

Technical Memorandum Number 794

**Expressing Inventory Control Policy in the Turnover
Curve**

by

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EXPRESSING INVENTORY CONTROL POLICY IN THE TURNOVER CURVE

ABSTRACT

Estimation of the inventory level for an entire class of items is a valuable time saver when control of inventories at the aggregate level, rather than the item level, is of interest. Inventory approximation by location, in supply chain network configuration and evaluation of inventory control policy shifts, are two examples of application. In this article, various popular inventory policies are related to a general function known as an inventory turnover curve that expresses inventory levels from the combined demand of multiple items. By knowing some basic item characteristics of representative items in a product class, the type of inventory policy being used, and the current aggregate inventory level, an inventory turnover curve can be constructed. This resulting turnover curve can be used to estimate inventory levels within 4.6%, on the average, of theoretically predicted ones.

INTRODUCTION

A simple function relating the volume passing through a facility (e.g., warehouse throughput) to the inventory level is used as an integral part of facility location analysis to estimate inventory levels at various stocking points. It allows inventory levels at the facilities to be approximated as customer demand is allocated to them. Once inventory levels are known, inventory carrying costs can be computed and placed in tradeoff with production/purchase; transportation; and facility fixed, handling, and storage costs, the sum of which is to be minimized. The function, called the inventory turnover curve, has the form $AIL = kV^\alpha$, where V is the volume passing through a facility, k and α are constants to be determined from a company's inventory turnover ratios at their stocking points, and AIL is the average inventory level at a stocking point.

The function has additional utility as an evaluative tool for inventory policy. It is an expression of a company's inventory policy application and provides a means for comparing performance. That is, various inventory policies can be represented by the turnover curve from which the resulting inventory levels can be compared to existing levels. The magnitude of the differences are determined that can serve as the basis for improvement.

Incomplete data and misapplication of policy can require that the function be established from the nature of the inventory policy itself. This requires that the relationship of inventory policy to the inventory turnover curve be understood. It is the intent of this article to develop the turnover curves resulting from various inventory policies applied under a variety of item characteristics, uncertainty conditions, and fill rates. Thus, by knowing basic information about item cost, demand, lead time, and inventory control methodology, a normative inventory turnover curve can be expressed.

BACKGROUND

Inventory policy

Inevitably, product flows in a supply channel simply because product is not available at the time and place desired and must be moved to meet demand. Depending on the

strategy that a firm adopts to manage the flows, some inventories will accumulate at various facilities in the channel or in the transportation system. A supply-to-demand, or just-in-time, strategy attempts to avoid inventories by matching supply to demand as demand occurs. On the other hand, a supply-to-stock strategy includes inventories with the objective of balancing them with product availability requirements. It is the control of these planned inventories that is considered in this research.

Planned inventories are controlled in numerous ways. Ford Harris [Harris (1913)] attempted to optimize inventory levels as early as 1913 with the development of the Economic Order Quantity (EOQ) model for determining replenishment order size in perpetual demand inventories. Subsequently, the EOQ became an integral part of reorder point and periodic review inventory control methodologies for controlling the combination of regular and safety stocks when demand and lead time are uncertain. Although demand is assumed constant and perpetual, practical application usually involves forecasting demand with one of the popular forecasting methods, such as exponential smoothing or moving average. The reorder point and periodic review methodologies are well described in most operations, logistics, and supply chain management textbooks; however for reference, they are repeated in the appendix.

Less discussed, but very popular with companies, is the stock-to-demand approach to inventory management, which personally has been observed when collecting data from numerous warehouse location studies. With this method, inventory levels are directly related to the demand forecast. The objective is to maintain inventory items at the level of, for example, 6-weeks demand or of a specific inventory turnover ratio. It is a form of the periodic review method, since inventory levels are examined at specified time intervals, perhaps monthly in conjunction with a forecast, except that replenishment quantities are *not* based on the EOQ and the review period is *not* economically determined and optimized. The method's popularity probably comes from its simplicity of understanding and application. It also is described in the appendix.

These previous methods are "pull" methods, meaning that stocking decisions are made locally to the inventory. In contrast, many companies prefer a "push" methodology. Under the latter, inventory decisions are made at the source point level in the channel based on the demand at more than one downstream stocking location. Stocking quantities shipped to the inventory locations are determined from the combined forecasted demands, the period for replenishment of all stocking points, and product lot sizes or vendor order minimums. Except for demand being forecasted from multiple stocking locations, the "push" method is similar to the stock-to-demand methodology.

As a practical matter, variations of these basic control methods are used. For example, orders from the same source may be composed of multiple product items, order quantities may be determined by vendor deals and quantity discounts, and there may be different policies applied to different stocking locations. Although there may be many possible inventory policies applied to specific situations, the analysis here will be limited to these popular methods for perpetual demand inventories. Just-in-time, or supply-to-demand, methods are not considered since their intent is to eliminate inventories rather than manage them. The "pull" methods are more likely to be applied by those members of the supply channel that experience perpetual item demand and regular demand patterns, namely retailers and distributors, but not limited strictly to these.

The conversion of inventory policy to the inventory turnover curve format is necessary when empirical data do not produce a true representation of a firm's inventory policy, when there is inadequate data to accurately construct the turnover curve, or when a different inventory policy from an existing one is to be represented for which there are no data. Otherwise, developing the turnover curve directly from empirical data would be reasonable.

The turnover curve

The inventory turnover curve perhaps finds its greatest usefulness in supply chain network planning, although it is useful for projecting inventory levels of individual items or all items collectively under various inventory policy scenarios. Numerous researchers such as Baumol and Wolfe (1958), Magee et al. (1958), Ballou (1984), Bender (1985), and Ho and Perl (1995), Shapiro (2001), and Daskin et al. (2002) recognized that inventory costs should be an integral part of location analysis because inventory levels change with the number of facilities and the demand allocated to them. A major problem with including inventory cost directly in location analysis rather than adding it after a solution is found is that inventories are *temporally* oriented whereas location is *spatially* oriented [Heskett (1966)]. It is difficult to merge the two into a single analysis because of the dimensional dissimilarities. However in location analysis, it is possible to estimate inventory levels in the aggregate by approximating them from facility throughput rather than through more traditional approaches involving lead times, demand levels, and relevant inventory costs. The inventory turnover curve serves the purpose, although others have used the square root law that associates inventory levels with the number of stocking points in a distribution network. The turnover curve is preferred because it combines both cycle and safety stock, and if applied to empirical inventory data, no assumption needs to be made about the underlying control policy. The development and application of the turnover curve has been described on several occasions [Ballou (1981), (2000)].

