for this supplier. Consequently, while a longer lead time of a supplier would decrease the expected profits of the assembler and the system, the longer lead time actually becomes a strategic leverage for this particular supplier to extract a larger share of the revenue for his own individual benefit.

In addition to component production costs and procurement lead times, demand uncertainty is also a major factor affecting the behavior of the assembler and the suppliers. We next provide some results from our numerical experiments to illustrate the impact of demand uncertainty. Specifically, consider the following 6-component example with $c_1 = 8, c_2 = 8, c_3 = 4, c_4 = 4, c_5 = 9, c_6 = 5$, $P^0 = 120$, $L_i = i$ and $P^i = P^0 - 10i$ for $i = 1, ..., 6$. Also, assume that the demand D is normally distributed with mean $\mu = 1000$ and standard deviation σ, with σ varying from 50 to 150. For this example, the optimal

Table 1: Expected Profits of suppliers, assembler, and total system for under different σ

σ	Π_1	Π_2	Π_3	Π_4	Π_5	Π_6	Π_0	Π_s
50	200	200	152	152	343	218	75293	76558
70	298	298	219	219	492	319	73267	75111
90	389	389	286	286	643	411	71431	73835
110	478	478	352	352	792	510	69745	72707
130	561	561	419	419	943	602	68182	71686
150	648	648	477	477	1072	690	66724	70735

cluster formation is given by $\{(1,2),(3,5),(6,6)\}$. We summarize the expected profits of the individual suppliers, the assembler and the system for different values of σ in Table 1.

Observe from the results in Table 1 that as the demand variability σ increases, the expected profits of all suppliers increase, while that of the assembler and the system decrease. This suggests that since the suppliers bear the full risk of over-production of their individual components, they require a larger share of the product revenue when faced with a higher demand uncertainty. Consequently, a higher uncertainty prompts the suppliers to demand for a higher expected profit to compensate for the higher underlying risk. While the expected profits of all suppliers increase as demand uncertainty increases, the expected profit of the assembler decreases. Furthermore, the decrease in the expected profit of the assembler is larger than the combined increase in profits of all suppliers such that the total expected profit of the system decreases as demand uncertainty increases. This observation holds for all our other numerical experiments.

7 Decentralized System Versus Centralized System

In our *decentralized* model, the assembler first selects the revenue-sharing scheme while each supplier then selects their individual production quantities of components, with firms each maximizing their individual expected profits. Alternatively, the assembler could purchase the components directly from the individual suppliers and bear the full risk of over-stocking the components. Also, when the assembler can buy the components from each supplier i at its unit production cost c_i, then the assembler essentially acts as the single decision maker of a *centralized* system. In this section we compare the production decisions and performances between the decentralized system and the centralized system. Later we will consider a situation when the assembler has to purchase the components from suppliers at prices higher than their production costs and compare the relative performance of the decentralized and centralized systems from the assembler's point of view.

We can model the centralized system as a special case analyzed by Hsu, et al. (2005a)

Table 2: Total System Profit for Different Values of σ

σ	50	70	90	110	130	150
Π_s^c	80136	79390	78644	77898	77153	76407
Π_s^d	76558	75111	73835	72707	71686	70735

with constant unit component procurement cost c_i and zero salvage value. Specifically, the assembler problem in the centralized system can be written as

$$\max_{0 \leq Q_1 \leq Q_2 \leq \cdots \leq Q_n} \Pi_0(\mathbf{Q}) = \sum_{i=1}^{n} G_i(Q_i) + (P^n - \sum_{i=1}^{n} c_i)E(D) \tag{21}$$

where

$$G_i(x) = (P^{i-1} - P^i)x - (P^{i-1} - P^i + c_i) \int_0^x F(y)dy$$

for all $i = 1, \ldots, n$.

Let $\mathbf{Q}^c = (Q_1^c, Q_2^c, \ldots, Q_n^c)$ denote the optimal solution of the centralized system and $\mathbf{Q}^d = (Q_1^d, Q_2^d, \ldots, Q_n^d)$ denote the equilibrium solution of the corresponding decentralized system. Also, let Π_s^c and Π_s^d denote the total expected profit of the centralized and decentralized systems, respectively. We shall show that the n suppliers will have the same cluster formation under the two systems. The component production quantities in a centralized system are, however, strictly larger than their counterparts in the decentralized system and consequently, the total expected profit of the centralized system is higher than that of the decentralized system. We summarize these results in the next proposition:

Proposition 6: *(a)* $\mathbf{Q}^c$ *and* $\mathbf{Q}^d$ *have the same cluster formation;*
(b) $Q_i^c > Q_i^d$ *for all* $i = 1, \ldots, n$*; and*
(c) $\Pi_s^c > \Pi_s^d$.

Proposition 6(c) shows that the total expected profit of the centralized system is always higher than that of the decentralized system. We next provide some results from our numerical experiments to illustrate the impact of demand uncertainty on both the centralized and decentralized systems. Table 2 summarizes the results for the same 6-component example provided in Section 6. Observe from Table 2 that as the demand uncertainty σ increases, the expected profits of both the centralized and decentralized systems decrease. However, the decrease in the expected profit of the decentralized system is more severe than that of the centralized system. In other words, increased demand uncertainty tends to harm the decentralized system more than that for the centralized system in terms of the total expected profit of the system. We observe similar results in all our numerical experiments.

Table 3: The Change-over Point α^* for Different Values of σ

σ	50	70	90	110	130	150
α^*	0.124	0.155	0.181	0.203	0.221	0.237

So far, we assume that the assembler can always buy the components from each supplier i at production cost c_i in the centralized system, so that the assembler can reap all the benefits while the suppliers make no profit. In this situation, the assembler would obviously prefer the centralized over the decentralized system. Suppose that each supplier i will now only sell the components to the assembler at a price above c_i in order to make a profit. Then, it is interesting to analyze the conditions under which the assembler would prefer the decentralized system over the centralized system.

To simplify our discussions for some useful insights, assume that all suppliers will mark up their components by the same percentage over their individual costs in the centralized system. That is, each supplier i charges the assembler a price equal to $(1+\alpha)c_i$. Denote the expected profit of the assembler by $\Pi_0^c(\alpha)$ in such a centralized system. Also, denote the expected profit of the assembler in the decentralized system by Π_0^d, which is a constant independent of α. When $\alpha = 0$, the assembler reaps all benefits of the centralized system and Proposition 6 shows that $\Pi_0^c(0) > \Pi_0^d$. Clearly, $\Pi_0^c(\alpha)$ decreases when α increases, as the assembler needs to pay for higher component prices. Let $\alpha^* > 0$ such that $\Pi_0^c(\alpha^*) = \Pi_0^d$. In other words, the value α^* represents the switch-over point where the assembler prefers the supplier managed inventory (decentralized) system over the centralized system in terms of her expected profit only when $\alpha > \alpha^*$. Then, a higher value of α^* means that the assembler is willing to pay a higher price premium for the components before she would switch to the decentralized system to share revenue with the suppliers.

Table 3 illustrates the impact of demand uncertainty σ on the change-over point α^*. Observe from Table 3 that when the demand uncertainty σ increases, α^* also increases, implying that the assembler is willing to pay a higher price premium for the components in the centralized system. For example, the assembler is willing to pay up to a 12.4% price premium for the components when $\sigma = 50$, and is willing to pay up to a 23.7% price premium when σ increases to 150 before she would switch to the decentralized system to share revenue with the suppliers. In other words, as demand uncertainty increases, the assembler is willing to pay more for the components in the centralized system to maximize her expected profit. This is consistent with the earlier observation that increased demand uncertainty tends to harm the decentralized system more than that for the centralized system in terms of the total expected profit of the system.

8 Conclusion

In this paper we study the optimal procurement strategy for an assembler who uses components from different independent suppliers to assemble a customized product for her client. The specific order quantity is uncertain and can only be confirmed by the client at some future time point. Timely delivery is essential, and any delay of a shipment will reduce the unit price that the assembler will receive for the product. The assembler uses a vendor-managed-inventory (VMI) scheme with revenue-sharing to contract with the different component suppliers, whereby the assembler specifies a revenue-sharing scheme with her component suppliers, and each supplier then makes his own decision in stocking the required components ahead of the actual order quantity. We model the system as a two-stage optimization problem. Using a game-theoretical framework, we analyze the equilibrium component stocking quantities of the suppliers under any revenue-sharing scheme chosen by the assembler and show that there always exists a unique Pareto-optimal equilibrium solution. We further devise an efficient algorithm for finding this Pareto-optimal equilibrium. We then analyze the optimal revenue-sharing scheme that maximizes the expected profit of the assembler and develop a simple algorithm to find the optimal revenue-sharing scheme for the assembler. Finally, we derive some sensitivity results that illustrate how different model parameters can affect the optimal solution of the assembler and the corresponding equilibrium solution of the suppliers.

