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**Demographic Structure and Entrepreneurial Activity**

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# **DEMOGRAPHIC STRUCTURE AND ENTREPRENEURIAL ACTIVITY**

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## **DEMOGRAPHIC STRUCTURE AND ENTREPRENEURIAL ACTIVITY**

Entrepreneurship promotes innovation and, by doing so, contributes to economic growth. Levels of entrepreneurial activity, however, vary considerably across countries. We show that, under certain conditions, the distribution of a population across age cohorts may have a significant effect on the aggregate level of entrepreneurship and that countries whose populations are excessively skewed toward old or young cohorts are likely to experience low levels of entrepreneurial activity. At a time when poorer countries confront unprecedented increases in population while several developed ones see their populations aging, our study provides important insights on the relationship between demographic structure and aggregate entrepreneurial activity.

*JEL: J10, M13, O10*

## **I. Theoretical Background**

Although the exact mechanism through which entrepreneurial activity influences economic growth is still under debate, most scholars agree that a significant amount of competition and innovation stem from the entrepreneurial sector (Acs and Audretsch 1990, Baumol 2002, Schumpeter 1934). Through a process of creative destruction, entrepreneurs challenge incumbent industries by introducing innovations that generate productivity increases (Michelacci 2003) and contribute to job creation (Bates 1990). The rates of entrepreneurial activity, however, differ significantly across countries. A variety of factors ranging from the availability of financing to the existence of supportive social norms have been shown to contribute to such differences. Among others, Evans and Jovanovic (1989) and Kihlstrom and Laffont (1979) have studied the role of financial resources, Leazar (2002) and Otani (1996) the role of prior experiences, Minniti (2005) the role of networks, and Hamilton (2000) that of labor market. To complement this literature, we argue that, under certain conditions, the distribution of the population across age cohorts is also an important determinant of a country's level of entrepreneurial activity.

Lindh and Malmberg (1999) provided evidence that economic growth depends on human resources and human needs, and that the age structure of the population shapes both of these factors. Becker et al. (1999) argued that the relationship between a population with a low average age and its per capita income depends on the net effects of the relative size of younger cohorts (which tends to reduce marginal product by increasing the supply of labor) and the potential for new markets (which tend to induce knowledge accumulation and competition). Denton et al. (1996) and Kremer (1993) have shown that productive capacity and output per capita are lower for both older and younger populations. Overall, this literature has shown that the age distribution of a population is important for per capita income, technological progress, and productivity.

Thanks to the spreading of basic medical care and the resulting decline in child mortality, many developing countries have experienced unprecedented population growth and a significant reduction of the average age of their populations (Becker et al. 1990, Deaton and Paxson 1997). In contrast, many richer countries have experienced declines in birth rates which, along with longer life expectancies, have

produced a significant increase of the average age of their populations (Weil 1999, Kosai et al. 1998). These changes raise a number of questions concerning the relationship between the age distribution of the population and macroeconomic performance (Bloom and Canning 2004, Holden et al. 2003), including the connection between aging and saving, size of the work force and growth, and economic incentives and immigration.<sup>1</sup> Variations in these relationships produce marked variations in important characteristics of the economy such as savings patterns and the capacity to innovate. Thus, the distribution of a population across age cohorts is likely to result also in skewed patterns of entrepreneurial activity.

Blanchflower (2004) has found evidence that the probability of starting a new business increases with age up to a threshold point and decreases thereafter. Similar evidence is presented in Figure 1 where the relative proportion of population engaged in entrepreneurial activity is depicted for a sample of more than 40,000 entrepreneurs across 18 countries. Figure 1 shows that the likelihood of being involved in entrepreneurship increases with age up to about 35 years and begins declining afterwards.<sup>2</sup> The existence of a systematic relationship between age and entrepreneurial behavior is also consistent with the literature on the life cycle of earnings showing that people invest in their human capital when they are young because by doing so they have a longer period over which they can receive income on their investment (Becker 1962, Ben-Porath 1967).

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Insert Figure 1 about here  
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In light of the systematic relationship between age and entrepreneurial behavior, we ask if, every thing else being the same, an economy with an *old* or a *young* population possesses a different capacity for generating new businesses and innovation than a population with a less skewed age distribution. Specifically, we complement existing literature by discussing what happens to the aggregate level of entrepreneurial activity when the age distribution of a population is significantly skewed toward younger

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<sup>1</sup> The 2003 special issue of the *Scandinavian Journal of Economics* Vol. 105 (3) and the volume edited by Sellon (2004) contain collections of important articles on these and related topics.

<sup>2</sup> Noticeably, the relationship between age and entrepreneurial activity shown in Figure 1 holds, with minor differences, also for each of the 18 countries in the sample.

or older cohorts, everything else being the same. The crucial point of our argument is the degree of substitutability in employment choices between different age groups in the work force.

Unlike most other professions that receive pre-agreed upon wages, entrepreneurship requires the willingness to commit to an uncertain income, as well as the willingness and ability to bear at least part of the risk of ownership, a characteristic distinguishing entrepreneurs from managers. In fact, the availability of financial resources, and wealth in particular, has been shown to be a crucial driver of entrepreneurship (Kihlstrom and Laffont 1979, Lindh and Ohlsson 1998). Individuals need resources to start new businesses. In countries with populations significantly skewed toward young individuals, many young individuals possess low accumulated wealth with which to reduce the uncertainty associated with new ventures. On the other hand, in countries with populations significantly skewed toward older individuals, many older individuals face high opportunity costs of entrepreneurship by having to forgo seniority wages in exchange for uncertain returns. In both cases, large portions of the population have relatively lower incentives to start new businesses.

In practice, wealth and its distribution across age cohorts are likely to be strongly influenced by institutional arrangements, such as legislation on retirement age and social security provisions. Lee et al. (2003), for example, showed how transfer systems may have significant effects on savings and capital accumulation. Also, using the example of OECD countries, Lindh (1999) showed that the age distribution of the population has pervasive effects on savings and economic growth. Clearly, the existence of appropriate institutions allowing for the frictionless redistribution of wealth across age cohort may mitigate (and possibly eliminate) these problems. In fact, in a country in which institutions allow the mobility of resources and in which factors are compensated with their market value, supply and demand for capital across age cohorts will automatically correct distortions. The self-correcting mechanism of the market, however, does not work when the mobility of wealth is limited, a problem more likely to emerge in poorer countries or when cross-border transfers are required as it would be the case for countries with heavily skewed population distributions.

In this paper we consider the interplay of age and wealth on entrepreneurial activity every thing else being the same. Consistently with life-cycle theories, we are interested in understanding under what conditions the non-linear relationship between age and wealth at the individual level has unintended consequences for entrepreneurship at the aggregate level and across countries. Since differences in age distributions are associated with differences in rates of savings (Bloom et al. 2003, Lee et al. 2003), and to the extent that entrepreneurial activity is responsible for much innovation and technical progress (Acs and Audretsch 1990, Baumol 2002), the omission of its relationship to age may have a significant impact on the validity of existing models depicting the relationship between demographic structure and economic growth. Our paper contributes to the elimination of this gap in the literature.