This different approach involving the square-root law relates system inventory levels to the number of facilities in the network. The basic relationship can be expressed as

$$AIL_1 = AIL_i N^\alpha \quad (1)$$

where

AIL_1 = the amount of inventory to stock, if consolidated into one location,

AIL_i = the average amount of inventory in each of N locations,

N = the number of stocking locations, and

$\alpha = 0.5$.

This equation can be converted to a throughput relationship by recognizing that N may be estimated from $N = V/v_i$, where V is the total volume passing through the system of stocking points and v_i is the average volume through a single facility. As stated, this relationship has several limitations not inherent in the inventory turnover curve. First, it is assumed that the throughput is the same for all stocking points. Second, it is constructed from an EOQ-based inventory policy. Third, demand variance/forecast errors are the same and independent among stocking points; however, the utility of the formulation has been extended by empirically determining α [Bender 1985)]. Based on

practice, Bender's suggestion is for α to be between 0.4 and 0.6. Varying α allows for some safety stock or the effect of inventory control methods other than EOQ-based ones, although there was no attempt to relate the value of α to either of these.

The turnover curve concept can be used to estimate inventory levels at a single location as well. Although the turnover curve can be determined for a single inventoried item, it is perhaps most useful for estimating inventories in the aggregate, i.e., a collection of multiple items. By establishing the turnover curve for a group of items, it is then possible to project the effect of policy changes on overall inventory levels. Top managers are concerned with the overall investment in inventories, and network planning involves the location of broad classes of inventoried items.

METHODOLOGY

The process to establish the turnover curve is to relate the shape factor α to the type of inventory policy used to control inventory levels. Then, once α is known, the curve can be fitted to the known inventory level for a particular group of inventoried items. What is not known is how the shape factor relates to inventory policy. Therefore, one of the objectives of this research is to determine the relationship. The shape factor is found for the three popular inventory control policies outlined in the appendix. Specifically, the policies to be evaluated are:

Reorder point (ROP)

- Order quantities optimally determined
- Specified order quantities

Periodic review (PR)

- Review time optimally determined
- Specified review times

Stock-to-demand (STD)

- Pull
- Push

These policies were selected because they are the ones most frequently described in the literature and observed in practice for perpetual demand patterns that are projected in the short run from historical time series.

Each inventory control policy was evaluated under a variety of inventory situations. These were:

- Items of varying values
- Perpetual demand with varying coefficients of variation
- Lead times of various lengths
- Customer service represented with various item fill rates
- Multiple items managed simultaneously within the same fill rate

A Monte Carlo type computer simulation was used to accurately establish the inventory levels that occur under various fill rates, cost, and demand-lead time conditions. Using simulation is especially appropriate as a research methodology (compared with calculating the inventory levels from theoretical formulations) because fill rates and, in

some cases safety stocks, can only be determined approximately through calculation. Simulation allows the action of an inventory policy to mimic, over time, actual inventory performance. The simulator used was SCSIM in the LOGWARE supplement to Ballou (2004) for a single supply chain echelon.

Simulation is an experimental methodology whereby complex problems, such as the inventory policies described previously, cannot be solved accurately using analytical techniques, usually because of the limiting assumptions required to achieve an analytical solution. When the problem contains probabilistic elements, such as random demand and lead times as in this case, random numbers and probability distributions are used to create a trial, which represents a specific level of demand on a given day or the lead time for a particular replenishment order. A trial is a specific outcome, which is an inventory level in our case. Multiple trials are averaged to generalize the outcomes, and the variation from multiple trials is expressed statistically. If different random numbers cause a substantial difference in the outcomes, repeating the simulation a large number of times may be needed to assure results that can be generalized. Determining the correct probability distributions and the often lengthy time required for building a simulation model can be a disadvantage. If the simulation closely mimics the problem in practice, testing various scenarios with a simulation can be a real economic advantage, since testing the scenarios under actual conditions that interrupt on-going operations can be avoided.

Each of the noted inventory policies were simulated using one-day increments for a period of 10 years. Demand was randomly generated from normal distributions for a single-location inventory. A variety of seed numbers used to initiate the simulation processes were tested to see if outcomes changed significantly due to the number selected. Although the outcomes generally changed little (less than 1%) when using different seed numbers, there was some increased variability in the outcomes from seed numbers as lead times increased, but this may not be statistically significant. Since the results were consistent for a given seed number and the variation in outcomes was not great, the same seed number was used throughout the study. Various run lengths were tested so as to ensure stable results. The first two simulated years were omitted from the results to eliminate start up aberrations in the simulation process; then, the results for years 3 through 10 were averaged. Run lengths beyond a 10-year simulated period made no significant difference in the results. A base case for item characteristics, demand, lead time, forecast smoothing constant, fill rate, and inventory related costs was established from which changes were made for testing purposes. This base case is outlined as follows:

Item value (C): \$3 per unit

Procurement cost (S): \$50 per order

Inventory carrying cost (I): 25% per year of item value

Item demand (d): 100 units per day

Demand variability (s_d): 10 units per day

Lead time (L): 1 day order filling and 4 days in transit

Item fill rate: 99%

Forecasting method: Simple exponential smoothing with a smoothing constant of 0.2

This base data case was the author's attempt to represent an inventoried item having medium to low value with a reasonable level of demand and demand variability. Other costs and factors were set at what would likely be experienced in practice. It was recognized that no single product description can be representative of all items in a variety of inventories so key factors were varied over a wide range to test many situations that might occur.

Average demand was varied from a low of 100 for the base case to high of 20,000 units per day. The forecast error ranged from zero to a coefficient of variation of 0.3, since larger values would violate the assumption of normality and the time series would be lumpy rather than perpetual. If larger forecast errors were to be experienced, a supply-to-demand policy rather than a supply-to-inventory policy would be more appropriate for lumpy demand.