Our sensitivity analysis results reveal several interesting insights on how demand uncertainty and component procurement lead times affect individual firms' performance using this supplier managed inventory system. These results have important practical implications for managing component inventory under this type of revenue-sharing scheme. Specifically, our results show that a lead time reduction of any individual supplier, while beneficial to the assembler and the system as a whole, does not increase the expected profit of this individual supplier. Similarly, a reduction of the component cost by any individual supplier, while beneficial to the assembler and the system, again may not necessarily increase the expected profit of this individual supplier. These two results imply that there is no incentive for any individual supplier to unilaterally reduce his lead time or cost. As such, it would be desirable for the assembler to provide some incentive for the suppliers to reduce their lead times or component costs, as either of such reduction is beneficial to the assembler. Also, our results would suggest that it is actually desirable for the suppliers to inflate their lead times or component costs. Therefore, it is also important for the assembler to devise effective mechanisms such that the suppliers would reveal their true lead times and component costs.

Our results also show that increased demand uncertainty benefits the component suppliers, but has an adverse effect on the assembler and the overall system. Furthermore, the higher the demand uncertainty, the more the assembler prefers a centralized system. This implies that while the objective of supplier managed inventory is to allow the assembler to share the underlying risk of the system due to demand uncertainty, it is still most beneficial to the assembler to reduce the demand uncertainty so as to maximize her expected profit under the revenue sharing scheme. On the other hand, component supplier will be able to charge a higher price premium for their components as demand uncertainty increases.

Acknowledgments

This research is supported in part by the National University of Singapore Research Grant R-314-000-064-112 for Kut C. So.

A Appendix

Proof of Proposition 1: Observe that for $Q_i \geq \min(Q_{i+1}, ..., Q_n)$,

$$P_i^i E[\min(Q_{i+1}, ..., Q_n, D) - Q_i]^+ = 0,$$

and the summation term in (2) can be written as

$$\sum_{k=0}^{n} P_i^k E[\min(Q_{k+1}, ..., Q_n, D) - Q_k]^+ = \sum_{k=0}^{i-2} P_i^k E[\min(Q_{k+1}, ..., Q_{i-1}, Q_{i+1}, ..., Q_n, D) - Q_k]^+$$

$$+P_i^{i-1} E[\min(Q_{i+1}, ..., Q_n, D) - Q_{i-1}]^+ + \sum_{k=i+1}^{n} P_i^k E[\min(Q_{k+1}, ..., Q_n, D) - Q_k]^+,$$

which is independent of Q_i. Also, the term $c_i E[\max(Q_i, D)]$ is increasing in Q_i. Therefore, the profit function of supplier i given in (2) is decreasing in Q_i for $Q_i \geq \min(Q_{i+1}, ..., Q_n)$. This implies that the optimal Q_i^* must be less than or equal to $\min(Q_{i+1}, ..., Q_n)$. *q.e.d.*

Proof of Lemma 1: (a) It follows directly from (5) that $\frac{dG_i^j(Q_i)}{dQ_i} = (P_i^j - P_i^i)\bar{F}(Q_i) - c_i F(Q_i)$ and $\frac{d^2 G_i^j(Q_i)}{dQ_i^2} = -(P_i^j - P_i^i + c_i) f(Q_i) < 0$. Thus, $G_i^j(Q_i)$ is strictly concave in Q_i. Solving $\frac{dG_i^j(Q_i)}{dQ_i} = 0$ gives the unique maximizer q_i^j given in (6). Using (1), $(P_i^j - P_i^i)$ is decreasing in j. Since $\bar{F}(x)$ is a decreasing function, it follows that q_i^j is decreasing in j.

(b) It follows from (4) and the result in part (a) that $\Pi_i(Q_i|Q_{-i})$ is continuous and strictly concave on $Q_{i-1} \leq Q_i \leq Q_{i+1}$ and each interval of $Q_j \leq Q_i \leq Q_{j+1}$, for all $j = 0, 1, ..., i-2$. Thus, it suffices to check the property on the breaking points Q_j for $j = 1, 2, ..., i-1$. First, it follows directly from (4) that, for $j = 1, 2, ..., i-1$,

$$\Pi_i(Q_j|Q_{-i}) = G_i^{j-1}(Q_j) + K_i^{j-1}(Q_{-i}) = G_i^j(Q_j) + K_i^j(Q_{-i}),$$

which implies that $\Pi_i(Q_i|Q_{-i})$ is continuous at each of these breaking points. Furthermore,

$$\frac{d\Pi_i(Q_i|Q_{-i})}{dQ_i}|_{Q_i=Q_j^-} = \frac{dG_i^{j-1}(Q_i)}{dQ_i}|_{Q_i=Q_j} = (P_i^{j-1} - P_i^i)\bar{F}(Q_j) - c_iF(Q_j)$$

and

$$\frac{d\Pi_i(Q_i|Q_{-i})}{dQ_i}|_{Q_i=Q_j^+} = \frac{dG_i^{j}(Q_i)}{dQ_i}|_{Q_i=Q_j} = (P_i^{j} - P_i^i)\bar{F}(Q_j) - c_iF(Q_j).$$

Since $P_i^{j-1} \geq P_i^j$, we have $\frac{d\Pi_i(Q_i|Q_{-i})}{dQ_i}|_{Q_i=Q_j^-} \geq \frac{d\Pi_i(Q_i|Q_{-i})}{dQ_i}|_{Q_i=Q_j^+}$. Thus, $\Pi_i(Q_i|Q_{-i})$ is concave at each breaking point $Q_i = Q_j$ for $j = 1, 2, ..., i-1$. This completes the proof. *q.e.d.*

Proof of Proposition 2: There are three *mutually exclusive and jointly exhaustive* cases for the value of right-hand side of (7): 1) q_i^k for some $k \in \{0, 1, \ldots, i-1\}$; 2) Q_k for some $k \in \{1, \ldots, i-1\}$; and 3) Q_{i+1}. Using the results in Lemma 1, we shall show that in each of these three cases, the value of right-hand side of (7) maximizes $\Pi_i(Q_i|Q_{-i})$.

For Case 1, $\min_{0\leq j\leq i-1}\left\{\max(Q_j, q_i^j), Q_{i+1}\right\} = q_i^k$ for some $k \in \{0, 1, \ldots, i-1\}$. In this case, since q_i^j is decreasing in j from Lemma 1(a) and Q_j is increasing in j, we must have $Q_k \leq q_i^k \leq Q_{k+1}$ (for $k = 0, 1, ..., i-2$) or $Q_{i-1} \leq q_i^k \leq Q_{i+1}$ (for $k = i-1$). Thus, q_i^k maximizes $\Pi_i(x|Q_{-i})$ for $Q_k \leq x \leq Q_{k+1}$ or $Q_{i-1} \leq x \leq Q_{i+1}$. It follows from Lemma 1(b) that $x = q_i^k$ maximizes $\Pi_i(x|Q_{-i})$ for $0 \leq x \leq Q_{i+1}$.

For Case 2, $\min_{0\leq j\leq i-1}\left\{\max(Q_j, q_i^j), Q_{i+1}\right\} = Q_k$ for some $k \in \{1, \ldots, i-1\}$. In this case, we must have $q_i^k \leq Q_k \leq q_i^{k-1}$. From the definition of q_i^j, $q_i^k \leq Q_k$ implies that $\Pi_i(x|Q_{-i})$ is decreasing for $x > Q_k$ and $Q_k \leq q_i^{k-1}$ implies that $\Pi_i(x|Q_{-i})$ is increasing for $x < Q_k$. It thus follows from Lemma 1(b) that $x = Q_k$ maximizes $\Pi_i(x|Q_{-i})$.