## **II. Age and Entrepreneurial Choices at the Individual Level**

In microeconomic models, entrepreneurship is traditionally described as an employment choice in which the availability of financing is a crucial determinant of an individuals' decision to become an entrepreneur (Blanchflower and Oswald 1998, Evans and Jovanovic 1989, Taylor 1996). Specifically, it has been shown that individuals with greater wealth are more likely to be involved in entrepreneurial activities. Consistently with this literature, we frame the formal decisional problem of the individual as an employment choice in which the expected income from entrepreneurship is a function of available resources. In addition, we introduce age as an important determinant of an individual's decision to become an entrepreneur.

In our model, individuals choose to be wage workers or entrepreneurs in order to maximize their utility, which is a function of income. At each unit of time, the individual chooses a working career. In our formulation,  $t$  indicates both the time period at which a career choice is made, i.e. the time lag from the beginning of an individual's working career, and the age of that individual. We use the latter interpretation for the remaining of the paper. At age  $t$ , if an individual works for a wage she receives a known net payment,  $w(t)$ . This known wage increases with respect to  $t$  as the individual gains seniority and experience. If, instead, the individual chooses to start an entrepreneurial venture, she commits her

working time to the exploitation of an opportunity and receives a random net payment,  $\tilde{\mathbf{p}}$ . Consistently with the literature, individuals are risk averse and the income from entrepreneurship, at age  $\mathbf{t}$ , is influenced by the individual's wealth,  $A(\mathbf{t})$ , which is assumed to accumulate with age.<sup>3</sup>

For any concave utility function  $u$ , an individual chooses to be an entrepreneur at time  $\mathbf{t}$  if

$$E_p u(\tilde{\mathbf{p}}(\mathbf{r}A(\mathbf{t})) + r\{1-\mathbf{r}\}A(\mathbf{t}), \mathbf{a}) = u(rA(\mathbf{t}) + w(\mathbf{t}), \mathbf{a}), \text{ and a wage worker if}$$

$$E_p u(\tilde{\mathbf{p}}(\mathbf{r}A(\mathbf{t})) + r\{1-\mathbf{r}\}A(\mathbf{t}), \mathbf{a}) = u(rA(\mathbf{t}) + w(\mathbf{t}), \mathbf{a}), \text{ where } \mathbf{a} \text{ represents the individual's degree}$$

of risk aversion,  $\mathbf{r} (\in [0,1])$  the proportion of wealth invested in entrepreneurial activity,  $r$  the interest rate

and  $E_p$  the expectation operator taken on the random variable  $\tilde{\mathbf{p}}$ . If an individual chooses at age  $\mathbf{t}$  to be

an entrepreneur, she invests a portion of her wealth  $\mathbf{r}A(\mathbf{t})$  into creating the business and receives a

random net payment of value  $\tilde{\mathbf{p}}(\mathbf{r}A(\mathbf{t}))$ .<sup>4</sup> In addition, that individual collects returns equal to  $r\{1-\mathbf{r}\}A(\mathbf{t})$

from the wealth she does not invest in the new business. If, instead, that individual chooses to be a wage

worker, in each time period she considers the returns from her wealth  $rA(\mathbf{t})$  and her wage  $w(\mathbf{t})$ .

Given a utility function for income  $y$  where  $u(y) = 1 - e^{-a \cdot y}$ , the degree of risk aversion  $\mathbf{a}$  of a randomly selected individual is a random variable following a Weibull probability density function of positive parameters  $\mathbf{g}$  and  $\mathbf{b}$ , where  $\mathbf{g}$  is a shape parameter and  $\mathbf{b}$  a scale parameter.<sup>5</sup> Wealth allows entrepreneurs to support their businesses, and more wealth reduces the variability of income associated with a new venture by increasing the likelihood of survival. As a result, entrepreneurial income  $\tilde{\mathbf{p}}$  is normally distributed with a positive mean  $\mathbf{m}_p$  and variance  $\mathbf{s}_p^2$ , where the variance diminishes as the

<sup>3</sup> This is the case because, consistently with the life-cycle theory of earnings (Ben-Porath 1967), older individuals are more likely to have accumulated wealth through equity in financial assets (e.g. stock market) and/or real assets (e.g. housing).

<sup>4</sup> As Kihlstrom and Laffont (1979), we model entrepreneurial income as an immediate (random) output derived from a production function in which, without loss of generality, the price of output is 1.

<sup>5</sup> The Weibull distribution is desirable because of its ability to accommodate any characterization of risk averse behavior.

portion of wealth invested,  $rA(t)$ , increases.<sup>6</sup> In an alternative to wealth, the reduction in the entrepreneurial income's variance over time can be explained by the fact that older individuals have accumulated more experience and other human capital resources and are, therefore, better able to reduce variations in their potential entrepreneurial income (Jovanovic 1994, Lazear 2002).<sup>7</sup>

Based on the above formulation, the likelihood of being an entrepreneur at age  $t$  is

$$l(t) = \Pr \left( E_p \left[ 1 - e^{-a[\tilde{p}(A(t)) + r(1-r)A(t)]} \mid \mathbf{a} \right] \geq 1 - e^{-a[rA(t) + w(t)]} \right) \quad (1)$$

which, from the moment generating function of a normal distribution of mean  $\mathbf{m}_p$  and variance  $\mathbf{s}_p^2(rA(t))$ ,

$$= \Pr \left( e^{-a r(1-r)A(t)} \cdot e^{-\mathbf{m}_p \mathbf{a} + \frac{\mathbf{s}_p^2(A(t)) \mathbf{a}^2}{2}} \leq e^{-a[rA(t) + w(t)]} \right) = \Pr \left( \mathbf{a} \leq \frac{2[\mathbf{m}_p - r r A(t) - w(t)]}{\mathbf{s}_p^2(r A(t))} \right). \quad (2)$$

Therefore, from the cumulative density function of a Weibull distribution,

$$l(t) = \begin{cases} 1 - e^{-\left\{ \frac{2[\mathbf{m}_p - r r A(t) - w(t)]}{\mathbf{s}_p^2(r A(t))} \right\}^g}, & \mathbf{m}_p - r r A(t) > w(t) \\ 0, & \mathbf{m}_p - r r A(t) \leq w(t), \end{cases} \quad (3)$$

where  $r r A$  represents the opportunity cost of the new business. That is, the returns the individual would have received had she not invested that portion of wealth in the new venture.

The tradeoff between risky entrepreneurial income and riskless wage labor changes with age. Thus, the likelihood of being an entrepreneur depends on the time unit  $t$  at which it is evaluated and, as an individual ages (or, equivalently, as  $t$  increases), the likelihood of being an entrepreneur increases up to a critical threshold age beyond which it begins declining. Formally,

<sup>6</sup> Typically, income from wage labor possesses a positive lower bound value and it is often modeled as following a Pareto distribution (e.g. Singh and Maddala 1976). Entrepreneurial income, on the other hand, can be negative since entrepreneurs may run out of cash and fail.

<sup>7</sup> It can be easily shown that our results hold even when entrepreneurial earnings are expected to increase with the portion of wealth invested in the new business.

PROPOSITION 1: If  $\frac{m_p - r\mathbf{r} A(0) - w(0)}{s_p^2(\mathbf{r} A(0))} > \frac{d[r\mathbf{r} A + w]}{dt}\Big|_{t=0} / \left( -\frac{ds_p^2}{d[\mathbf{r} A]} \frac{d[\mathbf{r} A]}{dt} \right)\Big|_{t=0}$ , then there

exists an age  $t^*$  before which the likelihood of being an entrepreneur increases with age and after which it decreases until it eventually reaches zero. (Proof is provided in the appendix.)

