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Reverse Logistics : Managing Returns on a Delivery Route

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Abstract

Designing pickup protocols for materials returning on a delivery route is the focus of this paper. Pickup strategies on a fixed route are influenced by variables such as the number of stops on the route, the variability of stop demand, delivery vehicle capacity, the use of outside carriers to supplement delivery vehicle capacity, the number of periods for planning, the penalty cost for not picking up returning materials promptly. Three special cases are identified where the problem is analytically tractable. For the general problem where customers have different penalty costs for materials not returned promptly, an efficient heuristic procedure is proposed. Several insightful rules for route management resulting from this analysis are offered.

1 Introduction

Reverse logistics issues are emerging concerns within supply chain management. For economic, environmental, and regulatory reasons, companies must manage to efficiently and effectively recover some or all parts of previously shipped products for possible reuse, recycling, reprocessing, or disposal. Planning the integrated flow of returning materials with deliveries is a more complex problem than planning each separately. This simultaneous decision problem frequently occurs in practice and is the subject of this work. A motivating example comes from the American Red Cross (ARC): Blood Services, which is the largest processor of volunteer blood donations in the country. Regional blood centers are the major blood suppliers to local hospitals. Depending on the size of the hospital, deliveries of blood products are made up to several times per week. Blood is delivered by vans in insulated boxes that are valuable enough to be recycled. They cannot be knocked down due to the rigidity of the insulation material, so the return boxes are as bulky as the delivered ones. Boxes cannot be returned immediately upon delivery due to the time required for unpacking but normally are available for pickup the following day. Ideally, the returning boxes are placed on the vans as deliveries are made, but space limitations of the vans and the imbalance between the delivery quantities and the quantities to be returned can cause returns not always to be made in a timely manner.

There are many additional examples throughout the economy where the type of reverse logistics described above exists. For example, firms delivering product on pallets often recover the pallets for reuse. Beer and soft drink bottlers mix deliveries with returning empty containers with the aid of specially designed rack trucks. Retail appliance stores pick up old appliances at the time of new appliance deliveries. Similarly, office leasing equipment firms delivering new equipment may pick up and recycle used equipment for refurbishing and resale. Products experiencing quality problems may need to be returned for inspection, rework, and possible resale, and they are picked up at the time that deliveries are made. Thus, it is good economic practice that recovered materials are handled in conjunction with deliveries since it is generally understood that combining delivery with pickups results in lower transportation costs when compared with handling them separately. In fact some 3PLs (Third Party Logistics providers) such as Roadway Transportation have set up separate divisions to efficiently handle returning products.

In addition to economical concerns, the volume of returning materials is growing because of increasing environmental regulations, which add to the importance of the reverse logistics problem. For example, the German packaging ordinance of 1991, motivated by environmental issues, requires industry to take back all sales packaging materials (see Fleischmann et al., 1997). Japan has similar legislation (see Jones, 1998) and in the USA with over 2000 bills on solid waste materials being introduced in Congress in the last decade.

Under these economic and environmental pressures requiring producers to take responsibility for their products throughout the whole product life cycle, the recovery of materials has increased significantly during the last decade. Recovered materials in general can be categorized depending on their recovery nature as follows.

- **Reuse or repair.** Products in their original structure are recovered, possibly for repair or direct reuse. Examples are defective products returned for repair or disposition and reusable shipping packages such as bottles, pallet, and containers.

- **Remanufacturing.** Only the valuable parts of products are returned after disassembly. Examples are aircraft and auto engine components, gold and other precious metals from products, and computer components.

- **Material recovery (Recycling).** The whole product or parts of it are returned without conserving the product structure. Examples are recyclable metal scrap, glass, plastic, and paper.

Some products such as pallets, reusable beverage bottles, and containers are returned quickly and are immediately reusable, possibly after minor processing such as cleaning. These products can be returned quickly since they are no longer required once the content has been delivered. In contrast to these examples, some products take a long time before recovery can occur. Examples are products returned due to failure, being at the end of product life cycle, the need for preventive maintenance, or being at the end of a lease contract. Products might be returned to original producers such as the reusable packaging material or to other places such as a paper waste processor.