For case 3, $\min_{0\leq j\leq i-1}\left\{\max(Q_j, q_i^j), Q_{i+1}\right\} = Q_{i+1}$. In this case, we must have $Q_{i+1} \leq q_i^{i-1}$, which implies that $\Pi_i(x|Q_{-i})$ is increasing for $Q_{i-1} \leq x \leq Q_{i+1}$. Lemma 1(b) implies that $x = Q_{i+1}$ maximizes $\Pi_i(x|Q_{-i})$. *q.e.d.*

Proof of Theorem 1: We first show that the solution $Q_i = q_i^{i-1}$ satisfies (3) and (7), and thus is a Nash equilibrium. Since $q_i^{i-1} < q_{i+1}^i$ for all $i = 1, \ldots, n-1$, (3) is clearly satisfied. To prove (7), we need to show that, for all $i = 1, 2, ..., n$,

$$\min_{1\leq j\leq i-1}\{\max(Q_0, q_i^0), \max(q_j^{j-1}, q_i^j), q_{i+1}^i\} = q_i^{i-1}. \tag{22}$$

By assumption, $q_{i-1}^{i-2} < q_i^{i-1}$ and so $\max(q_j^{j-1}, q_i^j) = q_i^{i-1}$ for $j = i-1$. Furthermore, since q_i^j is decreasing in j by Lemma 1(a) and $Q_0 \equiv 0$, we have $q_i^0 \geq q_i^{i-1}$ and for $1 \leq j < (i-1)$, $\max(q_j^{j-1}, q_i^j) \geq q_i^j \geq q_i^{i-1}$. Finally, $q_{i+1}^i > q_i^{i-1}$ by assumption. Therefore, (22) holds.

To show that the equilibrium is unique, we further demonstrate that $Q_i = q_i^{i-1}$ for all $i = 1, \ldots, n$ is the only solution satisfying (3) and (7). Let $(\tilde{Q}_1, ..., \tilde{Q}_n)$ be any Nash equilibrium satisfying (3) and (7). Equation (7) implies that $\tilde{Q}_1 = \min\{q_1^0, \tilde{Q}_2\} \le q_1^0$. Since $q_i^{i-1} < q_{i+1}^i$ for all i, we have $\tilde{Q}_1 \le q_1^0 < q_2^1$, which implies that $\max(\tilde{Q}_1, q_2^1) = q_2^1$. Continue this argument, we have $\max(\tilde{Q}_{i-1}, q_i^{i-1}) = q_i^{i-1}$ for all $i = 1, \ldots, n$, which implies that

$$\min_{0 \le j \le i-1} \{\max(\tilde{Q}_j, q_i^j), \tilde{Q}_{j+1}\} = \min_{0 \le j \le i-2} \{\max(\tilde{Q}_j, q_i^j), q_i^{i-1}, \tilde{Q}_{j+1}\} = \min\{q_i^{i-1}, \tilde{Q}_{j+1}\}, \quad (23)$$

where the last equality follows from the fact that $\max(\tilde{Q}_j, q_i^j) \ge q_i^j \ge q_i^{i-1}$ for $j < (i-1)$ since q_i^j is decreasing in j by Lemma 1(a). Using (23) for $i = n$, we have $\tilde{Q}_n = q_n^{n-1}$. Using (23) repeatedly from $i = n-1$ to $i = 1$ with $\tilde{Q}_{i+1} = q_{i+1}^i$, we prove that $\tilde{Q}_i = q_i^{i-1}$ for all i. *q.e.d.*

Proof of Lemma 2: Using (7) for suppliers i and $i+1$, we have

$$\tilde{Q}_i = \min_{0 \le j \le i-1} \left\{\max(\tilde{Q}_j, q_i^j), \tilde{Q}_{i+1}\right\} \quad (24)$$

$$\tilde{Q}_{i+1} = \min_{0 \le j \le i} \left\{\max(\tilde{Q}_j, q_{i+1}^j), \tilde{Q}_{i+2}\right\} \quad (25)$$

Assume that $\tilde{Q}_i < \tilde{Q}_{i+1}$. It follows from (25) that $\tilde{Q}_{i+1} \le \max(\tilde{Q}_i, q_{i+1}^i)$, which implies that $\tilde{Q}_{i+1} \le q_{i+1}^i$. It then follows from (3) that

$$\tilde{Q}_{i-1} \le \tilde{Q}_i < \tilde{Q}_{i+1} \le q_{i+1}^i \le q_i^{i-1}. \quad (26)$$

From (24) and (26), we have

$$\tilde{Q}_i = \min_{0 \le j \le i-2} \{\max(\tilde{Q}_j, q_i^j), q_i^{i-1}, \tilde{Q}_{i+1}\} = \min\{q_i^{i-1}, \tilde{Q}_{i+1}\},$$

where the last equality follows from the fact that $\max(\tilde{Q}_j, q_i^j) \ge q_i^j \ge q_i^{i-1}$ for $j < (i-1)$ since q_i^j is decreasing in j by Lemma 1(a). Since $\tilde{Q}_i < \tilde{Q}_{i+1}$, we must have $\tilde{Q}_i = q_i^{i-1}$. This leads to the fact that $\tilde{Q}_i = q_i^{i-1} \ge q_{i+1}^i \ge \tilde{Q}_{i+1}$, contradicting the assumption that $\tilde{Q}_i < \tilde{Q}_{i+1}$. Therefore, we prove that $\tilde{Q}_i = \tilde{Q}_{i+1}$. *q.e.d.*

Proof of Theorem 2: Observe that in each iteration, Algorithm 1 transforms the original m-cluster system into a new $(m-1)$-cluster system with cluster k and $(k+1)$ in the original m-cluster system merged into one single cluster. To establish the result of Theorem 2, we first need to define an equilibrium solution for any m-cluster system $\{(l_1, r_1), (l_2, r_2), \ldots, (l_m, r_m)\}$. To do so, let us define, for all $i = 1, 2, \ldots, m$, and $j = 0, 1, \ldots, i-1$,

$$q_{[i]}^j = \min_{l_i \le a \le r_i} \{q_a^{r_j}\}, \quad (27)$$

where $q_a^{r_j}$ is defined in (6). It follows from Lemma 1(a) that, for all $i = 1, \ldots, m$, $q_{[i]}^0 \ge q_{[i]}^1 \ge \ldots \ge q_{[i]}^{i-1}$. In view of (3) and (7), we define an equilibrium solution of the m-cluster system defined above, $(Q_{[1]}, Q_{[2]}, ..., Q_{[m]})$, as satisfying the following two equations:

$$Q_{[1]} \leq Q_{[2]} \leq \ldots \leq Q_{[m]} \tag{28}$$

$$Q_{[i]} = \min_{0 \leq j \leq i-1} \{\max(Q_{[j]}, q_{[i]}^{j}), Q_{[i+1]}\}. \tag{29}$$

where $Q_{[0]} \equiv 0$ and $Q_{[m+1]} \equiv \infty$.

For the m-cluster system, any equilibrium solution $(Q_{[1]}^m, ..., Q_{[m]}^m)$ must therefore satisfy the following equilibrium conditions as given by (27)-(29) and $Q_{[0]}^m \equiv 0$ and $Q_{[m+1]}^m \equiv \infty$:

$$Q_{[1]}^m \leq \cdots \leq Q_{[k-1]}^m \leq Q_{[k]}^m \leq Q_{[k+1]}^m \leq Q_{[k+2]}^m \leq \cdots \leq Q_{[m]}^m \tag{30}$$

$\forall i = 1, \ldots, k-1,$

$$Q_{[i]}^m = \min_{0 \leq j \leq i-1} \{\max[Q_{[j]}^m, \min_{l_i \leq a \leq r_i} (q_a^{r_j})], Q_{[i+1]}^m\} \tag{31}$$

$$i = k, k+1, Q_{[i]}^m = \min_{0 \leq j \leq i-1} \{\max[Q_{[j]}^m, \min_{l_i \leq a \leq r_i} (q_a^{r_j})], Q_{[i+1]}^m\} \tag{32}$$

$\forall i = k+2, \ldots, m,$

$$Q_{[i]}^m = \min_{0 \leq j \leq i-1} \{\max[Q_{[j]}^m, \min_{l_i \leq a \leq r_i} (q_a^{r_j})], Q_{[i+1]}^m\}. \tag{33}$$