Proposition 1 shows the existence of an inverted U-shaped relationship between age and entrepreneurial activity at the individual level. The left-hand side of the inequality measures the relative entrepreneurial premium. That is, it measures the weighted difference between expected entrepreneurial income and the income from wealth and wage at the beginning of an individual's career.<sup>8</sup> The right-hand side, instead, measures the relative increase in income from non-entrepreneurial activities over time also at the beginning of an individual's career. As long as the entrepreneurial premium exceeds the relative increase in income from non-entrepreneurial activity, the likelihood of being an entrepreneur increases with age up to a critical threshold point. Beyond that point, however, the entrepreneurial premium is smaller than the relative increase in income from non-entrepreneurial activity and the likelihood of an individual becoming an entrepreneur decreases with age until it eventually reaches zero. It is easy to show that the inverted U-shaped relationship between age and entrepreneurial activity holds true even if risk aversion is modeled as a monotone function of age.

### III. Age Distribution and Aggregate Entrepreneurial Activity

The individual model developed in the previous section provides the micro-foundation for our aggregate model of entrepreneurial activity. A population is composed of individuals of various ages. For any given age  $t$ , there exists a certain percentage of individuals from that population who are of that age. Let  $f(t;t)$  be the probability density at age  $t$  for that population, where  $t$  represents the time period at which this population is observed. Let  $L$  be a lower bound on the population's working age and  $U$  an upper bound. Individuals in a population are heterogeneous based on their degree of risk aversion, age and wealth. We have shown that for any age  $t \in [L, U]$  a randomly selected individual of that age possesses a likelihood

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<sup>8</sup> Without loss of generality,  $t = 0$  can be interpreted as the beginning of an individual's career as opposed to age 0.

$l(\mathbf{t})$  of being an entrepreneur. Hence, the aggregate level of entrepreneurial activity at time period  $t$ ,  $\Phi(t)$ , is obtained by adding individual entrepreneurial activities weighted by the probability density of the individuals' ages over the age interval  $[L,U]$ . Formally,

$$\Phi(t) = \int_L^U f(\mathbf{t};t)l(\mathbf{t})d\mathbf{t} . \quad (4)$$

It then follows from Proposition 1 that when the distribution of individuals across age cohorts is skewed toward old or young individuals, the aggregate level of entrepreneurial activity is low. Formally,

PROPOSITION 2: A population for which the distribution of individuals across age cohorts is skewed either toward old or toward young individuals will exhibit a low level of aggregate entrepreneurial activity.

Different countries possess different population distributions and no single distribution can approximate all countries (Horiuchi and Preston 1988, Kim 1986). Nevertheless, it is possible to fit most countries to a set of standard distributions which are capable of approximating the most common demographic structures (Keyfitz 1966). In countries exhibiting high net birth rates, for example, the probability density function of the population is likely to exhibit highest frequencies at lower age cohorts. On the other hand, in countries with low or negative net birth rates, the probability density function of the population is likely to exhibit highest frequencies somewhere over the mid-range of the working-age interval or possibly be skewed to the right. Consistently with those observations, we consider, in turn, cases in which the age distribution of the population follows an exponential, a normal, and a lognormal probability density function. We then use these general cases to study how different demographic structures may influence the level of entrepreneurial activity of that population, every thing else being equal.

In some countries, the probability density function of a population across working-age cohorts may be best approximated by an exponential function with mean  $1/I(t)$ . Such countries would include countries whose population's age density is likely to be skewed to the right, showing a high density for

young individuals and decreasing densities as cohorts of older individuals are considered. Figure 2 (a) and (b) shows that such countries include Kenya and Uganda.

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 Insert Figure 2 about here  
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Noticeably, since our analysis focuses on the working-age interval, the age distribution only needs to fit an exponential distribution over the age interval  $[L,U]$ . Thus, the probability density for any age  $t$  is approximated by  $f(t;t) = I(t)e^{-I(t)t}$  and the change in aggregate entrepreneurial activity is described by the first order derivative of Equation 4. Specifically,

$$\frac{d\Phi(t)}{dt} = \frac{dI(t)}{dt} \int_L^U \left[ \frac{1}{I(t)} - t \right] I(t) e^{-I(t)t} l(t) dt . \quad (5)$$

In other countries, the probability density function of a population across working-age cohorts may be best approximated by a normal density function with a mean  $m(t)$  and variance  $S^2$ . Figure 2 (c) and (d) shows that such countries may include Australia and the United States. In this case, the effects of age asymmetries on our results are expected to be of little significance. In those countries, since an individual's age can only be positive and finite, the probability density is truncated and approximated for any age  $t$  by  $f(t;t) = \frac{1}{\sqrt{2\pi S}} e^{-\frac{[t-m(t)]^2}{2S^2}}$ , and the change over time in a population's entrepreneurial activity is

$$\frac{d\Phi(t)}{dt} = \frac{dm(t)}{dt} \cdot \frac{1}{S^2} \int_L^U [t - m(t)] \frac{1}{\sqrt{2\pi S}} e^{-\frac{[t-m(t)]^2}{2S^2}} l(t) dt . \quad (6)$$

Finally, the probability density function of a working population across age cohorts may be best represented by a lognormal density function where the logarithm of the age variable has a normal distribution of mean  $m(t)$  and variance  $S^2$ . Figure 2 (e) and (f) shows that such countries include Iran and

Yemen whose population's age density is skewed to the right on the working-age interval. In this case,

$$f(\mathbf{t}; t) = \frac{1}{\sqrt{2ps t}} e^{\frac{-1}{2s^2}[\ln t - \mathbf{m}(t)]^2} \quad \text{and the change in a population's entrepreneurial activity is}$$

$$\frac{d\Phi(t)}{dt} = \frac{d\mathbf{m}(t)}{dt} \frac{1}{s^2} \int_L^U [\ln t - \mathbf{m}(t)] \frac{1}{\sqrt{2ps t}} e^{\frac{-1}{2s^2}[\ln t - \mathbf{m}(t)]^2} l(t) dt . \quad (7)$$

If a population's average age changes, its aggregate level of entrepreneurial activity changes too. Noticeably, this change may not be in the same direction as that of the average age because the integrand term in Equations 5, 6, or 7 can be both positive and negative. It is now possible to derive the sufficient conditions under which changes in the average age of a population lead to decreases in the aggregate level of entrepreneurial activity of that population. These conditions are based on changes in the population's average age, on how this average age compares to the ages  $L$  and  $U$  at which individuals begin and stop, respectively, their working careers, and, as per Proposition 1, on the critical threshold age,  $\mathbf{t}^*$ , at which an individual's probability of being an entrepreneur is maximized.

#### IV. Entrepreneurial Activity in Aging Countries and in Countries Getting Younger

##### IV.I Aging Countries

We identify *aging countries* as those countries whose average age is growing beyond the critical threshold age at which individual entrepreneurial activity peaks. Let  $d_U$  be the *career-exit differential*. That is, for any time period  $t$ , let  $d_U$  represents the difference between the age at which individuals exit the labor force and the population's average age. When age is exponentially distributed,  $d_U = U - [1/I(t)]$ , and when age is normally distributed,  $d_U = U - \mathbf{m}(t)$ . When age is lognormally distributed, instead,  $d_{\ln U} = \ln U - \mathbf{m}(t)$ . Then,

##### PROPOSITION 3

- (a) When age is *exponentially* or *normally* distributed, if  $[1/I(t)] - d_U > \mathbf{t}^*$  or  $\mathbf{m}(t) - d_U > \mathbf{t}^*$ , respectively,

or

(b) when age is *lognormally* distributed, if  $\mathbf{m}(t) - d_{\ln U} > \ln L + [t^* - L]$ ,

then the rate at which aggregate entrepreneurial activity changes possesses the opposite sign of the rate at which a population average age changes over time. (Proof is provided in the appendix.)