Lead times were assumed to be certain in this study. It is this author's experience that, in practice, lead time variability is rarely measured when planning inventories. Experimentation shows that, along with demand variability, the effect on inventory levels is difficult to portray with a single variable. Also, theoretical formulations involving both demand and lead time uncertainty were of questionable accuracy. As a practical matter, managers will increase desired in-stock probabilities or increase average lead times for the purpose of setting inventory levels as a way of compensating for not knowing lead time variability. Thus, the lead time variability effect is accounted for when managers take such actions. For testing purposes, average lead times ranged from 2 days to 800 days, or over two years.

Item value, procurement cost, inventory carrying cost, exponential smoothing forecast method with a smoothing constant of 0.2 [a typical range is 0.05 to 0.3 according to Flowers (1982)] were set at nominal values, although items values were varied up to \$10 per unit for validation checking purposes. Testing variations in their values is not needed since they don't affect the shape of the turnover curve, although the forecasting method can affect the degree of uncertainty in an inventory planning problem. However, the effect is captured within demand variability and is not tested further.

Although simulation results give the average inventory level for a particular data scenario, the information is used to estimate the shape factor α in the turnover curve. What is sought is a single, representative independent variable that can be used as a predictor of the shape factor. The best measure found was the coefficient of total variation (CV_T) when compared to other measures involving only demand, demand variability, or lead time. CV_T is a combined measure involving average demand, demand uncertainty, and average lead time. It is dependent on inventory policy where CV_T for the reorder point policy is

$$CV_T = \frac{s_d \sqrt{L}}{d} \quad (2)$$

and for the periodic review and stock-to-demand policies it is

$$CV_T = \frac{s_d \sqrt{T + L}}{d} \quad (3)$$

CV_T is computed when Q or T are optimized using the EOQ formula, otherwise, the shape factor α is known to be 1; thus, a CV_T calculation is not required.

Knowing average demand, the forecast error expressed as a standard deviation, and the average lead time for an inventoried item or groups of items, the shape factor can be estimated for a particular inventory control policy.

The methodology is to run the simulation multiple times varying demand, lead times, fill rates, and other item characteristics to determine the average inventory levels. From each series of runs where demand is the primary variable, all other variables are held constant; a power function (a curve of the form $AIL = kV^\alpha$) is fitted to the inventory level data to find the shape factor α . With repeated simulation runs, curves are developed that allow the shape factor α to be found for a variety of item characteristics.

RESULTS

Basic effects

The general effect of the inventory policies tested on inventory levels can be seen in Figure 1. The simulated results using the base data case show that there are economies of scale to be expected with reorder point and periodic review policies when the EOQ formula is used to find the order quantities and review times. These scale economies are not present with a stock-to-demand policy since review time parallels forecasted demand. There is certainty in both demand forecasting and lead times, which allows the performance of the different policies (as calculated from theory) to be seen as they actually might perform in practice (as simulated). Formulas that represent inventory policy are based on assumptions such as constant demand, fill rate, and the distribution of the demand-during-lead time that need not be made in the simulation; thus, a more accurate representation of reality is expected from the simulation against which theoretical results may be compared. The curves in the figure result from fitting a power function to the simulation results without the complications of demand uncertainty. It should be noted that stock-to-demand and reorder point policies mimic closely theoretical projections with their shape factors of $\alpha = 1$ and $\alpha = 0.5$ respectively. However, the periodic review policy has a shape factor of $\alpha = 0.56$, indicating fewer economies of scale and more inventory than theoretically would be projected. The expectation is that α will be 0.5 and have the same inventory levels as the reorder point policy. However, the periodic review policy formulation is approximate and the simulated results reflect that fact.

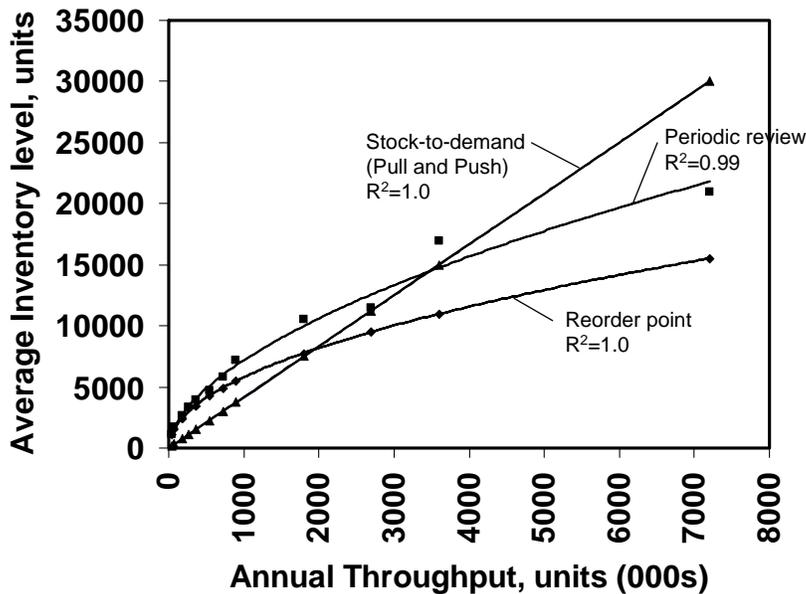


Figure 1 Simulated Inventory Levels using Various Inventory Control Policies under Certain Demand with Constant Lead Time

Although not specifically shown in a graph, when demand is held constant while lead times are varied, inventory levels increase but at a decreasing rate with increases in lead time. This is true for both reorder point and periodic review policies whose order quantities are determined through EOQ calculation. The economies of scale are not seen for the stock-to-demand policy where safety stock is determined from demand levels and not lead times. This is also true when the reorder point and periodic review policies are fixed. In the stock-to-demand case, inventory levels are accurately predicted from the theoretical formulations in the appendix.