In step 2 of Algorithm 1 when clusters k and $(k+1)$ are merged together, the new system consists of $(m-1)$ clusters where the i^{th} cluster is (l_i, r_i) for $i = 1, ..., k-1$, (l_i, r_{i+1}) for $i = k$, and (l_{i+1}, r_{i+1}) for $i = k+1, ..., m-1$. For this $(m-1)$-cluster system, any equilibrium solution $(Q_{[1]}^{m-1}, ..., Q_{[m-1]}^{m-1})$ must satisfy the following equilibrium conditions as given by (27)-(29) with $Q_{[0]}^{m-1} \equiv 0$ and $Q_{[m]}^{m-1} \equiv \infty$:

$$Q_{[1]}^{m-1} \leq \cdots \leq Q_{[k-1]}^{m-1} \leq Q_{[k]}^{m-1} \leq Q_{[k+1]}^{m-1} \leq \cdots \leq Q_{[m-1]}^{m-1} \tag{34}$$

$\forall i = 1, \ldots, k-1,$

$$Q_{[i]}^{m-1} = \min_{0 \leq j \leq i-1} \{\max[Q_{[j]}^{m-1}, \min_{l_i \leq a \leq r_i} (q_a^{r_j})], Q_{[i+1]}^{m-1}\} \tag{35}$$

$$Q_{[k]}^{m-1} = \min_{0 \leq j \leq k-1} \{\max[Q_{[j]}^{m-1}, \min_{l_k \leq a \leq r_{k+1}} (q_a^{r_j})], Q_{[k+1]}^{m-1}\} \tag{36}$$

$\forall i = k+1, \ldots, m-1,$

$$Q_{[i]}^{m-1} = \min_{\substack{0 \leq j \leq k-1 \\ k \leq j' \leq i-1}} \{\max[Q_{[j]}^{m-1}, \min_{l_{i+1} \leq a \leq r_{i+1}} (q_a^{r_j})], \max[Q_{[j']}^{m-1}, \min_{l_{i+1} \leq a \leq r_{i+1}} (q_a^{r_{j'+1}})], Q_{[i+1]}^{m-1}\} \tag{37}$$

With the above equilibrium conditions for each of the two systems being set up, we next proceed to establish two results regarding the equilibrium solutions of the m-cluster and $(m-1)$-cluster systems during each iteration of Algorithm 1. Suppose $\mathbf{Q^{m-1}} = (Q_{[1]}^{m-1}, \ldots, Q_{[m-1]}^{m-1})$

be any solution to the $(m-1)$-cluster system. We define $\mathbf{Q^m} = (Q^m_{[1]}, \ldots, Q^m_{[m]})$ with $Q^m_{[i]} = Q^{m-1}_{[i]}$ for $i = 1, ..., k-1$, $Q^m_{[k]} = Q^m_{[k+1]} = Q^{m-1}_{[k]}$ and $Q^m_{[i]} = Q^{m-1}_{[i-1]}$ for $i = k+2, ..., m$ be the corresponding m-cluster solution. Similarly, suppose $\mathbf{Q^m} = (Q^m_{[1]}, \ldots, Q^m_{[m]})$ with $Q^m_{[k]} = Q^m_{[k+1]}$ be any solution to the m-cluster system. We define $\mathbf{Q^{m-1}} = (Q^{m-1}_{[1]}, \ldots, Q^{m-1}_{[m-1]})$ with $Q^{m-1}_{[i]} = Q^m_{[i]}$ for $i = 1, ..., k-1$, $Q^{m-1}_{[k]} = Q^m_{[k]}$, and $Q^{m-1}_{[i]} = Q^m_{[i+1]}$ for $i = k+1, ..., m-1$ be the corresponding $(m-1)$-cluster solution. We shall use the following identity in our proof, which can be easily verified. For any real numbers Q, a and b,

$$\min\{\max(Q, a), \max(Q, b)\} = \max\{Q, \min(a, b)\}. \tag{38}$$

Proposition A1: *The corresponding solution of any equilibrium solution of the $(m-1)$-cluster system is also an equilibrium solution of the m-cluster system.*

Proof of Proposition A1: Suppose that $\mathbf{Q^{m-1}} = (Q^{m-1}_{[1]}, \ldots, Q^{m-1}_{[m-1]})$ is an equilibrium solution of the $(m-1)$-cluster system, so it satisfies (34)-(37). We want to show that the corresponding m-cluster solution $\mathbf{Q^m} = (Q^m_{[1]}, \ldots, Q^m_{[m]})$ satisfies (30)-(33) and thus is an equilibrium solution of the m-cluster system. Clearly, (34)-(35) imply (30)-(31). Since

$$\min_{l_k \le a \le r_{k+1}} (q_a^{r_j}) = \min[\min_{l_k \le a \le r_k} (q_a^{r_j}), \min_{l_{k+1} \le a \le r_{k+1}} (q_a^{r_j})],$$

it follows from (38) that for all $j = 0, 1, ..., k-1$,

$$\max\{Q^{m-1}_{[j]}, \min_{l_k \le a \le r_{k+1}} (q_a^{r_j})\} = \min\{\max[Q^{m-1}_{[j]}, \min_{l_k \le a \le r_k} (q_a^{r_j})], \max[Q^{m-1}_{[j]}, \min_{l_{k+1} \le a \le r_{k+1}} (q_a^{r_j})]\}. \tag{39}$$

In conjunction with the condition of $q^{k-1}_{[k]} \ge q^k_{[k+1]}$ as given in Step 2 of Algorithm 1, i.e. $\min_{l_k \le a \le r_k}(q_a^{r_{k-1}}) \ge \min_{l_{k+1} \le a \le r_{k+1}}(q_a^{r_k})$, we can show that (36) implies (32). Since $Q^m_{[k]} = Q^m_{[k+1]} = Q^{m-1}_{[k]}$ and $q_a^{r_{k+1}} \le q_a^{r_k}$ for all a by Lemma 1(a), it follows from (38) that for all $i = k+1, ..., m-1$,

$$\begin{aligned}\max\{Q^{m-1}_{[k]}, \min_{l_{i+1} \le a \le r_{i+1}} (q_a^{r_{k+1}})\} &= \max\{Q^{m-1}_{[k]}, \min[\min_{l_{i+1} \le a \le r_{i+1}} (q_a^{r_k}), \min_{l_{i+1} \le a \le r_{i+1}} (q_a^{r_{k+1}})]\} \\ &= \min\{\max[Q^m_{[k]}, \min_{l_{i+1} \le a \le r_{i+1}} (q_a^{r_k})], \max[Q^m_{[k+1]}, \min_{l_{i+1} \le a \le r_{i+1}} (q_a^{r_{k+1}})]\}.\end{aligned}$$

Hence, (37) implies (33). This proves the result. *q.e.d.*

Proposition A2: *Suppose that the corresponding $(m-1)$-cluster solution of an equilibrium solution of the m-cluster system, say $\mathbf{Q^m}$, is not an equilibrium solution of the $(m-1)$-cluster system. Then, there always exists an equilibrium solution of the $(m-1)$-cluster system such that its corresponding m-cluster solution dominates $\mathbf{Q^m}$ in maximizing the profit of every supplier.*

Proof of Proposition A2: Let $\mathbf{Q^m} = (Q^m_{[1]}, \ldots, Q^m_{[m]})$ be an equilibrium solution of the m-cluster system, since k is the smallest index such that $q^{k-1}_{[k]} \geq q^k_{[k+1]}$ as given in Step 2 of Algorithm 1, it follows from Lemma 2 that $Q^m_{[k]} = Q^m_{[k+1]}$. Suppose the corresponding $(m-1)$-cluster solution of $\mathbf{Q^m}$, denoted by $\mathbf{Q^{m-1}} = (Q^{m-1}_{[1]}, \ldots, Q^{m-1}_{[m-1]})$, is not an equilibrium solution of the $(m-1)$-cluster system. By assumption, $\mathbf{Q^m}$ satisfies (30)-(33). From (30)-(31) and (33), it is straightforward to see that $\mathbf{Q^{m-1}}$ also satisfies (34)-(35) and (37). Also, (32) implies that

$$Q^m_{[k]} \leq \min_{0 \leq j \leq k-1} \{\max[Q^m_{[j]}, \min_{l_k \leq a \leq r_k} (q^{r_j}_a)]\}$$

and

$$Q^m_{[k+1]} \leq \min_{0 \leq j \leq k-1} \{\max[Q^m_{[j]}, \min_{l_{k+1} \leq a \leq r_{k+1}} (q^{r_j}_a)], Q^m_{[k+2]}\}.$$