Thus, a population facing an increase in its average age would also face a decrease of its aggregate level of entrepreneurial activity. Regardless of how the population is distributed, the left-hand side of the conditions required by Proposition 3 measures, on average, the *maturity* of the work force, where maturity describes how far into their working career people are or, in other words, how many years they are from retiring.<sup>9</sup> If that number exceeds the critical threshold age (or some transformation of it for the lognormal case) at which the likelihood of becoming an entrepreneur is maximized (shown on the right-hand side of the conditions), then that population will face a decrease in its aggregate level of entrepreneurial activity. Consider, for example, a population in which the average age is 50, the retirement age is 65, and the likelihood of becoming an entrepreneur is maximized at age 32. If that population is aging, the level of entrepreneurial activity will decline because the proportion of individuals who are now 35 or older, an age which exceeds the most propitious “entrepreneurial age” of 32, is growing. The rate of decline in entrepreneurial activity will depend, of course, on the specific distribution of this population across age cohorts. The sufficient conditions described in Proposition 3 are consistent with situations in which either individuals maximize their likelihood of being entrepreneurs when they are still very young or the age at which individuals retire is not too far ahead from the population’s average age. Aging populations are clearly more likely to fit the latter situation.

#### IV.II. *Countries getting younger*

We identify as *countries getting younger* those countries whose average age is decreasing below the critical threshold age  $\mathfrak{a}$  which individual entrepreneurial activity peaks. Let  $d_L$  be the *career-entry differential*. That is, for any time period  $t$ , let  $d_L$  represent the difference between the population’s average

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<sup>9</sup> Notice that the *maturity* of a population depends also on  $U$ , the retirement age.

age and the age at which individuals enter the labor force. When age is normally distributed,  $d_L = \mathbf{m}(t) - L$ , and when age is lognormally distributed,  $d_{\ln L} = \mathbf{m}(t) - \ln L$ . Finally, when age is exponentially distributed, we simply compare its average to the age at which individuals enter the labor force. Then,

PROPOSITION 4

(a) When age is *exponentially* distributed, if  $1/\mathbf{I}(t) < L$ ;

or

(b) when age is *normally* distributed, if  $\mathbf{m}(t) + d_L < \mathbf{t}^*$ ;

or

(c) when age is *lognormally* distributed, if  $\mathbf{m}(t) + d_{\ln L} < \ln L + [\mathbf{t}^* - L]$ ,

then the rate at which aggregate entrepreneurial activity changes possesses the same sign as the rate at which a population average age changes over time. (Proof is provided in the appendix.)

Thus, a population facing a decrease in its average age would also face a decrease of its aggregate level of entrepreneurial activity. The intuitive meaning of Proposition 4 is analogous to that of Proposition 3. The left-hand side of the conditions required by Proposition 4 (b) and (c) measures the maturity of the work force. If that number is below the critical threshold age at which the likelihood of becoming an entrepreneur is maximized, then the population will experience a decrease in its aggregate level of entrepreneurial activity. Noticeably, when the population is exponentially distributed, the condition in Proposition 4 (a) only requires comparing the average age of the population with the age at which individuals enter the labor force. Similarly to the case of aging countries, and regardless of how the population is distributed, the rate of decline in entrepreneurial activity depends on the specific distribution of this population across age cohorts. In this case, however, the rate of entrepreneurial activity declines because of the growing proportion of individuals in the population who have not yet reached the critical threshold level at which the likelihood of being an entrepreneur is maximized. In fact, before the critical threshold is reached, involvement in entrepreneurial activity decreases as progressively younger individuals are considered. Since the critical age at which individuals maximize their likelihood of being

an entrepreneur occurs later in their working career, the aggregate level of entrepreneurial activity of a young population that grows younger decreases as time goes by. The sufficient condition described in Proposition 4 are consistent with situations in which either individuals maximize their likelihood of being entrepreneurs when they are relatively old or the age at which individuals begin their working career is not too far behind the population's average age. Populations that are getting younger are more likely to fit the latter situation.

When the sufficient conditions required by Propositions 3 and 4 are violated, the sign of the corresponding integral in Equations 5-7 cannot be univocally determined. As a result, the behavior of a population's aggregate level of entrepreneurial activity as changes occur in its average age cannot be predicted. It is important noticing that, for both aging countries and countries getting younger, our conclusions hold true even if the shape of the age distribution of the population should change over time. In fact, for the families of distributions discussed above, our sufficient conditions rely only on the relationship between the population's average age, the ages at which individuals begin and stop their working career, and the critical threshold age at which an individual's probability of being an entrepreneur is maximized. What changes in net birth rate are consistent with the age density functions we have analyzed is, of course, an empirical, country specific question (Keyfitz 1966).

## **V. Population Characteristics and Aggregate Entrepreneurial Activity**

Economic growth is fueled by innovation, competition and productivity increases (Baumol 1993, 2002). Entrepreneurs play a significant role in each of these characteristics of the economy (Acs and Audretsch 1990, Wennekers and Thurik 1999). A population lacking a sufficient proportion of entrepreneurs may suffer a reduced growth rate (Baumol 2002, Schumpeter 1934). We have shown that the distribution of a population across age cohorts matters because entrepreneurial decisions are not neutral with respect to age. At the individual level, the opportunity costs of entrepreneurship are higher at early and advanced ages. Younger individuals possess lower accumulated wealth with which to reduce the uncertainty associated with new ventures, whereas older individuals have much more to lose by forgoing seniority wages in

favor of uncertain returns. This inverted U-shape relationship between age and entrepreneurial attitude is compounded at the aggregate level. As a result, *young* and *old* countries are likely to exhibit low levels of entrepreneurship. In addition, countries whose average age is moving further away (either above or below) from the critical threshold age at which an individual's likelihood of being an entrepreneur is maximized are also likely to experience a decline in their aggregate level of entrepreneurial activity.

In a country where productive factors are allowed to move freely and are compensated according to the value of their marginal product, market forces eliminate discrepancies between demand and supply for entrepreneurial activity by allowing financial resources as well as people of various ages to relocate appropriately. Unfortunately, institutional rigidities often exist that prevent or reduce significantly cross-country and even cross-generational exchanges. As a result, the market for entrepreneurship does not clear and, under the conditions highlighted in Propositions 2, 3, and 4, a shortage of entrepreneurial activity may result.

In aging countries, immigration can provide a good guarantee against long-run declines in entrepreneurial activity. Arthur and Espenshade (1988) have shown that, in the United States and in countries where fertility rates are below replacement levels, both population size and age composition respond to variation in immigrants' ages. Espenshade et al. (1982) and Coale (1987) have also shown the importance that immigration has for the age structure of a population, among other things, because of its effects on productivity. In addition, Storesletten (2003) has shown that the fiscal effects of immigration are extensive and depend strongly on the age of the immigrants. Specifically, Storesletten shows that immigrants who are 20-30 years of age give rise to large gains, whereas older immigrants represent large net costs to the government. In fact, new immigrants tend to be concentrated in age brackets favorable to entrepreneurial behavior and relatively new immigrant groups tend to be associated to high rates of new firm formation. Unfortunately, policy makers seldom pay attention to the demographic and economic implications of influencing the ages at which immigrants are admitted in a country. Only recently, some countries such as the United States, Australia and Canada, have adopted age as one of the criteria determining admission. In those countries, policy questions center on which ages should be favored and

on how much weight age should receive in relation to other factors likely to influence the productivity of new immigrants.