Demand uncertainty, as represented by the coefficient of variation of demand CV_D , has a significant effect on the inventory levels as demand changes for an item, as illustrated in Figure 2 for a reorder point policy and a fill rate (FR) of 99 percent. Greater demand uncertainty or poorer forecasting has the effect of flattening the curve, thus resulting in fewer scale economies with increasing throughput. The same phenomenon holds for the periodic review policy, except that there is slightly more inventory held under this policy than with the reorder point policy, as can be seen in Figure 3. Stock-to-demand, reorder point with a specified order quantity, and periodic review with a specified order interval are policies that show inventory levels to be proportional to demand levels, regardless of the degree of uncertainty. The increased linearity of the inventory turnover curve with greater demand uncertainty is a result of safety stock becoming a higher proportion of total stock (regular plus safety) for a known lead time and a given fill rate.

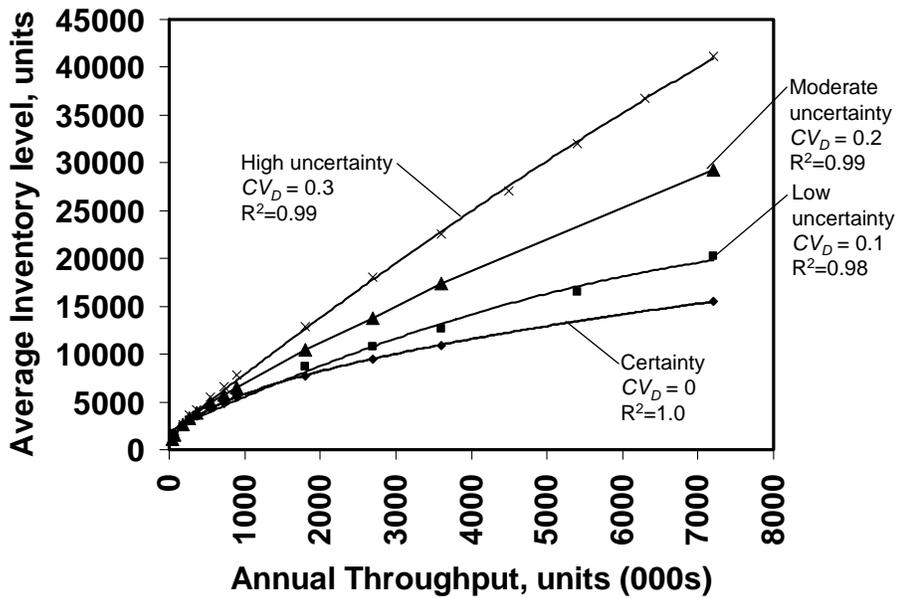


Figure 2 Simulated Results for a Reorder Point Policy under Various Levels of Demand Uncertainty and $FR = 99\%$

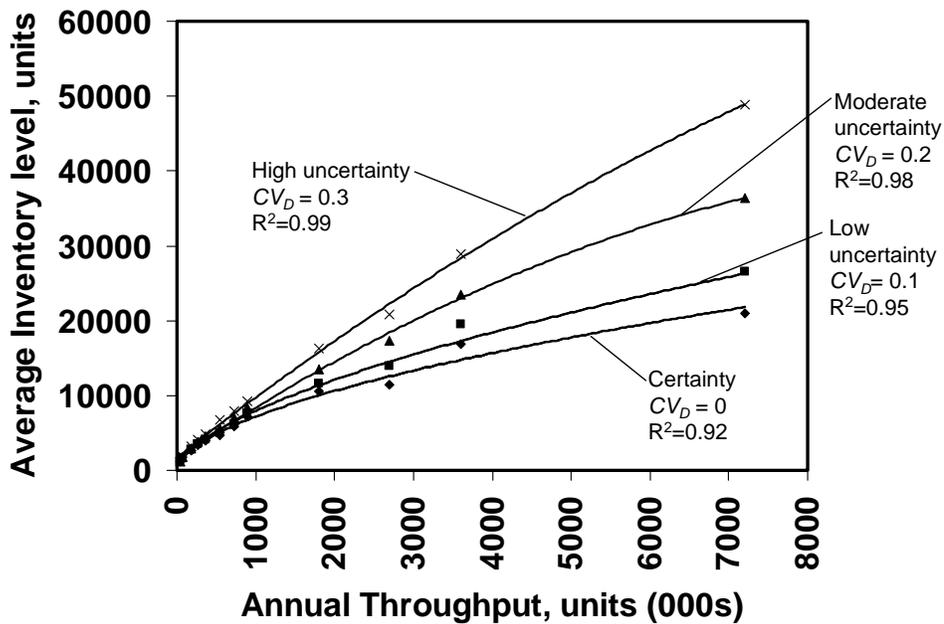


Figure 3 Simulated Results for a Periodic Review Policy under Various Levels of Demand Uncertainty and $FR = 99\%$

The effect of changes in lead time can be seen when the average demand level is held constant but with a $CV_D = 0.3$ and lead times that range from 2 to 800 days. Table 1 shows the simulated and calculated results for a daily demand level of 100 units and a demand standard deviation of 30 units for both ROP and PR policies. Whether the inventory levels are simulated or calculated from theoretical formulas, the estimates are similar. The STD policy will show similar results as long as safety stocks are determined from a square root function of the lead time, e.g., as in Equations A4 and A8. There are economies of scale as safety stocks increase at a decreasing rate with increasing lead times. Regular (cyclical) stock remains constant. The scale effect diminishes for lower fill rates and when safety stock is tied to demand levels rather than to lead time. In all cases, calculating inventory levels is satisfactory for the purposes here.