Then, it follows from (39) and $Q^m_{[k]} = Q^m_{[k+1]}$ that

$$Q^m_{[k]} \leq \min_{0 \leq j \leq k-1} \{\max[Q^m_{[j]}, \min_{l_k \leq a \leq r_{k+1}} (q^{r_j}_a)], Q^m_{[k+2]}\}.$$

Since by assumption that $\mathbf{Q^{m-1}}$ is not an equilibrium of the $(m-1)$-cluster system, we must have

$$Q^m_{[k]} < \min_{0 \leq j \leq k-1} \{\max[Q^m_{[j]}, \min_{l_k \leq a \leq r_{k+1}} (q^{r_j}_a)], Q^m_{[k+2]}\}.$$

Let $\mathbf{\bar{Q}^{m-1}} = (\bar{Q}^{m-1}_{[1]}, \ldots, \bar{Q}^{m-1}_{[m-1]})$ be the same as $\mathbf{Q^{m-1}}$ except with

$$\bar{Q}^{m-1}_{[k]} = \bar{Q}^m_{[k]} = \min_{0 \leq j \leq k-1} \{\max[Q^m_{[j]}, \min_{l_k \leq a \leq r_{k+1}} (q^{r_j}_a)], Q^m_{[k+2]}\} > Q^m_{[k]} = Q^{m-1}_{[k]}. \tag{40}$$

Then, $\mathbf{\bar{Q}^{m-1}}$ satisfies (34)-(37), and thus is an equilibrium of the $(m-1)$-cluster system. Proposition A1 implies that its corresponding m-cluster solution, denoted by $\mathbf{\bar{Q}^m}$, is also an equilibrium solution of the m-cluster system.

To complete the proof, all we need is to show that $\mathbf{\bar{Q}^m}$ dominates $\mathbf{Q^m}$. To that end, we can first rewrite the profit function of supplier i given in (2), for any $Q = (Q_1, \ldots, Q_n)$ with $Q_1 \leq Q_2 \leq \ldots \leq Q_n$, as

$$\Pi_i(Q) = \sum_{j=1}^{n} (P^{j-1}_i - P^j_i) \int_0^{Q_j} \bar{F}(x)dx - c_i \int_0^{Q_i} F(x)dx + (P^n_i - c_i)E(D). \tag{41}$$

Observe that $\bar{Q}^m_{[i]} = Q^m_{[i]}$ for all i except for $i = k, k+1$, with $\bar{Q}^m_{[k]} = \bar{Q}^m_{[k+1]}$ and $Q^m_{[k]} = Q^m_{[k+1]}$. Therefore, for all $i < l_k$ or $i > r_{k+1}$,

$$\Pi_i(\mathbf{\bar{Q}^m}) - \Pi_i(\mathbf{Q^m}) = \sum_{j=l_k}^{r_{k+1}} (P^{j-1}_i - P^j_i) \int_0^{\bar{Q}^m_{[k]}} \bar{F}(x)dx - \sum_{j=l_k}^{r_{k+1}} (P^{j-1}_i - P^j_i) \int_0^{Q^m_{[k]}} \bar{F}(x)dx.$$

Since $\bar{Q}^m_{[k]} = \bar{Q}^{m-1}_{[k]} > Q^{m-1}_{[k]} = Q^m_{[k]}$, $\Pi_i(\mathbf{\bar{Q}^m}) - \Pi_i(\mathbf{Q^m}) > 0$ for all $i < l_k$ or $i > r_{k+1}$.

For $l_k \le i \le r_{k+1}$, we have

$$\begin{aligned}\Pi_i(\bar{\mathbf{Q}}^{\mathbf{m}}) - \Pi_i(\mathbf{Q}^{\mathbf{m}}) &= \Big\{\sum_{j=l_k}^{r_{k+1}} (P_i^{j-1} - P_i^j) \int_0^{\bar{Q}^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{\bar{Q}^m_{[k]}} F(x)dx\Big\} \\ &\quad - \Big\{\sum_{j=l_k}^{r_{k+1}} (P_i^{j-1} - P_i^j) \int_0^{Q^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{Q^m_{[k]}} F(x)dx\Big\} \\ &= \Big\{(P_i^{l_k-1} - P_i^{r_{k+1}}) \int_0^{\bar{Q}^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{\bar{Q}^m_{[k]}} F(x)dx\Big\} \\ &\quad - \Big\{(P_i^{l_k-1} - P_i^{r_{k+1}}) \int_0^{Q^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{Q^m_{[k]}} F(x)dx\Big\}.\end{aligned}$$

It is easy to verify that the function

$$(P_i^{l_k-1} - P_i^{r_{k+1}}) \int_0^{Q} \bar{F}(x)dx - c_i \int_0^{Q} F(x)dx$$

is concave in Q and reaches its maximum at $\bar{F}^{-1}\left(\frac{c_i}{P_i^{l_k-1} - P_i^{r_{k+1}} + c_i}\right)$. From (40),

$$\bar{Q}^m_{[k]} \le \max[Q^m_{[k-1]}, \min_{l_k \le a \le r_{k+1}} (q_a^{r_{k-1}})].$$

Since $\bar{Q}^m_{[k-1]} = Q^m_{[k-1]} \le Q^m_{[k]} < \bar{Q}^m_{[k]}$ by (28) and (40), we have

$$\bar{Q}^m_{[k]} \le \min_{l_k \le a \le r_{k+1}} (q_a^{r_{k-1}}) = \min_{l_k \le a \le r_{k+1}} (q_a^{l_k-1}),$$

where the equality follows from the definition of $r_{k-1} = l_k - 1$. Therefore, (1) and (6) imply that for all $l_k \le i \le r_{k+1}$,

$$Q^m_{[k]} < \bar{Q}^m_{[k]} \le q_i^{l_k-1} \le \bar{F}^{-1}\left(\frac{c_i}{P_i^{l_k-1} - P_i^{r_{k+1}} + c_i}\right).$$

This implies that

$$(P_i^{l_k-1} - P_i^{r_{k+1}}) \int_0^{Q^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{Q^m_{[k]}} F(x)dx < (P_i^{l_k-1} - P_i^{r_{k+1}}) \int_0^{\bar{Q}^m_{[k]}} \bar{F}(x)dx - c_i \int_0^{\bar{Q}^m_{[k]}} F(x)dx.$$

Hence, $\Pi_i(\bar{\mathbf{Q}}^{\mathbf{m}}) - \Pi_i(\mathbf{Q}^{\mathbf{m}}) > 0$ also for all $l_k \le i \le r_{k+1}$. This completes the proof of Proposition A2. *q.e.d.*

The first part of Theorem 2 is obvious in view of Theorem 1. If Step 2 of Algorithm 1 is executed, the m-cluster system is reduced to an $(m-1)$-cluster system with clusters k and $(k+1)$ merged. Proposition A1 shows that any equilibrium solution of the $(m-1)$-cluster system corresponds to an equilibrium solution of the m-cluster system. Since the original n-supplier system corresponds to the n-cluster system at the start of the Algorithm, Proposition 1 implies that any equilibrium solution of the $(m-1)$-cluster system corresponds to an equilibrium solution of the original n-supplier problem. Furthermore, the result of Proposition A2 shows that any equilibrium solution in the m-cluster system will correspond to or be dominated by an equilibrium solution of the $(m-1)$-cluster. Propositions A1 and A2 guarantee that the Algorithm always retains the Pareto-optimal equilibria of the original n-supplier problem at each iteration, i.e., any equilibrium solution in the original solution will be dominated by some equilibrium solution in the $(m-1)$-cluster solution. Since the last iteration of the Algorithm always ends up with a unique equilibrium solution, this unique solution must dominate all the equilibrium solutions of the original n-supplier problem, and therefore is the unique Pareto-optimal equilibrium solution. This proves Theorem 2. *q.e.d.*

Proof of Proposition 3: It follows from (6) and (11) that for all $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$,

$$q_k^j = \begin{cases} Q_{[i]}, & 0 \leq j \leq l_i - 1 \\ 0, & l_i \leq j \leq n \end{cases}$$

satisfies (10). Therefore, the revenue-sharing scheme (11) induces the equilibrium solution $(Q_{[1]}, \ldots, Q_{[m]})$ under Algorithm 1.