Furthermore, it has been shown that the negative consequences of population decline can be avoided, for example, if mechanisms aiming at increasing technological innovation and efficiency are sufficiently effective (Beaudry and Collard 2003, Kosai et al. 1998). The intuition is that, although an aging population and work force may lead to less innovation and, therefore, lower economic growth, the constraint by itself stimulates efficiency-augmenting technological changes by tightening labor and capital resource constraints.

In young countries or in countries that are getting younger, on the other hand, the problem of sustaining entrepreneurial activity may be more difficult to address. New growth theory has allowed research on demographic structure to evolve and improve by progressively treating key demographic variables as endogenous, rather than exogenous, to the growth process (Ehrlich and Lui 1997). In this context, Aghion and Howitt (1992) and Howitt (1999) have shown that economic growth is possible even when population size increases. Similarly, Bloom and Freeman (1988) have shown how some developing countries have been able to shift their work force from agriculture to the industry and service sectors thereby increasing productivity despite the rapid growth in their populations. In many of these countries, however, a large number of young individuals compete for a limited amount of resources. Typical issues of moral hazard and adverse selection set in mechanisms that, because of financial intermediaries' inability to identify entrepreneurial quality, lead to excessive lending and investment in low-return projects (de Meza and Webb 1999). Consistently, the inability of people without sufficient credit and collateral to obtain credit becomes one of the most serious impediments to the creation of new businesses and to the exploitation of opportunities for self-employment.

In general, the age distribution of the population is a structural parameter of the economy and cannot be changed quickly. To alter the way in which individuals of different ages think about themselves and their role in society takes a long time. As a result, short term intervention may not be an effective way to support the level of entrepreneurial activity in a country. Entrepreneurship is, to a large extent, an

embedded phenomenon and may flourish only when the appropriate institutions are in place. Many of the institutions that mold and encourage entrepreneurial decisions are not written but, rather, inherited and learned through direct observation. As in the case of other embedded phenomena, intergenerational attritions reduce the ability of a population to perpetuate institutions and routines.

Lindh (1999) argued that policies that ignore the macroeconomic implications of the age distribution are likely to be faulty and potentially costly decisions. If age is indeed a triggering factor of entrepreneurship, it is important to take its influence into account when considering what policies and institutional settings are more conducive to entrepreneurial behavior. In general, programs aimed at increasing the level of entrepreneurial activity by changing socio-economic incentives such as, for example, taxation or subsidies to small businesses are likely to be less effective if the age distribution of the population is heavily skewed. Depending on the age distribution of the population, policies affecting a population's average age (e.g. immigration laws) and the age at which individual entrepreneurial activity peaks (e.g. unionized wage contracts and unemployment benefits) may be more effective in influencing the level of aggregate entrepreneurial activity over time. For example, in aging countries, it seems plausible that people would begin to think beyond the 65-year-old benchmark as the endpoint of their working lives. This has implications for governments who are thinking of imposing or removing mandatory retirement laws.

Finally, in this paper, we focus on monetary rewards because we are interested in understanding the relationship between aggregate entrepreneurial activity and the age structure of the population, a macroeconomic characteristic of a country that, because of its implications for the distribution of resources, has an unavoidable effect on the aggregate level of entrepreneurship when market rigidities are present. In our model, age is concomitant to acquiring wealth and with the accumulation of human capital and work experiences as reflected by changes in the opportunity cost of entrepreneurial activity at the individual level. Of course, other variables such as personal preferences for independence or education are also important for entrepreneurial decisions. Depending on specific cultural factors and on the level of development of a country, the effect of the age distribution of the population on entrepreneurial activity

may be augmented or reduced by the influence of factors not included in our model. We do acknowledge their potential importance, their effect on entrepreneurship, however, is beyond the scope of this paper. In our model, their influence is implicitly captured by the specific shape of the age density function and by the country specific upper and lower bounds defining exit and entry in the labor force.

## **VI. Suggestions for Further Research**

Our study can be expanded in several ways. First, we prove our results for populations distributed following an exponential, normal, or lognormal distribution with respect to the age of their inhabitants. Clearly, other probability density functions are possible and perhaps more accurate for specific countries. Fitting data of one particular country to a specific distribution is an exercise useful in determining the effect of changes in the distribution of the population across age cohorts for that country. This is not, however, the purpose of this paper. Our purpose is to develop a general theoretical point showing that, under certain conditions, the distribution of individuals across age cohorts matters for the aggregate level of entrepreneurial activity and has potential implications for economic growth. The validation of our theory for specific countries is an empirical issue that we hope will generate further research.

Second, our theoretical model can be extended to describe the market dynamics between supply and demand for entrepreneurial activity. In fact, the equilibrium level of entrepreneurship in a country is the outcome of the relative strength of supply and demand forces. Baumol (2002) has provided a detailed analysis of aggregate conditions influencing nascent entrepreneurship and argued that technology, level of economic development, culture, and institutions all influence the demand for entrepreneurship by creating opportunities available for start-ups. He has also suggested that cultural and institutional conditions have an impact on the supply of entrepreneurship because of their ability to influence the skills, resources and preferences of individuals within the population. The links between allocation of resources across generations and demographic structure are complex, and our *ceteris paribus* model provides only a partial description of their relevance for entrepreneurial activity. The purpose of our model, however, is to point

out the importance of demographic structure for aggregate entrepreneurship by showing how the former matters even when all other factors are assumed unchanged.

Third, appropriately modified, our model could be used to link demographic structure to the determination of a country's optimal rate of entrepreneurship. The innovations entrepreneurs generate often convey increasing returns for the rest of society, making everyone else more productive. Thus, high economic growth may require some segment of the population to be entrepreneurs, but it may not be necessary for that segment to be very large. Beaudry and Collard (2003), for example, have shown that the role played by technology and innovation is crucial in understanding differences in economic performance across countries whose populations are getting younger. Specifically, they use cross-country data to show why differences in rates of growth in working-age population may be a key to understanding differences in economic performance across industrialized countries. In particular, they argue that countries with lower rates of adult population growth adopted new capital-intensive technologies more quickly than their high population growth counterparts, therefore allowing them to reduce their work time without deterioration of growth in output-per-adult.

Overall, our model suggests that the aggregate level of entrepreneurial activity is indeed influenced by the age distribution of the population and, in particular, that such influence is negative for very young and very old populations, as well as for populations whose average age is increasing above or decreasing below the critical threshold age at which individual entrepreneurial activity is maximized. Within this context, we speculate that, in the presence of institutional rigidities preventing the redistribution of resources across cohorts and countries, attempts to increase the substitutability between groups of individuals from different age cohorts may have negligible effects on the aggregate level of entrepreneurial activity. To the extent that entrepreneurial activity contributes to innovation and technological progress, as well as development and the elimination of poverty, the omission of its relationship to age may cause significant inaccuracy in models depicting the relationship between demographic structure and economic growth.