Table 1 Simulated versus Calculated Inventory Levels for ROP, PR, and STD Policies with Variations in Lead Times

Lead Time, days	ROP at $FR = 99\%$		PR at $FR = 99\%$		STD at $FR = 99\%$	
	Simulated	Calculated	Simulated	Calculated	Simulated*	Calculated
2	1,106	1,149	1,129	1,193	2,012	NC**
5	1,122	1,167	1,150	1,205	2,114	NC
10	1,156	1,194	1,182	1,222	2,218	NC
15	1,194	1,217	1,221	1,241	2,372	NC
30	1,293	1,277	1,285	1,287	2,738	NC
60	1,423	1,369	1,374	1,372	3,422	NC
90	1,533	1,447	1,504	1,444	4,250	NC
120	1,657	1,518	1,603	1,510	4,648	NC
180	1,725	1,639	1,661	1,624	6,030	NC
360	1,889	1,928	1,957	1,910	8,423	NC
720	2,360	2,376	2,324	2,353	10,682	NC
800	2,420	2,458	2,358	2,437	11,009	NC

* Review time set at 30 days

** Not Calculated since the m parameter in Equation A9 is arbitrarily set by the inventory manager

Determining the shape factor

Now that the basic effects of inventory policy on inventory levels have been established, attention is directed toward developing the turnover curve for inventory estimating purposes. Recall that the turnover curve is a power function of the form $AIL = kV^\alpha$. To define a curve, two parameters k and α are needed. It is possible to generalize the shape factor α for a given inventory policy and then k can be determined from a specific inventory situation. Using the coefficient of total variation CV_T as defined in Equations 2 and 3 and through repeated simulations involving average demand, demand variation, and lead time, it is possible to construct the turnover curve and observe its shape factor for a particular fill rate. In Figure 4 for a ROP policy with EOQ-determined order quantities, many turnover curves were found by fitting power function curves to simulated results. The simulated results are shown for the $FR = 99\%$ case.

Once the simulated shape factors were known for $FR=99\%$, the shape factors for $FR=99\%$ were determined mathematically by calculating the inventory levels for various demand levels (see Equations A1 through A4 in the appendix) and fitting a power function to these results. The computed shape factors were plotted in Figure 4 and are

shown in contrast to the simulated results. A smooth curve through the computed α values showed a reasonable approximation to the simulated results. With this good correspondence between simulated and calculated results for the $FR=99\%$ case, similar shape factor curves confidently can be calculated for other FR s without the need for further simulations, which saves time-consuming simulations and allows curves for fill rates not shown to be easily approximated. The shape factor curves for selected FR s are shown in Figure 4.

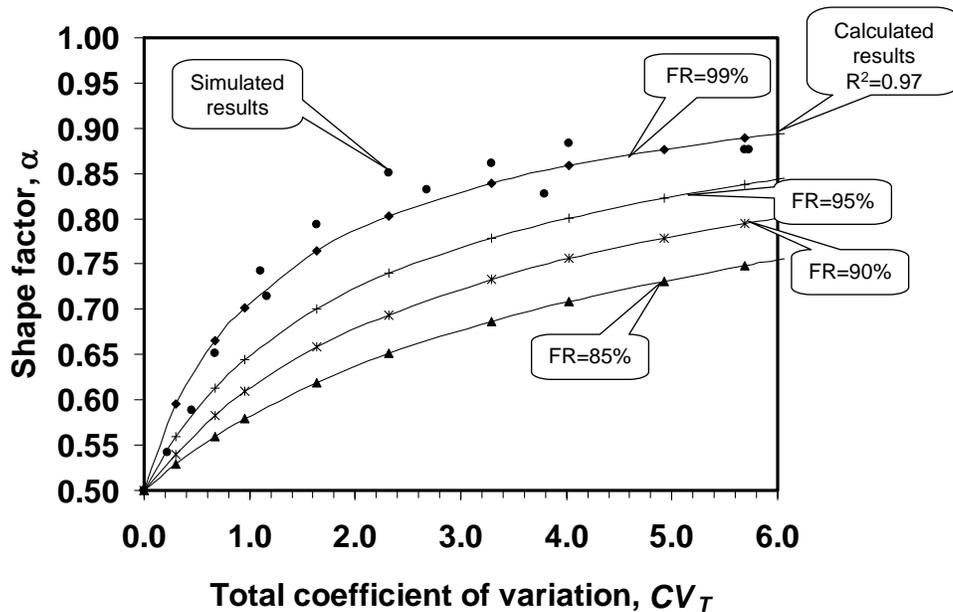


Figure 4 Reorder Point Policy Shape Factors for Various CV_T , Fill Rates, and Optimized Order Quantity

The shape factors for a periodic review policy where the order interval is optimally computed are shown in Figure 5. The PR simulated results do not track as well with the computed results as they did for the ROP policy. On the other hand, the shape factor curve varies little over a wide fill rate range. Because the simulated results likely are more accurate than the calculated ones due to the approximations within the theoretical PR formulations (see Equations A5 through A8 in the appendix) and the marginal difference caused by variations in fill rate levels, the simulated shape factor curves represented by the bold line for $FR=99\%$ in Figure 5 are preferred. The second bold line for $FR=85\%$ is approximated by extrapolating from the position of the calculated $FR=85\%$ line to the calculated $FR=99\%$ line. Because the shape factor α cannot theoretically be less than 0.5, the curves are truncated at that value for any CV_T less than about 1.7.

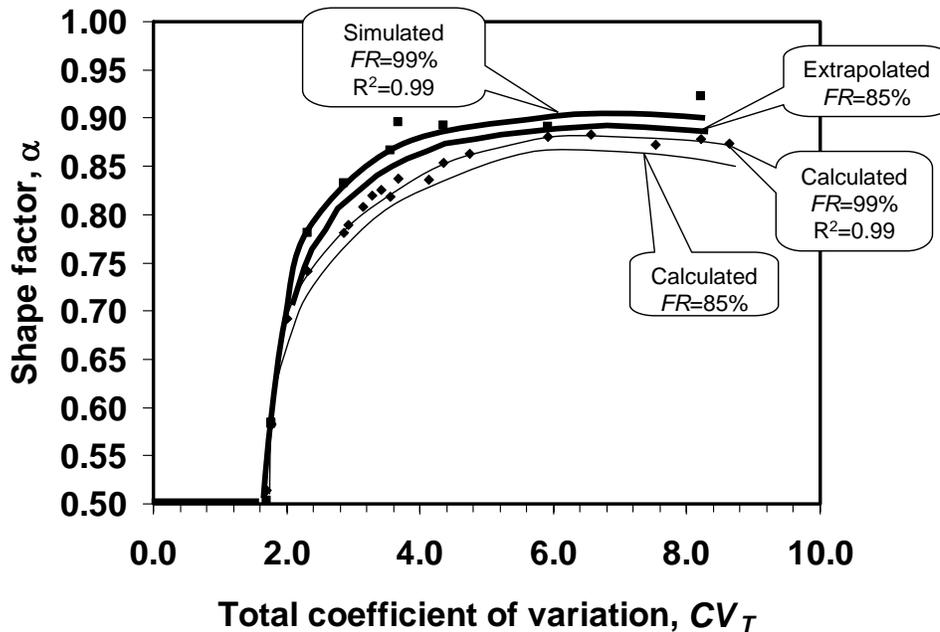


Figure 5 Periodic Review Policy Shape Factors for Various CV_T , Fill Rates, and Optimized Review Interval

Shape factors for the stock-to-demand policies of both push and pull are known to be 1 and are therefore not plotted. That is, inventory levels directly follow demand levels. Although different fill rates will change the overall inventory level, they will not cause the shape factor to deviate from 1. When the order quantity in the ROP policy is specified to be in proportion to item demand and the order interval in the PR policy is specified, the shape factor for these policies will be 1 as well. For a given lead time, none of these policies will show inventory levels that increase at a decreasing rate with increasing demand levels.