To show that the revenue-sharing scheme given by (11) maximizes the assembler's expected profit, we shall demonstrate that this revenue-sharing scheme is component-wise smaller than any other feasible revenue-sharing scheme $\{\tilde{P}_i^t\}$ that can induce the same equilibrium solution under Algorithm 1, i.e., for all $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$, $P_k^j \leq \tilde{P}_k^j$ for all $0 \leq j \leq n$.

First, it is obvious from (1) and (11) that for any $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$, we have $P_k^j = c_k \leq \tilde{P}_k^j$ for all $l_i \leq j \leq n$. Furthermore, it follows from (11) that

$$Q_{[i]} = \bar{F}^{-1}\left(\frac{c_k}{P_k^0}\right)$$

for all $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$. Since the revenue-sharing scheme $\{\tilde{P}_i^t\}$ also induces the same equilibrium solution $(Q_{[1]}, \ldots, Q_{[m]})$ under Algorithm 1, it follows from (6) and (10) that

$$Q_{[i]} = \min_{l_i \leq k \leq r_i} \{q_k^{r_i-1}\} = \min_{l_i \leq k \leq r_i} \left\{\bar{F}^{-1}\left(\frac{c_k}{\tilde{P}_k^{r_i-1} - \tilde{P}_k^k + c_k}\right)\right\}.$$

From the above two equations and the fact that $\bar{F}^{-1}(x)$ is decreasing in x, we have $P_k^0 \leq \tilde{P}_k^{r_i-1} - \tilde{P}_k^k + c_k$, which implies that $P_k^0 \leq \tilde{P}_k^{r_i-1}$ since $\tilde{P}_k^k \geq c_k$. Therefore, for any given $0 \leq j \leq l_i - 1$,

$$P_k^j \leq P_k^0 \leq \tilde{P}_k^{r_i-1} = \tilde{P}_k^{l_i-1} \leq \tilde{P}_k^j,$$

as both P_k^j and $\tilde{P}_k^j$ are decreasing in j as given in (1). Thus, for all $i = 1, \ldots, m$ and $l_i \leq k \leq r_i$ we have $P_k^j \leq \tilde{P}_k^j$ for $0 \leq j \leq l_i - 1$ as well. This completes the proof. *q.e.d.*

Proof of Lemma 3: (a) Direct differentiations of (13) give

$$\frac{dM_{(a,b)}(Q)}{dQ} = \left[P^{a-1} - P^b + \sum_{k=a}^{b} c_k \left(1 - \frac{1}{\bar{F}(Q)}\right)\right] \bar{F}(Q) - \sum_{k=a}^{b} c_k R(Q)$$

and

$$\frac{d^2 M_{(a,b)}(Q)}{dQ^2} = -\left(P^{a-1} - P^b + \sum_{k=a}^{b} c_k\right) f(Q) - \sum_{k=a}^{b} c_k \cdot \frac{dR(Q)}{dQ}.$$

Since $R(Q)$ is increasing in Q, we have $\frac{dR(Q)}{dQ} \geq 0$, which implies that $\frac{d^2 M_{(a,b)}(Q)}{dQ^2} \leq 0$. Hence, $M_{(a,b)}(Q)$ is concave, and its maximum $\hat{q}_{(a,b)}$ is given by $\frac{dM_{(a,b)}(Q)}{dQ} = 0$, or equivalently, (15).
(b) Since $R(Q)$ is increasing in Q and $\bar{F}(Q)$ is decreasing in Q, it follows that the right-side of (15) is increasing in Q. The result follows immediately from the definition of $\hat{q}_{(a,b)}$. *q.e.d.*

Proof of Theorem 3: Observe that the assembler problem (17) has the same structure as the optimal component procurement problem studied in Hsu, et al. (2005a), which can be expressed as

$$\max_{0 \leq Q_1 \leq Q_2 \leq \dots \leq Q_n} \sum_{i=1}^{n} G_i(Q_i),$$

where $G_i(.)$ is concave. Using the results of Lemma 3, it is straightforward to adapt the proof of Proposition 3 in Hsu, et al. (2005a) to establish the desired result. We omit the details here. *q.e.d.*

Proof of Proposition 4: (a) The proof of Proposition 6 in Hsu et.al (2005a) can be directly applied to establish the desired result here. We omit the details here.
(b) From (18), we obtain

$$\frac{d\Pi_i(Q_i)}{dQ_i} = \frac{c_i f(Q_i) \int_o^{Q_i} \bar{F}(x)dx}{\bar{F}(Q_i)^2} > 0.$$

Since $\tilde{c}_i = c_i$ except for $i = k$, the result follows from part (a).
(c) It is clear from (19) that $\Pi_0(\mathbf{Q})$ is strictly decreasing in c_k for all $k = 1, ..., n$. Since $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$ is the same as $\{\mathbf{P}, \mathbf{c}\}$ except $\tilde{c}_k < c_k$ for some $1 \leq k \leq n$, we have $\tilde{\Pi}_0(\mathbf{Q}^*) > \Pi_0(\mathbf{Q}^*)$. Since $\tilde{\mathbf{Q}}^*$ is optimal for the n-component system with $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$, we have $\tilde{\Pi}_0(\tilde{\mathbf{Q}}^*) \geq \tilde{\Pi}_0(\mathbf{Q}^*)$. Hence, $\tilde{\Pi}_0(\tilde{\mathbf{Q}}^*) \geq \tilde{\Pi}_0(\mathbf{Q}^*) > \Pi_0(\mathbf{Q}^*)$.
(d) Consider any equilibrium solution $\mathbf{Q} = (Q_1, ..., Q_n)$ forming m clusters $\{(l_1, r_1), ..., (l_m, r_m)\}$. Let $(Q_{[1]}, Q_{[2]}, ..., Q_{[m]})$ denote the corresponding equilibrium production quantity for each cluster, i.e., $Q_a^* = Q_{[i]}$ for all $l_i \leq a \leq r_i$ and $i = 1, ..., m$. We can then express the total expected profit of the system given by (20) as

$$\Pi_s(\mathbf{Q}) = \sum_{j=1}^{m} \left[(P^{l_j-1} - P^{r_j}) \int_0^{Q_{[j]}} \bar{F}(x)dx + \sum_{i=l_j}^{r_j} c_i \left(\int_0^{Q_{[j]}} \bar{F}(x)dx - Q_{[j]} - E(D)\right)\right] + P^n E(D).$$

Using (15) and (16) in Lemma 3, the quantity $Q_{[j]}$ satisfies

$$\frac{P^{l_j-1}-P^{r_j}}{\sum_{i=l_j}^{r_j} c_i}+1=\frac{1}{\bar{F}(Q_{[j]})}+R(Q_{[j]}).$$

Substituting the above equation into $\Pi_s(\mathbf{Q})$, we obtain

$$\begin{aligned}\Pi_s(\mathbf{Q}) &= \sum_{j=1}^{m}\left[(P^{l_j-1}-P^{r_j})\left(\int_0^{Q_{[j]}}\bar{F}(x)dx+\frac{\int_0^{Q_{[j]}}\bar{F}(x)dx-Q_{[j]}-E(D)}{R(Q_{[j]})+\bar{F}(Q_{[j]})^{-1}-1}\right)\right]+P^nE(D)\\ &= \sum_{j=1}^{n}\left[(P^{j-1}-P^{j})\left(\int_0^{Q_j}\bar{F}(x)dx+\frac{\int_0^{Q_j}\bar{F}(x)dx-Q_j-E(D)}{R(Q_j)+\bar{F}(Q_j)^{-1}-1}\right)\right]+P^nE(D),\end{aligned}$$

which is independent of any c_i. Define

$$H_j(Q)\equiv\int_0^{Q}\bar{F}(x)dx+\frac{\int_0^{Q}\bar{F}(x)dx-Q-E(D)}{R(Q)+\bar{F}(Q)^{-1}-1}.$$

Differentiating $H_j(Q)$ with respect to Q yields

$$H_j'(Q)=\frac{[f(Q)+R'(Q)\bar{F}(Q)^2]\left[E(D)+\int_0^Q F(x)dx\right]+R(Q)\bar{F}^2(Q)\left[F(Q)+R(Q)\bar{F}(Q)\right]}{\left[\bar{F}(Q)R(Q)+F(Q)\right]^2}$$

Since $R(Q)$ is increasing in Q, $H_j'(Q)>0$ for any Q. Hence, it follows from the result of part (a) that $H_j(\tilde{Q}_j^*)\geq H_j(Q_j^*)$ for all $j=1,..,n$. Consequently, $\tilde{\Pi}_s(\tilde{\mathbf{Q}}^*)\geq\Pi_s(\mathbf{Q}^*)$.