## Appendix

PROOF OF PROPOSITION 1. The first order derivative of the likelihood of being an entrepreneur is

$$\frac{dl(t)}{dt} = \begin{cases} \frac{2g}{b[s_p^2(rA(t))]^2} e^{-\left\{\frac{2[m_p - rA(t) - w(t)]}{bs_p^2(rA(t))}\right\}^g} \left\{\frac{2[m_p - rA(t) - w(t)]}{bs_p^2(rA(t))}\right\}^{g-1} \times \\ \left\{ [m_p - rA(t) - w(t)] \left( -\frac{ds_p^2}{d[rA]} \right) r \frac{dA}{dt} - s_p^2(rA(t)) \left[ r r \frac{dA}{dt} + \frac{dw}{dt} \right] \right\}, & m_p - rA(t) > w(t) \\ 0, & m_p - rA(t) \leq w(t). \end{cases} \quad (\text{A1})$$

Hence, when  $m_p - rA(t) > w(t)$  the sign of the first order derivative of this likelihood equals to the sign of

$$[m_p - rA(t) - w(t)] \left( -\frac{ds_p^2}{d[rA]} \right) r \frac{dA}{dt} - s_p^2(rA(t)) \left[ r r \frac{dA}{dt} + \frac{dw}{dt} \right]. \quad (\text{A2})$$

Since we assumed that  $\frac{m_p - rA(0) - w(0)}{s_p^2(rA(0))} > \frac{d[rA + w]}{dt} \Big|_{t=0} / \left( -\frac{ds_p^2}{d[rA]} \frac{d[rA]}{dt} \right)_{t=0}$ , this

expression is positive for  $t$  small enough. But this expression becomes negative because both the opportunity cost of the new business  $rA(t)$  and the wage  $w(t)$  increase with  $t$  to eventually reach expected entrepreneurial income  $m_p$ . Once the wage exceeds expected entrepreneurial income net of opportunity cost, the likelihood and its first order derivative with respect to  $t$  equal zero.

PROOF OF PROPOSITION 3. We begin with the case where the distribution of a population across

age cohorts is exponential. If  $t^* < \frac{1}{I(t)} < U$  we distinguish between  $t^* < \frac{1}{I(t)} - d_U$  and

$t^* > \frac{1}{I(t)} - d_U$ , where  $d_U = U - \frac{1}{I(t)}$ . In the first case the integral in Equation 5 is broken into three

parts: from  $L$  to  $\frac{1}{I(t)} - d_U$ , from  $\frac{1}{I(t)} - d_U$  to  $\frac{1}{I(t)}$ , and from  $\frac{1}{I(t)}$  to  $U$ . Since  $l$  is always positive, the

integral from  $\frac{1}{I(t)} - d_U$  to  $\frac{1}{I(t)}$  is positive and the integral from  $\frac{1}{I(t)}$  to  $U$  is negative. Let

$$Q(t) \equiv \left[ \frac{1}{I(t)} - t \right] l(t) e^{-l(t)t}. \text{ Since } l \text{ is decreasing after } \frac{1}{I(t)} - d_U, \text{ we note that}$$

$$\left| \int_{1/I(t)}^U Q(\mathbf{t})l(\mathbf{t})d\mathbf{t} \right| < \left| \int_{1/I(t)}^U Q(\mathbf{t})l\left(\frac{1}{I(t)} - \left[\mathbf{t} - \frac{1}{I(t)}\right]\right)d\mathbf{t} \right|. \quad (\text{A3})$$

Since  $\mathbf{t}$  and  $\left(\frac{1}{I(t)} - \left[\mathbf{t} - \frac{1}{I(t)}\right]\right)$  are at the same distance from the mean and the exponential density function is smaller for the former than for the latter, we also note that

$$\left| \int_{1/I(t)}^U Q(\mathbf{t})l\left(\frac{1}{I(t)} - \left[\mathbf{t} - \frac{1}{I(t)}\right]\right)d\mathbf{t} \right| < \left| \int_{1/I(t)}^U Q\left(\frac{1}{I(t)} - \left[\mathbf{t} - \frac{1}{I(t)}\right]\right)l\left(\frac{1}{I(t)} - \left[\mathbf{t} - \frac{1}{I(t)}\right]\right)d\mathbf{t} \right|. \quad (\text{A4})$$

By changing variables, the right-hand side equals  $\left| \int_{\frac{1}{I(t)} - d_U}^{1/I(t)} Q(\mathbf{t})l(\mathbf{t})d\mathbf{t} \right|$ . Therefore, the sum of the last two integrals is positive. The first integral is also positive. However, if  $\mathbf{t}^* > \frac{1}{I(t)} - d_U$ , we cannot draw such a conclusion because  $l$  begins increasing and may not balance off the negative integral from  $\frac{1}{I(t)}$  to  $U$ .

This also implies that, as  $\frac{1}{I(t)}$  increases and hence  $I(t)$  decreases, the sufficient condition

$$\mathbf{t}^* < \frac{1}{I(t)} - d_U \text{ holds true and } \frac{dI(t)}{dt} < 0 \text{ leads to } \frac{d\Phi(t)}{dt} < 0.$$

We use the same approach for the case where the distribution of a population across age cohorts is normal. If  $\mathbf{t}^* < \mathbf{m}(t) < U$  we distinguish between  $\mathbf{m}(t) - d_U > \mathbf{t}^*$  and  $\mathbf{m}(t) - d_U < \mathbf{t}^*$ , where  $d_U = U - \mathbf{m}(t)$ . In the first case the integral in Equation 6 is broken into three parts: from  $L$  to  $\mathbf{m}(t) - d_U$ , from  $\mathbf{m}(t) - d_U$  to  $\mathbf{m}(t)$ , and from  $\mathbf{m}(t)$  to  $U$ . Since  $l$  is always positive, the integral from  $\mathbf{m}(t) - d_U$  to  $\mathbf{m}(t)$  is negative and the integral from  $\mathbf{m}(t)$

to  $U$  is positive. Let  $Q(\mathbf{t}) \equiv [\mathbf{t} - \mathbf{m}(t)] \frac{1}{\sqrt{2ps}} e^{-\frac{[\mathbf{t} - \mathbf{m}(t)]^2}{2s^2}}$ . Since  $l$  is decreasing after  $\mathbf{m}(t) - d_U$ , we note that

$$\left| \int_{\mathbf{m}(t)}^U Q(\mathbf{t})l(\mathbf{t})d\mathbf{t} \right| < \left| \int_{\mathbf{m}(t)}^U Q(\mathbf{t})l(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])d\mathbf{t} \right|, \quad (\text{A5})$$

which, by anti-symmetry of  $Q(\mathbf{t})$  with respect to  $\mathbf{m}(t)$ ,

$$= \left| \int_{\mathbf{m}(t)}^U Q(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])l(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])d\mathbf{t} \right|, \quad (\text{A6})$$

and, by changing variables,

$$= \left| \int_{\mathbf{m}(t)-d_U}^{\mathbf{m}(t)} Q(t)l(t)dt \right|. \quad (\text{A7})$$

Therefore, the sum of the first two integrals is negative. The third integral is also negative. However, if  $\mathbf{m}(t)-d_U < \mathbf{t}^*$ , we cannot draw such a conclusion because  $l$  begins increasing and may not balance off the positive integral from  $\mathbf{m}(t)$  to  $U$ . This means that, as  $\mathbf{m}(t)$  increases, the sufficient condition  $\mathbf{m}(t)-d_U > \mathbf{t}^*$  holds true and  $\frac{d\mathbf{m}(t)}{dt} > 0$  leads to  $\frac{d\Phi(t)}{dt} < 0$ .