Aggregating items

The theoretical formulations for inventory policies are for individual items. The concept of the turnover curve is to estimate inventory as a collection of items that form an item classification. Items within a classification are likely to have the same fill rate, inventory carrying cost, and order processing cost but will differ according to demand rate, demand variability, lead time, and item value. For the purposes of determining a shape factor, combining the coefficients of total variation CV_T for multiple items is appropriate. Among multiple choices, creating a combined CV_T through demand weighting is simple and effective. Other choices are to combine individual shape factors, demand variances, or item characteristics, but they offered less estimating accuracy than did a demand-weighted CV_T , probably because demand level, and the forecast thereof, is the primary driving force for determining inventory levels.

VALIDATION

The accuracy of projecting inventory in the aggregate using various inventory control policies and the turnover curve needs to be established. Forty-two tests were conducted where two items having the base case characteristics had average demand levels of 100 and 200 sales units per day respectively. Five inventory policies were used: ROP with the order quantity calculated from the EOQ, ROP with the order quantity specified as a fraction of demand, PR with the order interval calculated from the EOQ, PR with a specified order interval, and STD with the order interval and the safety stock as a fraction of demand. Testing involved computing the CV_T values for each product and then determining the combined, weighted value for the two products. With this CV_T , the shape factor α was found from the curves of Figure 4 or 5. For some policies, the shape factor is known to be 1 and requires no lookup from a graph. After the shape factor was determined, the k parameter in the turnover curve was found, given the average inventory level AIL for the combined products. This combined inventory level was established from the theoretical estimates for AIL , as given from the formulas in the appendix. After changing demand variability (10% to 30% of average demand), average lead time (60 to 120 days), fill rate (90 to 99%), and product value (\$3 to \$10 per unit), demand for the two items was decreased by 50% or increased by 100%. By comparing the inventory level from theoretical formulas with that determined from the turnover curve, an estimate of the expected deviation was projected.

Test results showed that the aggregate inventory projected from a ROP policy with a calculated Q will have an average difference of 8.3% with a range of 3.4% to 17%. Using the same policy but with Q as a fraction of demand showed no difference between the two approaches. For the PR policy with a calculated T , the average difference using the turnover curve was 9.5% with a difference range of 2.8% to 14.8%. If the T interval was specified, the average difference was 4.5% with a range of 1.3% to 8.5%. There was no error using a STD policy. Overall, the average expected difference was 4.6% using the turnover curve approach to aggregate inventory estimation rather than summing the inventory levels as determined through theoretical calculation.

USING THE RESULTS

The use of the results is now illustrated in two ways: estimating the inventory level changes as the number of stocking points is changed and estimating the effect of an inventory control policy change. Additional uses of the turnover curve include comparing theoretical control policy results with current inventory performance, approximating inventory levels where there currently is no inventory, developing an inventory estimating relationship when only one inventory point exists, and projecting inventory level changes as critical factors such as demand, lead time, and fill rate are changed.

Inventory split between two locations

In a practical situation, many items make up inventory. First, the items are to be separated into a class where the fill rate is the same, such as A items, B items, or C items, if a classification scheme is used. Second, because many items typically will be in a class, the items will be sampled to be representative of all items in the class. For illustrative purposes, three items are used that are assumed to represent 90% of the items

having a fill rate of 95% (the *B* items). The three items have the characteristics shown in Table 2. The control policy is ROP with EOQ-determined order quantities.

The inventory is to be divided between two locations, where 60% of the demand is allocated to one location and 40% to the other. How much inventory can be expected in the two locations versus one? This kind of question might be the raised when designing a supply chain network.

Table 2 Item Characteristics and Inventory Levels for Three Items Sampled from the *B*-Class Items in an Inventory with a 95% Fill Rate

Product characteristics	Item 1	Item 2	Item 3	Total
Daily demand, d	100	50	200	350
Demand std. dev., s_d	30	5	40	
Lead time in days, L	30	15	30	
Item value in \$/unit, C	3	10	5	
Order processing cost in \$/order, S	50	50	50	
Inventory carrying cost in %/yr., I	25	25	25	25
Average inventory level in units, AIL	2,900	1,075	5,800	9,775
Average inventory level in \$, AIL	8,700	10,750	29,000	48,450

A coefficient of total variation CV_T is computed for each sampled item according to Equation 2. Hence,

$$CV_{T_1} = \frac{30\sqrt{30}}{100} = 1.64 \quad CV_{T_2} = \frac{5\sqrt{15}}{50} = 0.39 \quad CV_{T_3} = \frac{40\sqrt{30}}{200} = 1.10$$

The average CV_T for the sampled group is the weighted average, where CV_T is weighted by the average item demand. That is,

$$CV_{T_r} = 1.64 \frac{100}{350} + 0.39 \frac{50}{350} + 1.10 \frac{200}{350} = 1.15$$

From Figure 4 on the 95% curve, the shape factor $\alpha = 0.65$ for $CV_{T_r} = 1.15$. The value of the sampled *B* items is \$48,450 (90% of total inventory) and the total value of all *B* items is \$48,450/0.9 = \$53,833 and the total demand is 350/0.9 = 389. Now, k in the turnover curve can be estimated as

$$k = \frac{53,833}{389^{0.65}} = 1115.79$$

With both k and α now estimated, the inventory in two locations with a 60/40 demand split can be determined as follows:

$$AIL_1 = 1115.79(389 \times 0.6)^{0.65} = \$38,623 \quad AIL_2 = 1115.79(389 \times 0.4)^{0.65} = \$29,675$$

The system-wide inventory is $\$38,623 + \$29,675 = \$68,298$, an increase of $\$68,298 - 53,833 = \$14,465$ in inventory investment from the one-location design. It is assumed that the lead times are the same for two locations as for one location, however, the inventory estimation is quite insensitive to this assumption. For example, a 50 percent change in the lead time causes only a 9 percent change in the system-wide average inventory level.