To establish the strict inequality, it suffices to show that $\tilde{Q}_j^*\neq Q_j^*$, which implies that there exists some i such that $\tilde{Q}_i^*>Q_i^*$. If $\tilde{Q}_j^*$ and Q_j^* do not have the same cluster formation, then the result is trivial. Assume that they have the same cluster formation. Since $\{\tilde{\mathbf{P}},\tilde{\mathbf{c}}\}$ is the same as $\{\mathbf{P},\mathbf{c}\}$ except $\tilde{c}_k<c_k$ for some $1\leq k\leq n$, equation (16) Lemma 3(b) implies that the cluster containing supplier k have a strictly higher production quantity under the system with $\{\tilde{\mathbf{P}},\tilde{\mathbf{c}}\}$ than that with $\{\mathbf{P},\mathbf{c}\}$. This completes the proof. *q.e.d.*

Proof of Proposition 5: (a) Let $\mathbf{Q}^A=(Q_1^A,\ldots,Q_k^A)$ denote the optimal solution to the following k-component subproblem

$$\max_{0\leq Q_1\leq Q_2\leq\ldots\leq Q_k}\sum_{i=1}^{k}M_{(i,i)}(Q_i) \tag{42}$$

with cost parameters $\{\mathbf{P},\mathbf{c}\}$. Similarly, let $\mathbf{Q}^B=(Q_{k+1}^B,\ldots,Q_n^B)$ denote the optimal solution to the other $(n-k)$-component subproblem

$$\max_{0\leq Q_{k+1}\leq Q_{k+2}\leq\ldots\leq Q_n}\sum_{i=k+1}^{n}M_{(i,i)}(Q_i), \tag{43}$$

except with $P^0 = P^k$. Let $\tilde{\mathbf{Q}}^A$ and $\tilde{\mathbf{Q}}^B$ denote the corresponding values with cost parameters $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$. Since $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$ is the same as $\{\mathbf{P}, \mathbf{c}\}$ except $P^{k-1} \geq \tilde{P}^k > P^k$, it follows from the definition (16) that $\tilde{m}_{(a,b)} \leq m_{(a,b)}$ for all $1 \leq a \leq b \leq k$. We can use the same argument as in the proof of Proposition 5 in Hsu, et al. (2005a) to show that $\tilde{Q}_i^A \leq Q_i^A$ for all $i = 1, ..., k$. Similarly, we can establish the result that $\tilde{Q}_i^B \geq Q_i^B$ for all $i = k+1, ..., n$.

Now consider the following three possible cases:

(i) $Q_k^A \leq Q_{k+1}^B$.
In this case, $\mathbf{Q}^* = (\mathbf{Q}^A, \mathbf{Q}^B)$. Furthermore, $\tilde{Q}_k^A \leq Q_k^A \leq Q_{k+1}^B \leq \tilde{Q}_{k+1}^B$. Therefore, $\tilde{\mathbf{Q}}^* = (\tilde{\mathbf{Q}}^A, \tilde{\mathbf{Q}}^B)$, and the result holds.

(ii) $Q^A > Q_{k+1}^B$ and $\tilde{Q}_k^A \geq \tilde{Q}_{k+1}^B$.
In this case, we have $Q_k^* = Q_{k+1}^*$ and $\tilde{Q}_k^* = \tilde{Q}_{k+1}^*$. Under the condition $Q_k = Q_{k+1}$, we can write the assembler problem (17) as

$$\max_{\substack{0 \leq Q_1 \leq \cdots \leq Q_n \\ Q_k = Q_{k+1}}} \sum_{\substack{i=1 \\ i \neq k, k+1}}^{n} M_{(i,i)}(Q_i) + M_{(k,k+1)}(Q_k),$$

which is independent of P^k. Hence, $\tilde{\mathbf{Q}}^* = \mathbf{Q}^*$ in this case and the result holds.

(iii) $Q_k^A > Q_{k+1}^B$ and $\tilde{Q}_k^A < \tilde{Q}_{k+1}^B$.
Consider the unit product price received at time $(t^0 + L_k)$ as a variable, denoted by y_k. Use the notation $Q_i^A(y_k)$ and $Q_i^B(y_k)$ to indicate the dependence of Q_i^A and Q_i^B on y_k such that we have $Q_k^A(P^k) > Q_{k+1}^B(P^k)$ and $Q_k^A(\tilde{P}^k) < Q_{k+1}^B(\tilde{P}^k)$. Since $Q_k^A(y_k)$ is continuous and decreasing in y_k and $Q_{k+1}^B(y_k)$ is continuous and increasing in y_k, there exists some x with $P^k < x < \tilde{P}^k$ such that $Q_k^A(\tilde{P}^k) < Q_k^A(x) = Q_{k+1}^B(x) < Q_{k+1}^B(\tilde{P}^k)$. Since $Q_k^A(x) = Q_{k+1}^B(x)$, when $y_k = x$, the optimal solution to the assembler problem (17) is $(\mathbf{Q}^A(x), \mathbf{Q}^B(x))$. Furthermore, for any $P^k \leq y_k \leq x$, the optimal solution to the assembler problem (17) has the property that $Q_k = Q_{k+1} = Q_k^A(x)$ and is independent of the value of y_k. In particular, for all $1 \leq i \leq k$, $Q_i^* = Q_i^A(x)$, and for all $k+1 \leq i \leq n$, $Q_i^* = Q_i^B(x)$. It follows directly from $x < \tilde{P}^k$ that for all $1 \leq i \leq k$, $\tilde{Q}_i^A \leq Q_i^A(x)$, and for all $k+1 \leq i \leq n$, $\tilde{Q}_i^B \geq Q_i^B(x)$. In conjuction with $\tilde{\mathbf{Q}}^* = (\tilde{\mathbf{Q}}^A, \tilde{\mathbf{Q}}^B)$ due to $\tilde{Q}_k^A < \tilde{Q}_{k+1}^B$, the result follows immediately.

(b) Differentiating (18) with respect to Q_i yields

$$\frac{d\Pi_i(Q_i)}{dQ_i} = \frac{c_i f(Q_i) \int_o^{Q_i} \bar{F}(x)dx}{\bar{F}(Q_i)^2} > 0.$$

Since $\tilde{c}_i = c_i$ for all $i = 1, ..., n$, it follows from the result of part (a) that $\tilde{\Pi}_i^* \leq \Pi_i^*$ for all $1 \leq i \leq k$ and $\tilde{\Pi}_i^* \geq \Pi_i^*$ for all $k+1 \leq i \leq n$.