Finally, we consider the case where the age distribution is lognormal. Let  $Z$  be a random variable representing the logarithm of the age variable. Then, for any age  $t$  there exists a realization  $z$  with  $t = e^z$ . As a result,  $dt = e^z dz$ , and  $t = L$  leads to  $z = \ln L$  whereas  $t = U$  leads to  $z = \ln U$ . By assumption,  $Z$  is normally distributed with mean  $\mathbf{m}(t)$  and variance  $\mathbf{s}^2$ . Consequently, Equation 7 is equivalent to

$$\int_{\ln L}^{\ln U} [z - \mathbf{m}(t)] \frac{1}{\sqrt{2ps}} e^{\frac{-1}{2s^2}[z - \mathbf{m}(t)]^2} l(e^z) dz. \quad (\text{A8})$$

The only differences with the proof for a normal age distribution are (1) the likelihood  $l(z)$  of being an entrepreneur at age  $z$  is substituted by  $l(e^z)$  and (2) the integral is taken on the interval  $[\ln L, \ln U]$  rather than  $[L, U]$ . In Proposition 1 we show that  $l$  increases in its component  $t$  up until a critical value  $\mathbf{t}^*$  and then decreases thereafter. Hence, with  $t = e^z$ ,  $\frac{dl}{dz} = \frac{dl}{dt} \frac{dt}{dz} = \frac{dl}{dt} e^z$  and, as a result,  $l(e^z)$  also increases in its component  $z$  up until a critical value  $z^*$  and then decreases thereafter. We note that the critical value  $z^* = \mathbf{t}^*$  since  $\frac{dl}{dz} = 0$  if and only if  $\frac{dl}{dt} = 0$ . Therefore, it is straightforward to verify that the proof for the normal age distribution applies to the right-hand side of Equation A8 where  $L, U, \mathbf{t}^*$  and  $d_U = U - \mathbf{m}(t)$  are substituted by, respectively,  $\ln L, \ln U, \ln L + [\mathbf{t}^* - L]$  and  $d_{\ln U} = \ln U - \mathbf{m}(t)$ . This also implies that, as a population average age increases so does the mean of the logarithm of age  $\mathbf{m}(t)$ , and the sufficient condition  $\ln L + [\mathbf{t}^* - L] < \mathbf{m}(t) - d_{\ln U}$  holds true and  $\frac{d\mathbf{m}(t)}{dt} > 0$  leads to  $\frac{d\Phi(t)}{dt} < 0$ .

**PROOF OF PROPOSITION 4.** We begin with the case where the distribution of a population across age cohorts is exponential. If  $\frac{1}{\mathbf{I}(t)} < L$ , then the integral in Equation 5 is negative. As a result, the sign

of  $\frac{d\Phi(t)}{dt}$  is the opposite of that of  $\frac{d\mathbf{I}(t)}{dt}$ . This implies that, as a population average age  $\frac{1}{\mathbf{I}(t)}$  decreases,

$\mathbf{I}(t)$  increases and thus  $\frac{d\Phi(t)}{dt} < 0$ .

Next, we look at the case where the distribution of a population across age cohorts is normal. If  $L < \mathbf{m}(t) < \mathbf{t}^*$ , we distinguish between  $\mathbf{m}(t) + d_L < \mathbf{t}^*$  and  $\mathbf{m}(t) + d_L > \mathbf{t}^*$ , where  $d_L = \mathbf{m}(t) - L$ . In the first case the integral in Equation 6 is broken into three parts: from  $L$  to  $\mathbf{m}(t)$ , from  $\mathbf{m}(t)$  to  $\mathbf{m}(t) + d_L$ , and from  $\mathbf{m}(t) + d_L$  to  $U$ . Since  $l$  is always positive, the integral from  $L$  to  $\mathbf{m}(t)$  is negative and the integral from  $\mathbf{m}(t)$  to  $\mathbf{m}(t) + d_L$  is

positive. Again, let  $Q(\mathbf{t}) \equiv [\mathbf{t} - \mathbf{m}(t)] \frac{1}{\sqrt{2ps}} e^{-\frac{[\mathbf{t} - \mathbf{m}(t)]^2}{2s^2}}$ . Since  $l$  is increasing up until  $\mathbf{m}(t) + d_L$ , we note

that

$$\left| \int_{\mathbf{m}(t)}^{\mathbf{m}(t)+d_L} Q(\mathbf{t})l(\mathbf{t})d\mathbf{t} \right| > \left| \int_{\mathbf{m}(t)}^{\mathbf{m}(t)+d_L} Q(\mathbf{t})l(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])d\mathbf{t} \right|, \quad (\text{A9})$$

which, by anti-symmetry of  $Q(\mathbf{t})$  with respect to  $\mathbf{m}(t)$ ,

$$= \left| \int_{\mathbf{m}(t)}^{\mathbf{m}(t)+d_L} Q(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])l(\mathbf{m}(t) - [\mathbf{t} - \mathbf{m}(t)])d\mathbf{t} \right|, \quad (\text{A10})$$

and, by changing variables,

$$= \left| \int_L^{\mathbf{m}(t)} Q(\mathbf{t})l(\mathbf{t})d\mathbf{t} \right|. \quad (\text{A11})$$

Therefore, the sum of the first two integrals is positive. The third integral is also positive. However, if  $\mathbf{m}(t) + d_L > \mathbf{t}^*$ , we cannot draw such a conclusion because  $l$  begins decreasing and may not balance off the non-positive integral from  $L$  to  $\mathbf{m}(t)$ . This means that, as  $\mathbf{m}(t)$  decreases, the sufficient condition  $\mathbf{m}(t) + d_L < \mathbf{t}^*$  holds true and  $\frac{d\mathbf{m}(t)}{dt} < 0$  leads to  $\frac{d\Phi(t)}{dt} < 0$ .

When the age distribution is lognormal, we proceed as per the proof of Proposition 3 for this case. The proof for the normal age distribution just offered applies where  $L$ ,  $U$ ,  $\mathbf{t}^*$  and  $d_L = \mathbf{m}(t) - L$  are substituted by, respectively,  $\ln L$ ,  $\ln U$ ,  $\ln L + [\mathbf{t}^* - L]$  and  $d_{\ln L} = \mathbf{m}(t) - \ln L$ . This also mean that, as a population average age decreases so does the mean of the logarithm of age  $\mathbf{m}(t)$ , and the sufficient condition  $\ln L + [\mathbf{t}^* - L] > \mathbf{m}(t) + d_{\ln L}$  holds true and  $\frac{d\mathbf{m}(t)}{dt} < 0$  leads to  $\frac{d\Phi(t)}{dt} < 0$ .