Policy change

A firm currently has inventory at three locations and uses a STD control policy that produces the linear relationship of inventory level to annual demand as shown in Figure 6. The product characteristics are the same as those in Table 2 and three items are taken as representative of the entire *B*-class inventory and are 80 percent of total class inventory. The same item mix is stocked at all locations. Will a policy change to PR with an optimized order interval T perform better than the current policy?

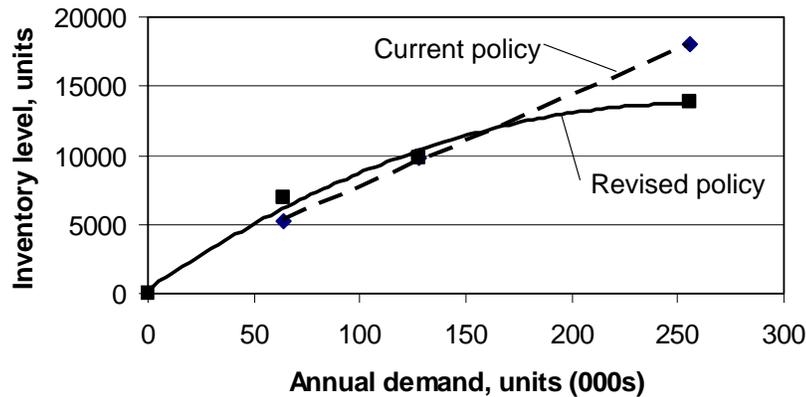


Figure 6 A Comparison of the Current STD Inventory Control with a Proposed PR Policy for Three Stocking Locations

The items for inventory are purchased centrally for the three locations with a sales and inventory distribution given in Table 3. Each item has a common order interval T and, under the revised policy, it is to be determined optimally. The optimal T s can be found by applying Equations A1 and A5. Hence, the T values for each item's total sales are calculated as follows.

$$T_1 = \frac{\sqrt{\frac{2(350 \times 365)(50)}{0.25(3)}}}{350} = 11.79 \quad T_2 = \frac{\sqrt{\frac{2(175 \times 365)(50)}{0.25(10)}}}{175} = 9.13 \quad T_3 = \frac{\sqrt{\frac{2(700 \times 365)(50)}{0.25(5)}}}{700} = 6.46$$

Because for the PR policy the CV_T is dependent on T , a combined CV_T is computed by weighting the individual CV_T values. Applying Equation 3, the individual item CV_T values are:

$$CV_{T_1} = \frac{105\sqrt{11.79 + 30}}{350} = 1.94 \quad CV_{T_2} = \frac{17.5\sqrt{9.13 + 15}}{175} = 0.49 \quad CV_{T_3} = \frac{140\sqrt{6.46 + 30}}{700} = 1.21$$

The s_d for the combined sales for the three locations is the same coefficient of variation (s_d / d) as shown in Table 2. For example, s_d for item 1 is $350 \times (30/100) = 105$.

Table 3 Daily Item Sales and Inventory at Three Locations in Units

	Location 1	Location 2	Location 3	Total Sales
Item 1	100	50	200	350
Item 2	50	25	100	175
Item 3	<u>200</u>	<u>100</u>	<u>400</u>	<u>700</u>
Total Sales	350	175	700	1,225
Inventory	9,775	5,200	18,000	

Combining the CV_{T_i} s with a sales-weighting average yields:

$$CV_{T_r} = 1.94 \frac{350}{1225} + 0.49 \frac{175}{1225} + 1.21 \frac{700}{1225} = 1.31$$

This value is used to find the shape factor α from Figure 5, which is $\alpha = 0.5$. In this case, construction of the k factor in the turnover curve requires some judgment. Using the midpoint sales among the locations (i.e., location 1) seems reasonable. Hence,

$$k = \frac{9775}{350^{0.5}} = 522.5$$

with the turnover curve now expressed as $AIL = 522.5d^{0.5}$ with AIL expressed in units. Applying the curve to the total sales for each location gives the policy comparison shown in Figure 6. The PR policy yields a net reduction of 2,464 units compared with the current policy, or $2,464/0.80 = 3,080$ units for the total inventory. Using a sales-weighted average value of \$5.14 per unit, the reduction in inventory investment is projected to be $3,080 \times 5.14 = \$15,831$ for *B*-class items.

The application of the inventory turnover curve rests on several assumptions. First, demand is perpetual, that is, non-terminating over the planning horizon. Second, it is expected that the items in inventory that are the subject of this application are in the middle stages of their life cycles. Third, demand is expected to show a “regular” pattern whereby its variability is not great relative to its average. Fourth, demand is reasonably forecasted from historical patterns. Fifth, lead times do not vary greatly from one stocking location to another, which is probably reasonable where transportation time is not a high proportion of total replenishment cycle time. Sixth, lead times are constant although lead time variability can be accounted for by increasing the fill rate level. Seventh, demand forecasting error is approximately normally distributed. It is not expected that each of these conditions will be precisely met in every application;

however, the robustness of the methodology suggests that reasonable results can be achieved in most applications.