1.1 Research Theme

Material recovery results in a reverse product flow from downstream channel locations to reprocessing points. An important part of reverse logistics is designing the logistics network in a way that efficiently incorporates this return flow of recovered products. The goal of this study is to integrate the forward and reverse material flows at the routing level where products in their original form are recovered in a short time and returned to the point where deliveries originate. Instead of sending vehicles exclusively for collecting return materials, a logistics cost reduction might be obtained from efficiently utilizing the capacity of vehicles to deliver new products as well as to simultaneously collect returning materials. This research focuses on the situation where the returning materials from delivered products are available for return the day following their delivery. In case on a given day, when there is not enough space to accommodate all materials to be recovered, certain pick ups can be postponed to subsequent days but with a penalty cost. Hence, there are two interrelated concerns.

- **The vehicle routing problem.** From the depot, determine the sequence in which stops are to be visited during a delivery period.
- **The returns strategy.** To determine the quantity of the returning materials to pick up at each stop on the delivery route during a given delivery period.

The vehicle routing problem is well known to be NP-complete. The return problem is also difficult, especially in combination with the delivery problem. Therefore, it is not computationally reasonable to simultaneously solve both problems optimally. Alternately, an approach is to devise a procedure for finding an optimal pickup strategy for a given routing stops sequence. The procedure then can be incorporated into an iterative algorithm that finds the optimal, or near optimal, combined routing-pickup strategy. With this approach in mind, it is assumed that the route is fixed from period to period and the focus is on finding pickup strategies for the returns problem. Limiting the research to this sub problem is necessary due to publication space restrictions while adequately presenting the research for a problem of this scope. However in the companion paper of Alshamrani et al. (2004), the work is extended to solving the combined delivery routing and pickup problem, wherein the results of this research provide the basis for dealing with this more complex problem.

1.2 Problem, Scope, and Limitations

A depot (distribution center, terminal, plant, etc.) serves a number of geographically dispersed stops to which deliveries are made. During a period (for example, a day), stops place orders requesting certain quantities (stop volume) of a product, termed delivery products, from the depot. *These stop volumes must be delivered in the same period and cannot be postponed.* Products in their original form (dimensions), termed returning materials, are recovered from delivery points and returned to the depot. A product delivered to a stop in one period is assumed to have its associated materials ready for recovery the following period.

A single *company-owned* vehicle of limited capacity is operated that performs a single combined delivery/pickup trip every period starting and ending at the depot and visiting each stop only once. Orders that cannot be accommodated in the vehicle or are received after the vehicle leaves the depot are delivered by some other means (henceforth termed *for-hire* carriers). However, the company-owned vehicle needs to collect all returning products regardless of how they are delivered. Materials that are ready for return can be delayed to subsequent periods but will incur a penalty cost per period.

In this research, only the single vehicle case is considered. As was the case with the American Red Cross: Blood Services, it is assumed, that based on geographical or other considerations, the stops have been partitioned into sectors. Each sector is served by a single vehicle. Beullens, et al. (2003) provides an excellent survey of sector design models in reverse logistics.

The reverse logistics problem is considered within a stochastic framework. At the beginning of a period, the company vehicle dispatcher knows only the delivery volume to be sent by the company-owned vehicle to each delivery point in that period and the quantities of returning materials ready to be picked up in that period, since the returns can be derived from past deliveries. Future stop volumes, which are the quantities of product to be delivered by the company-owned vehicle or a for-hire carrier in future periods, are uncertain. But the probabilistic behavior of both stop volumes is known and is the same in each period. For the ease of exposition, it is assumed that these two random variables for each stop are independent; however, all the results and the methodology of this paper are still valid when their behavior has a dependent joint distribution. The stochastic assumption pertains only to the stop volumes. Everything else is assumed deterministic and known: the travel time (distance) between any two points and the fixed time span of one period between delivering a product and its associated returning materials availability.

The problem is considered within a multi-period planning horizon. This raises the question of whether to use the same or a different stop routing sequence each period or to use the same stop sequence. For ease of implementation, some dispatchers prefer a fixed daily stop routing sequence. Fixed routing has the advantage of enhancing regularity of service and increasing driver performance through familiarity. Unless stated otherwise, the sequence of visited stops by a vehicle, once determined, is assumed the same each period. Also in a given period, one may choose not to visit a stop even if there is no delivery volume but has some returning materials. This additional decision adds another level of complexity to an already difficult problem. To keep the problem tractable, it is assumed that each stop is visited every period. Moreover, in the motivating example (ARC), all hospitals have a positive demand most of the days. Hence for making the strategic policy, it is believed that not much is lost with this assumption. Operationally, one can simply skip a stop with no demand and no returning materials to be picked up.