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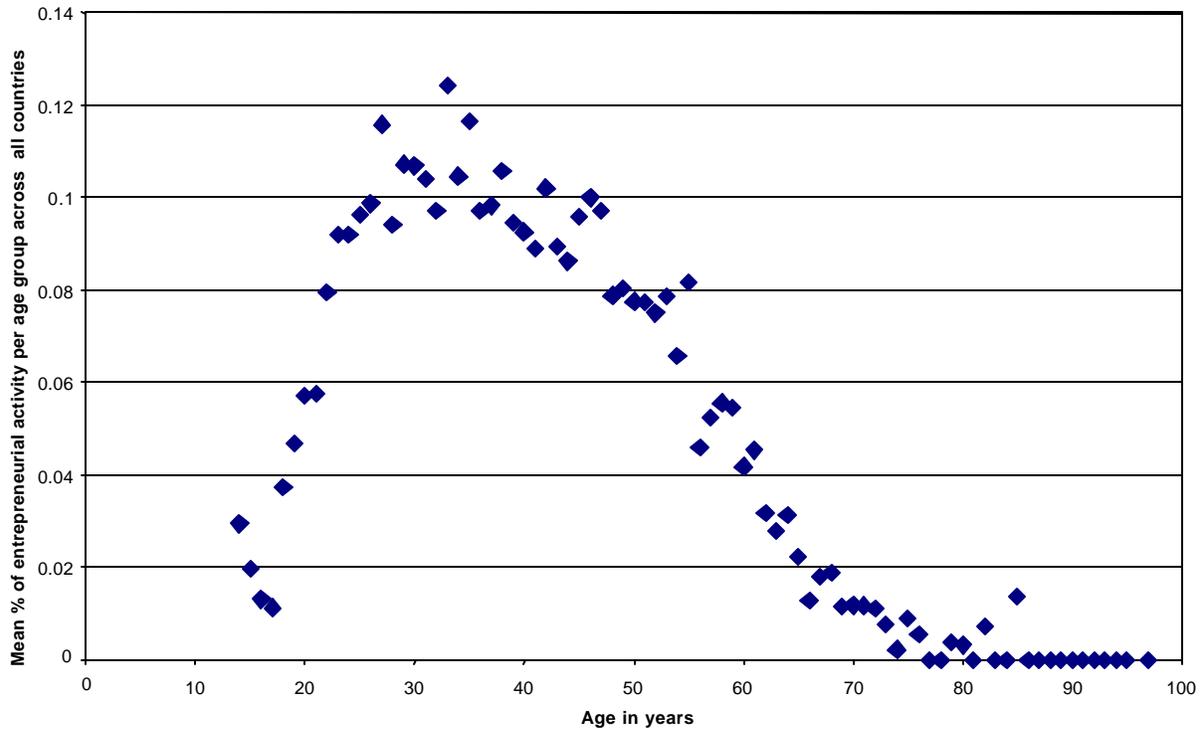
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Figure 1

*Relative Proportion of Population Engaged in Entrepreneurial Activity across 18 Countries\**

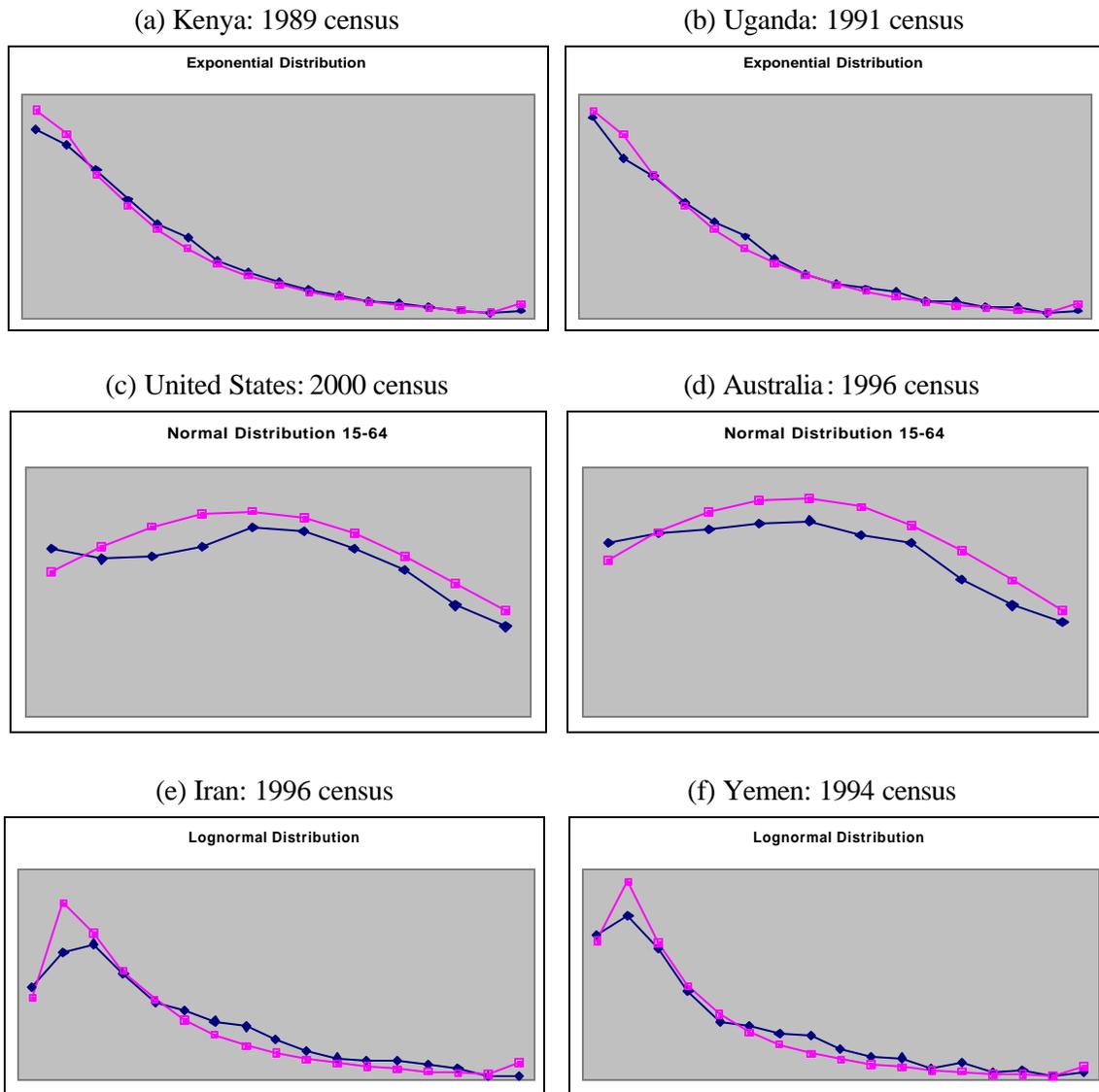


\* *Data source:* Global Entrepreneurship Monitor (GEM) Project at [www.gemconsortium.org](http://www.gemconsortium.org).

Data were collected, weighted and harmonized in spring 2001 through a standardized survey administered to a stratified representative sample of the population age 14 and above. Countries shown include Argentina, Canada, Denmark, Finland, Germany, Hungary, India, Israel, Italy, Japan, New Zealand, Poland, Portugal, Russia, Singapore, South Korea, Sweden, and the United States.

Figure 2

*Observed and Expected Age Distributions for a Sample of Countries\**



◆ Observed frequencies    † Frequencies estimated on the basis of the hypothesized density functions  
 Age categories: 0-4, 5-9, 10-14, 15-19, 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80+

\* Observed and estimated frequency distributions are constructed using data provided by the U.S. Bureau of the Census, International Data Base. ([www.census.gov/ipc/www/idbprint.html](http://www.census.gov/ipc/www/idbprint.html)).