(c) Since $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$ is the same as $\{\mathbf{P}, \mathbf{c}\}$ except for $\tilde{P}^k > P^k$, it follows from (19) that

$$\tilde{\Pi}_0(\mathbf{Q}^*) - \Pi_0(\mathbf{Q}^*) = (\tilde{P}^k - P^k)\left(\int_0^{Q^*_{k+1}} \bar{F}(x)dx - \int_0^{Q^*_k} \bar{F}(x)dx\right) > 0,$$

where for notational convenience, define $Q^*_{n+1} \equiv \infty$. Since $\tilde{\mathbf{Q}}^*$ is optimal for the n-component system with $\{\tilde{\mathbf{P}}, \tilde{\mathbf{c}}\}$, we have $\tilde{\Pi}_0(\tilde{\mathbf{Q}}^*) \geq \tilde{\Pi}_0(\mathbf{Q}^*)$. Hence, $\tilde{\Pi}_0(\tilde{\mathbf{Q}}^*) \geq \tilde{\Pi}_0(\mathbf{Q}^*) > \Pi_0(\mathbf{Q}^*)$. *q.e.d.*

Proof of Proposition 6: (a) It has been shown in Hsu, et al. (2005a) that the value of x_i^* maximizing $G_i(x)$ is given by

$$F(x_i^*) = (P^{i-1} - P^i)/(P^{i-1} - P^i + c_i). \tag{44}$$

Furthermore, Proposition 4 in Hsu, et al. (2005a) shows that the value x_i^* is increasing in the ratio $(P^{i-1} - P^i)/c_i$. This implies that the condition of $x_i^* > x_{i+1}^*$ in Step 2 of the Decompose-and-Combine Algorithm in Hsu, et al. (2005a) is equivalent to the condition of $m_{(l_i,r_i)} < m_{(l_{i+1},r_{i+1})}$ in Step 2 of Algorithm 2. Therefore, we can apply Algorithm 2 to solve both systems with the same cluster formation.
(b) For each cluster i, it follows from (44) and the definition of $m_{(l_i,r_i)}$ that the optimal production quantity in the centralized system $Q^c_{[i]}$ is given by

$$m_{(l_i,r_i)} + 1 = \frac{1}{\bar{F}(Q^c_{[i]})}.$$

Also, it follows from Lemma 3(a) that the optimal production quantity in the decentralized system $Q^d_{[i]}$ is given by

$$m_{(l_i,r_i)} + 1 = \frac{1}{\bar{F}(Q^d_{[i]})} + R(Q^d_{[i]}).$$

Since $R(x) > 0$ for $x > 0$ and $\bar{F}(x)$ is decreasing in x, we have $Q^c_{[i]} > Q^d_{[i]}$.
(c) Since $Q_i^d > Q_i^c$ for all $i = 1, 2, ..., n$, the result follows directly by comparing (20) and (21). *q.e.d.*

References

[1] Barlow, R.E. and F. Proschan, *Mathematical Theory of Reliability*, John Wiley and Sons, New York, 1965.

[2] Bernstein, F. and G. A. DeCroix, "Decentralized Pricing and Capacity Decisions in a Multi-Tier System with Modular Assembly," *Management Science*, **50**, pp.1293-1308 (2004a).

[3] Bernstein, F. and G. A. DeCroix, "Inventory Policies in a Decentralized Assembly System," working paper, The Fuqua School of Business, Duke University, (2004b).

[4] Carr, S.C. and U.S. Karmarkar, "Competition in Multiechelon Assembly Supply Chains," *Management Science*, **51(1)**, pp.45-59 (2005).

[5] Chu, C., J. M. Proth and X. Xie, "Supply Management in Assembly Systems," *Naval Research Logistics*, **40**, pp.933-950 (1993).

[6] Fu, K., V. N. Hsu and C. Y. Lee, "Inventory and Production Decisions for an Assemble-To-Order System with Uncertain Demand and Limited Assembly Capacity," Working Paper (2005).

[7] Gerchak, Y., Y. Wang and C.A. Yano, "Lot Sizing in Assembly Systems with Random Component Yields," *IIE Transactions*, **26 (2)**, pp.19-24 (1994).

[8] Gerchak, Y. and Y. Wang, "Revenue-Sharing vs. Wholesale-Price Contracts in Assembly Systems with Random Demand," *Production and Operations Management*, **13(1)**, pp.23-33 (2004).

[9] Granot, D. and S. Yin, "Competition and Cooperation in a Multi-Manufacturer Single-Retailer Supply Chain with Complementary Products", working paper, Sauder School of Business, University of British Columbia, Vancouver, Canada, (2004).

[10] Gurnani, H., R. Akella and J. Lehoczky, "Optimal Order Policies in Assembly Systems with Random Demand and Random Supplier Delivery," *IIE Transactions*, **28**, pp. 865-878 (1996).

[11] Gurnani, H. and Y. Gerchak, "Coordination in Decentralized Assembly Systems with Uncertain Component Yields," Dept. of Management Sciences, University of Waterloo, (1998).

[12] Hopp, W. and M. Spearman, "Setting Safety lead times for Purchased Components in Assembly Systems," *IIE Transactions*, **25**, pp.2-11 (1993).

[13] Hsu, V. N., C. Y. Lee and K. C. So, "Optimal Component Stocking Policy for Assemble-To-Order Systems with Leadtime-Dependent Component and Product Pricing," *Management Science,* forthcoming (2005a).

[14] Hsu, V. N., C. Y. Lee and K. C. So, "Managing Components for Assemble-To-Order Products with Leadtime-Dependent Pricing: The Full-Shipment Model," Working Paper (2005b).

[15] Kumar, A., "Component Inventory Costs in an Assembly problem with Uncertain Supplier Lead-Times," *IIE Transactions*, **21**, pp.112-121 (1989).

[16] Shore, H., "Setting Safety Lead-times for Purchased Components in Assembly Systems: A general Solution Procedure," *IIE Transactions*, **27**, pp.634-637 (1995).

[17] Song, J. S., C. A. Yano and P. Lerssrisutiya, "Contract Assembly: Dealing with Combined Supply Lead Time and Demand Quantity Uncertainty," *Manufacturing and Service Operations Management*, **2**, pp. 287-296 (2000).

[18] Song, J. S. and P. Zipkin, "Supply Chain Operations: Assemble-to-Order," in *Handbooks in Operations Research and Management Science*, Vol. 30, edited by A. de Kok and S. Graves, North-Holland, Amsterdam, the Netherlands (2003).

[19] Tomlin, B., "Capacity Investments in Supply Chains: Sharing the Gain Rather Than Sharing the Pain," *Manufacturing and Service Operations Management*, **5(4)**, pp.317-333 (2003).

[20] Wang, Y., "Joint Pricing-Production Decisions in Supply Chains of Complementary Products with Uncertain Demand," working paper, Weatherhead School of Management, Case Western Reserve University, Cleveland, Ohio, (2004).

[21] Wang, Y. and Y. Gerchak, "Capacity Games in Assembly Systems with Uncertain Demand," *Manufacturing and Service Operations Management*, **5**, pp. 252-267 (2003).

[22] Yano, C., "Stochastic Leadtimes in Two-level Assembly Systems," *IIE Transactions*, **19**, pp. 371-378 (1987).

[23] Zhang, F., "Competition, Cooperation and Information Sharing in a Two-echelon Assembly System," working paper, Graduate School of Management of University of California at Irvine, (2004).

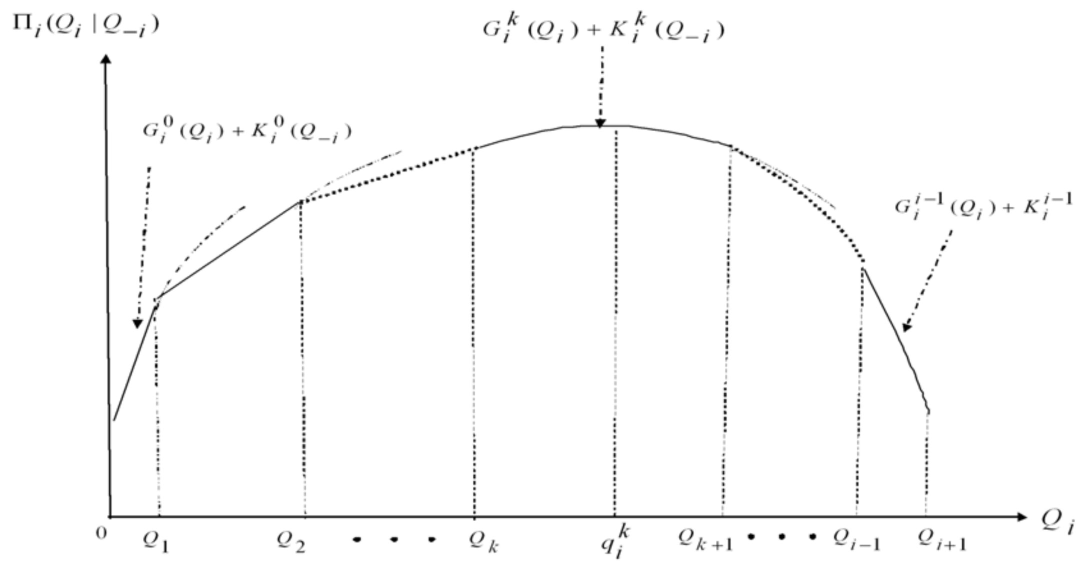

Figure 1: *Properties of the Profit Function of Supplier i.*