The returns strategy depends on the cost of leaving returning materials at stops. The penalty cost is assumed linear, that is, the cost of leaving e_i returning materials at stop i for one period is $a_i e_i$, where the unit penalty cost a_i is estimated based on explicit as well as implicit factors. An obvious explicit factor is the direct charge for delaying material returns. Some of the implicit factors could be customer dissatisfaction with unreturned materials accumulating at the stop resulting in lost sales and the cost associated with damaged or stolen materials left at a stop.

The research problem is a strategic planning one that can now be stated precisely as: *Given a sequence in which a vehicle of finite capacity visits n stops with known probabilistic behaviors of daily volume delivered by a company-owned vehicle as well as by a for-hire vehicle, design a return policy that specifies the number of returning materials (which are the delivered items in the previous periods) to pick up at each stop. The objective is to minimize over a finite planning horizon K, the cost of expected penalty cost for not picking up returning materials promptly.*

1.3 Background

Much of the literature about reverse logistics is qualitative and exploratory in nature. It emphasizes the importance and need for reverse logistics planning and seeks to provide a general framework for it. Carter and Ellram (1998) and Fleischmann et al. (1997) provide surveys of recent literature in reverse logistics; the latter in particular reviews it from a quantitative point of view (distribution planning, inventory control, and production planning). Bloemhof-Ruwaard et al. (1999) review the distribution issues in reverse logistics such as the location of reverse channel activities, form of collection systems, and routing.

The quantitative models for reverse logistics are mostly for network design that consider location of joint facilities of forward and reverse networks and allocation of returning materials to open facilities. Jayaraman et al. (1999) proposed a 0-1 mixed integer-programming model giving the set of distribution/remanufacturing locations to open, the quantities of product to stock, the quantities of product to ship from an open facility to a customer, and the quantities of recovered product to ship from a customer to an open facility. However, designing vehicle routes was not considered. A book by Dekker et al. (2003) provides a compilation of recent research that considers issues like collection and distribution, network design, inventory control, and routing in a reverse logistics systems context.

A problem similar to that studied in this research is the Vehicle Routing Problem with Backhauling (VRPB). In the VRPB, the customers are of two types: delivery customers who receive a given quantity of product from a company's depot, and backhaul customers who send a given quantity of product to a company's depot. There are two major variations of VRPB. The first is where the returning materials are picked up only after delivering products to all customers. The second type, Vehicle Routing Problem with Backhauling of Mixed Loads (VRPBM), is where pickup of returning materials can be performed before the last delivery is made thus resulting in a mixed load of pickups and deliveries in the vehicle. Compared with the pure delivery or pure pickup vehicle routing problem (VRP), there is scant literature about the VRPB. Moreover, most papers assume that the backhaul points are visited only after all deliveries have been made. A few papers [for example, Min (1989), Desaulniers et al. (2002), Dethloff (2001), and Toth and Vigo (1997)] consider mixed loads. See Casco, et al. (1988), Savelsbergh (1995), and Toth and Vigo (2002) for a survey of this and related problems. However, there are two characteristics of the proposed problem that distinguishes it from the VRPB. First, most of the published research has studied the VRPB under the assumption that demand is deterministic and the planning horizon is one period. Secondly, there is no correspondence between returning materials and delivery products, where as in the proposed research, today's delivery is tomorrow's pickup.

An outline of this work is as follows. In the next section, a general stochastic model is developed for finding the optimal pickup policy for a known fixed vehicle route. In Section 3, optimal solutions for some special cases solvable in polynomial time are derived. For the general case, a heuristic pickup policy is proposed in Section 4. Section 4 also contains the results of an extensive computational study designed to evaluate the solution quality of the heuristic procedure. The conclusion highlights some decision rules used for returns planning.

2 Notation and Mathematical Model

Some general notation used throughout this paper and a model of the material returns problem is introduced, assumming that the vehicle route is known. Without loss of generality, it is assumed that $0 \to 1 \to 2 \ldots \to n \to 0$ is a given vehicle route, where stop 0 represents the depot. At the beginning of a period, let

$\underline{e} = (e_1, e_2, \ldots, e_n)$ be the known (integer) vector representing amount of returning materials available at each stop and

$\underline{d} = (d_1, d_2, \ldots, d_n)$ be the known (integer) vector of stop volumes to be delivered by the vehicle in this period.