CONCLUSION

It has been the intent of this research to develop a process whereby aggregate inventory levels can be estimated from individual product characteristics and the inventory policy that controls them. The inventory turnover curve with the form $AIL = kV^\alpha$ is used as the medium in which to relate policy to inventory levels. Its major benefit is that the combined inventory levels of multiple items can be projected without the need to calculate and then sum the inventories for many individual items, which can be time consuming for many items stocked at numerous locations. Once the turnover curve is constructed, inventory levels can be projected in situations where none exist. For many top-level managers, financial control of inventory levels in the aggregate is of more interest than the control of individual items; and for strategic planning, projection of overall inventory levels is preferred to that of single items. This aggregate control, and the inventory turnover curve, is useful in the following applications:

- Planning supply chain networks
- Evaluating inventory policy change
- Comparing theoretical inventory control policy with current inventory performance
- Approximating inventory levels where there now is no inventory
- Estimating inventory levels for a range of circumstances when only one inventory point currently exists
- Projecting inventory level changes as critical factors such as demand, lead time and fill rate change

Three classes of inventory policies were examined: reorder point, periodic review, and stock-to-demand. It cannot be assumed that theoretical formulations of inventory policy would produce the inventory levels experienced in practice even when the policy was faithfully and precisely applied. At least one policy is approximate and the relationship between fill rates and inventory levels for all policies cannot be stated exactly. Therefore, the inventory policies were replicated through Monte Carlo simulation to better approximate actual inventory levels and to relate them to the inventory predicted by theory. Through this process, the shape factor α in the turnover equation was estimated, which relates directly to the type of inventory policy used. Then, from known inventory levels, the k factor was found to complete the turnover curve formulation.

It was found that the turnover curve can be a reasonable substitute for theoretical estimation. The average difference between turnover curve estimation and theoretical calculation was 4.6% when tested under a variety of item characteristics and inventory control policies.

Future research might well center around relaxing the assumptions made in this analysis. These would include (1) treating variable lead times exactly, (2) testing non-normality distributions for demand forecast error and lead time, (3) using forecasting methods other than exponential smoothing such as collaborative forecasting or regression

analysis, (4) testing the robustness of the turnover curve when demand is not “regular” and predictable, and (5) evaluating inventory policies not contained in those identified here. Approximating the k coefficient in the turnover curve when no inventory exists would be particularly useful. Finally, reporting on applications of the turnover curve in a variety of situations would help to establish its accuracy and utility.

NOTES

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APPENDIX

For reference, the three inventory control methods tested in this paper are given below. They all assume that item demand is perpetual and is estimated through forecasting, forecast error is normally distributed, and lead time is constant. Demand is filled from a single site. Inventory replenishment quantities are determined at the time a replenishment order is needed and is filled from a single source.

Reorder point method (ROP)

Under the reorder point method, inventory levels are controlled through the size of the replenishment order and the timing of the order. The replenishment order size is determined from the economic order quantity (Q), or

$$Q = \sqrt{\frac{2DS}{IC}} \quad (A1)$$

where

Q = the economic order size, units

D = forecasted item average annual demand, units

S = order procurement cost, \$ per order

I = inventory carrying cost, annual percent of item value

C = item value in inventory, \$ per unit

The order Q is placed when the inventory level falls to the reorder point quantity (ROP). ROP is

$$ROP = d(L) + z(s'_d) \quad (A2)$$

where:

d = forecasted item daily demand, units

L = average lead time, days

z = number of standard deviations on distribution of demand during lead time for a particular in-stock probability

s_d = standard deviation of the normally distributed forecast error, units

s'_d = standard deviation of the demand during lead time distribution, or $s'_d = s_d \sqrt{L}$

The z value for a given item fill rate (FR) is found through the unit normal loss integral $E_{(z)}$ where

$$E_{(z)} = \frac{(1 - FR)Q}{s'_d} \quad (A3)$$

and z is found from tabled $E_{(z)}$ values [see Ballou (2004), pp. 763-765]. The item fill rate is the fraction of demand filled on first request.

The average inventory level (AIL) is computed as

$$AIL = \frac{Q}{2} + z s'_d \quad (A4)$$

Alternately, Q and ROP may be specified rather than computed.

Periodic review method (PR)

The periodic review method is similar to the reorder point method except that the replenishment quantity to order is determined as the difference between the item inventory quantity on hand and a target inventory level called MAX . The timing for order placement is determined from the order interval T . The review interval is found from

$$T = \frac{Q}{d} \quad (A5)$$

where T is in days and Q and d are previously defined. The target level MAX is mathematically impractical to compute exactly and is usually estimated as

$$MAX = d(T + L) + z(s'_d) \quad (A6)$$

where s'_d for periodic review is

$$s'_d = s_d \sqrt{T + L} \quad (A7)$$

The average inventory level is computed as

$$AIL = \frac{Td}{2} + z s'_d \quad (A8)$$

where z is found from Equation A3 recognizing that $Q = T \times d$.

Alternately, T and MAX may be specified rather than computed.

Stock-to-demand method (STD)

Stock-to-demand is an empirical approach to inventory control whereby a forecast is made at specified intervals based on such factors as convenience, requirements of multiple items in inventory, workload scheduling when orders emanate from multiple inventory locations, and supplier order-size or product lot-size minimums. Then, inventory levels are managed according to desired goals, such as a particular turnover ratio or number of days of inventory. It is usually executed in a manner similar to the periodic review method with the exception that most of the parameters of the method are set based on judgment, experience, and goals for inventory. For example, the forecasting and inventory review interval T might be synchronized at one month, or 4 weeks. The MAX level is a multiple of the forecast and is based on a target level for inventory and an increment for uncertainties in demand and the length of the lead time. If the desired peak

inventory level is 6-weeks demand and a week's worth of demand is added for safety stock, the *MAX* level would be computed as the monthly forecast times $(6 + 1)/4$. The number of weeks to add for safety stock can be approximated from the safety stock quantity term $z(s'_d)$ in equation A6, or, more frequently, it is set from experience as a multiple of the demand forecast.

The average inventory level is computed as Equation A8 in the case where safety stock reflects demand variability; or if safety stock is a multiple of demand, then

$$AIL = \frac{Td}{2} + md \quad (A9)$$

where m is a multiplier used to reflect a specified lead time and demand uncertainty.

Push inventory control is a variant of the STD policy. Rather than replenishment orders originating at the location where the inventory is held, they originate from a source point such as a plant that serves the stocking points. Order size is planned based on forecasted demand levels from multiple locations and safety stocks are a function of the forecast errors and lead times for each location. Order intervals would be the same for all locations, but the *MAX* level can be different for each location to reflect varying demand and lead time characteristics.