The problem is to find a vector $\underline{z} = (z_1, z_2, \ldots, z_n)$ representing the number of returning units to be picked up at each stop on a route so that the total penalty cost of postponed pickups over K periods is minimized. This problem can be modeled as a *Markov Decision Process (MDP)* as follows.

Let

D_i^c = the set of all permissible (integer) volumes for stop i to be delivered by a company-owned vehicle,

D_i^p = the set of all permissible (integer) volumes for stop i to be delivered by a for-hire carrier,

$D^c = \{\, \underline{d}^c = (d_1^c, d_2^c, \ldots, d_n^c) : d_i^c \in D_i^c, i = 1, 2, \ldots, n \,\}$,

$D^p = \{\, \underline{d}^p = (d_1^p, d_2^p, \ldots, d_n^p) : d_i^p \in D_i^p, i = 1, 2, \ldots, n \,\}$,

$E = \{\, \underline{e} = (e_1, e_2, \ldots, e_n) \in R^n \,\}$,

$S = \{\, (\underline{e}, \underline{d}) : \underline{e} \in E \text{ and } \underline{d} \in D^c \,\}$,

a_i = the penalty cost of delaying pick up of one unit of returning material at stop i for one period,

Q = capacity of a company-owned vehicle, and

$\underline{z}_k^{(\underline{e},\underline{d})} = (z_{1,k}^{(\underline{e},\underline{d})}, z_{2,k}^{(\underline{e},\underline{d})}, \ldots, z_{n,k}^{(\underline{e},\underline{d})})$, the decision vector, where $z_{j,k}^{(\underline{e},\underline{d})}$ is the number of returning units to be picked up from the jth stop in period k, given the state vector $(\underline{e}, \underline{d}) \in S$.

Defining:

$f_k(\underline{e}, \underline{d})$ = the minimum expected cost of returning units left at stops from period k to period K given the state vector $(\underline{e},\underline{d})$ at the beginning of period k.

The stochastic dynamic programming model is

$$
\begin{aligned}
f_k(\underline{e},\underline{d}) \quad = \quad & \min_{\underline{z}_k^{(\underline{e},\underline{d})} \in Z^{(\underline{e},\underline{d})}} \left\{ \sum_{i=1}^{n} a_i \left(e_i - z_{i,k}^{(\underline{e},\underline{d})} \right) + \right. \\
& \left. \sum_{\substack{\underline{d}^c \in D^c \\ \underline{d}^p \in D^p}} P^{\underline{d}^p} P^{\underline{d}^c} f_{k+1}\left(\underline{e} - \underline{z}_k^{(\underline{e},\underline{d})} + \underline{d} + \underline{d}^p, \underline{d}^c \right) \right\} \\
& k = 1, 2, \ldots, K < \infty, \ \underline{e} \in E, \ \text{and } \underline{d} \in D^c \qquad (1)
\end{aligned}
$$

with boundary conditions

$$
f_{K+1}(\underline{e},\underline{d}) \quad = \quad 0, \quad \forall (\underline{e},\underline{d}) \in S \qquad (2)
$$

where

$$
\begin{aligned}
Z^{(\underline{e},\underline{d})} \quad & = \quad \Big\{ \underline{z} : \ z_1 \le e_1, \ z_2 \le e_2, \ldots, \ z_n \le e_n, \\
& \qquad\qquad z_1 \le d_1 + C_{\underline{d}}, \ z_1 + z_2 \le d_1 + d_2 + C_{\underline{d}}, \ldots, \\
& \qquad\qquad \textstyle\sum_{i=1}^{n} z_i \le \sum_{i=1}^{n} d_i + C_{\underline{d}} \\
& \qquad\qquad \underline{z} \ge 0 \text{ and integer } \Big\}, \\
C_{\underline{d}} \quad & = \quad Q - \textstyle\sum_{i=1}^{n} d_i, \\
P^{\underline{d}^c} \quad & = \quad \text{the probability of occurrence of the demand vector } \underline{d}^c, \text{ and} \\
P^{\underline{d}^p} \quad & = \quad \text{the probability of occurrence of the demand vector } \underline{d}^p.
\end{aligned}
$$

Note that boundary conditions in Equation (2) implies an inherent assumption that there is no cost of any returning items left after period K. The justification of this assumption is that if K is large, the one-time cost of picking up all the remaining returning units by any other practical means should be negligible compared to the total cost incurred over K periods. This cost should not have any significant influence in determining a pickup policy. However, it is worth mentioning that all results of this paper are still valid if the salvage cost is proportional to $\sum_{i=1}^{n} a_i e_i$ rather than 0.

The solution to the MDP model, in general, is intractable due to the dimensionality of its state vectors. Moreover, even when an optimal or near optimal policy might be derived or conjectured, it is not easy to find the exact cost of this policy. The focus was to develop the pickup (return) policies. For a given policy, the associated expected cost was estimated using simulation. In particular, the given policy was simulated for K periods and the associated penalty cost recorded. This simulation was replicated an appropriate number of times and the average cost was used as an estimate of the expected cost. In Section 3, some special cases are considered where the optimal policies can be easily derived.

3 Optimal Return Policies for Some Special Cases

Optimal pickup policies for three special cases are presented: (1) Stops with non-increasing service priorities, (2) the dynamic routing case where routing cost is insignificant (hence, in each period, stops can be visited in a different sequence), and (3) the deterministic stop volume case where all future demand is known a priori.

3.1 Stops with Non-increasing Service Priorities

In this case, the penalties of postponing a pickup for one period are in non-increasing order along the vehicle route. That is, $a_1 \geq a_2 \geq \ldots \geq a_n$. To derive an optimal policy, following definition is needed.

Definition For any two non-negative integer n-vectors $\underline{a}$ and $\underline{b}$, define

$$\underline{a} \succ \underline{b} \quad \text{if} \quad \sum_{i=1}^{j} a_i \geq \sum_{i=1}^{j} b_i, \quad j = 1, 2, \ldots, n$$

Using this definition, the following Theorem 3.1 (see Appendix A for the proof) provides an optimal pickup policy.

Theorem 3.1 *Given a fixed delivery route, say,* $0 \to 1 \to 2, \ldots, \to n \to 0$, *and* $a_i \geq a_{i+1}$, $i = 1, 2, \ldots, (n-1)$, *if* Z^k *is the decision space for pickup vector* $\underline{z}$ *in period* k*, then there exists a vector* $\underline{z}^* \in Z^k$ *such that* $\underline{z}^* \succ \underline{z}$ *for all* $\underline{z} \in Z^k$*. Moreover, vector* $\underline{z}^*$ *represents the optimal pickup policy.*

This theorem provides an intuitive optimal myopic policy: *Pick as many returning units at each stop along the route as available vehicle space allows.* A trivial corollory to the above theorem is the following result.

Corollary 3.1.1 *Given a fixed delivery route, if $a_1 = a_2 = \ldots = a_n$, the optimal policy is myopic:* Pick up as many returning units at each stop as available vehicle space allows.

3.2 Dynamic Routing

In this case, assume that in each period, a vehicle can visit stops in any desired sequence and the objective is simply to minimize the cost of delaying pick up of returning materials. This may be reasonable if the penalty cost of leaving returning units is much greater than the routing cost. For ease of notation, the stops are labeled in a decreasing order of their unit penalty costs, that is, $a_1 \geq a_2 \geq \ldots \geq a_n$. Consider a following two step myopic policy (Policy Dynamic) that determines, at the beginning of each period with state $(\underline{e}, \underline{d})$, first a vehicle route and then the number of returning units to be picked up at each stop on the route.

Policy Dynamic:

Step 1: **(Vehicle Route)** Given the state $(\underline{e}, \underline{d})$, partition all stops into following two sets:

$$\begin{aligned} S_1 &= \{ i : d_i > e_i \} \\ S_2 &= \{ i : d_i \leq e_i \} \end{aligned}$$

The vehicle route for this period is first to visit the stops in set S_1 in non-increasing order of a_i and then stops in set S_2 in the non-decreasing order of a_i.

Step 2: **(Returns)** Given the route derived in Step 1, the pickup vector z is determined iteratively by first assigning the largest possible value to z_1 (the number of units of returning materials to be picked up from the stop with highest unit penalty cost a_1), then z_2 (the number of units of returning materials to be picked up from the stop with second highest unit penalty cost a_2), and so on.

The optimality of this policy is stated as Theorem 3.2. Appendix B contains a detailed proof.

Theorem 3.2 *When stops can be visited in any desired sequence in each period, the policy* **Dynamic** *described above is optimal. That is, it results in the least expected penalty cost of postponing pickups of returning materials.*

3.3 Deterministic Stop Volumes

At the beginning of the planning horizon, all future stop volumes are assumed known. This includes products to be delivered by company-owned vehicles as well as for-hire carriers. This problem is important in its own right; however, the main motivation behind this special case is that it can be used to evaluate any heuristic procedure developed for the general stochastic problem. For any instance of the stochastic problem, the solution to the associated deterministic problem, assumming that all future demand is known at the beginning of period 1 provides a lower bound. To model the problem of finding the optimal pickup decisions over K periods, let

d_{ij}^c = the known volume for stop i in period j to be delivered by the company-owned vehicle,

d_{ij}^p = the known volume for stop i in period j to be delivered by a for-hire carrier,

e_i = the number of returning units ready for pick up at stop i at the beginning of the first period,

$C_{\underline{d}_j}$ = $Q - \sum_{i=1}^n d_{ij}^c$, is the available empty capacity when the company-owned vehicle leaves the depot in period j, and

z_{ij} = the decision variable representing the number of returning units to be picked up at stop i in period j.

Using this notation, the cost F_i of delaying pickup of returning materials at stop i over K periods is:

$$\begin{aligned} F_i &= a_i(e_i - z_{i1}) + a_i(e_i - z_{i1} + d_{i1}^c + d_{i1}^p - z_{i2}) + \\ &\quad a_i(e_i - z_{i1} + d_{i1}^c + d_{i1}^p - z_{i2} + d_{i2}^c + d_{i2}^p - z_{i3}) + \ldots + \\ &\quad a_i\left(e_i - \sum_{j=1}^{K} z_{ij} + \sum_{j=1}^{K-1} d_{ij}^c + \sum_{j=1}^{K-1} d_{ij}^p\right) \\ &= -a_i\left(\sum_{j=1}^{K}(K-j+1)z_{ij} - Ke_i - \sum_{j=1}^{K-1}(K-j)d_{ij}^c - \sum_{j=1}^{K-1}(K-j)d_{ij}^p\right) \end{aligned}$$

Hence, minimizing the total cost $\sum_{i=1}^n F_i$ is equivalent to maximizing

$$\sum_{i=1}^{n}\sum_{j=1}^{K} a_i(K+1-j)z_{ij} \tag{3}$$

The following pure integer linear program (ILP) finds the optimal decisions z_{ij}, $i = 1, 2, \ldots, n,$ and $j = 1, 2, \ldots, K$:

$$\text{Maximize} \quad \sum_{i=1}^{n}\sum_{j=1}^{K} a_i \,(K+1-j)\, z_{ij} \tag{4}$$

s.t.:

Availability Constraints:

$$z_{r1} \quad \leq \quad e_i \qquad r = 1, 2, \ldots, n \tag{5}$$

$$\sum_{j=1}^{l} z_{rj} \quad \leq \quad e_i + \sum_{j=1}^{l-1}\left(d_{rj}^{c} + d_{rj}^{p}\right), \qquad r = 1, 2, \ldots, n; \; l = 2, 3, \ldots, K \tag{6}$$

Capacity Constraints:

$$\sum_{i=1}^{r} z_{il} \quad \leq \quad C_{\underline{d}_l} + \sum_{i=1}^{r} d_{il}^{c}, \qquad l = 1, 2, \ldots, K; \; r = 1, 2, \ldots, n \tag{7}$$

$$z_{rl} \geq 0 \text{ and integers}, \quad r = 1, 2, \ldots, n; \;\; l = 1, 2, \ldots, K \tag{8}$$

In each period, the *availability* constraints guarantees that no more is picked up than is available. For example, consider stop r. In period 1,

$$z_{r1} \leq e_r$$

In period 2,

$$z_{r2} \leq \begin{pmatrix} \text{Unpicked} \\ \text{Units from} \\ \text{Period 1} \end{pmatrix} + \begin{pmatrix} \text{Delivered Units} \\ \text{in Period 1} \end{pmatrix}$$

That is,

$$\begin{aligned} z_{r2} &\leq (e_r - z_{r1}) + (d_{r1}^{c} + d_{r1}^{c}) \\ \text{or} \quad z_{r1} + z_{r2} &\leq e_r + (d_{r1}^{c} + d_{r1}^{c}) \end{aligned}$$

Repeating this process for subsequent periods derives constraints (6) in (ILP).

The *capacity* constraints enforces the company-owned vehicle's capacity limitations at each stop. Consider period l. At stop 1,

$$z_{1l} \leq \begin{pmatrix} \text{Space Available in Company-Owned} \\ \text{Vehicle After Delivering at Stop 1} \end{pmatrix}$$

or

$$z_{1l} \le C_{\underline{d}_l} + d^c_{1l}$$

At stop 2,

$$z_{2l} \le \begin{pmatrix} \text{Empty Space} \\ \text{On Leaving} \\ \text{Stop 1} \end{pmatrix} + \begin{pmatrix} \text{Delivered Units} \\ \text{at Stop 2} \end{pmatrix}$$

That is,

$$\begin{array}{rcl} z_{2l} & \le & \left(C_{\underline{d}_l} + d^c_{1l} - z_{1l}\right) + (d^c_{2l}) \\ \text{or} \quad z_{1l} + z_{2l} & \le & C_{\underline{d}_l} + (d^c_{1l} + d^c_{2l}) \end{array}$$

Repeating this process for subsequent stops derives the constraints in Equation (7) in ILP.

In general, the above large scale integer linear program (ILP) cannot be solved efficiently. However, by exploiting the structure of the above ILP model, it can be solved in an efficient manner.

By adding slack variables x_{rl} and y_{rl} and a dummy constraint $0 = 0$, the ILP formulation can be rewritten as:

$$\text{Maximize} \quad \sum_{i=1}^{n} \sum_{j=1}^{K} a_i \,(K + 1 - j)\, z_{ij} \tag{9}$$

s.t.:

$$z_{r1} + x_{r1} = e_r \qquad r = 1, 2, \ldots, n; \tag{10}$$

$$\sum_{j=1}^{l} z_{rj} + x_{rl} = e_r + \sum_{j=1}^{l-1} \left(d^c_{rj} + d^p_{rj}\right), \qquad r = 1, 2, \ldots, n; \; l = 2, 3, \ldots, K \tag{11}$$

$$-\sum_{i=1}^{r} z_{il} - y_{rl} = -C_{\underline{d}_l} - \sum_{i=1}^{r} d^c_{il}, \qquad l = 1, 2, \ldots, K; \; r = 1, 2, \ldots, n \tag{12}$$

$$0 = 0 \tag{13}$$

$$z_{rl} \ge 0 \text{ and integers}, r = 1, 2, \ldots, n; \;\; l = 1, 2, \ldots, K \tag{14}$$

Now, perform the following elementary row operations on the constraints in Equations (10) - (13):

1. From the dummy constraint in Equation (13), subtract constraints corresponding to $l = K$ and $r = 1, 2, \ldots, n$ in Equation (11) and $r = n$ and $l = 1, 2, \ldots, K$ in Equation (12). This transforms the constraints in Equation (13) to:

$$\begin{aligned} -\sum_{r=1}^{n} x_{rK} + \sum_{l=1}^{K} y_{nl} &= -\sum_{r=1}^{n} e_r - \sum_{r=1}^{n}\sum_{j=1}^{K-1}\left(d^c_{rj} + d^p_{rj}\right) + \\ & \quad \sum_{l=1}^{K} C_{\underline{d}_l} + \sum_{l=1}^{K}\sum_{i=1}^{n} d^c_{il} \end{aligned}$$

2. For each r in Equations (10) and (11), subtract constraint $l-1$ from l, for $l = 2, 3, \ldots, K$. This results in:

$$\begin{aligned} z_{r1} + x_{r1} &= e_r & & r = 1, 2, \ldots, n \\ z_{rl} - x_{r,l-1} + x_{rl} &= d^c_{r,l-1} + d^p_{r,l-1} & & r = 1, 2, \ldots, n \quad l = 2, 3, \ldots, K \end{aligned}$$

3. For each l in Equation (12), subtract constraint $r-1$ from r, for $r = 2, 3, \ldots, n$. This results in:

$$\begin{aligned} -z_{1l} - y_{1l} &= -C_{\underline{d}_l} - d^c_{1l} & & l = 1, 2, \ldots, K \\ -z_{rl} + y_{r-1,l} - y_{rl} &= -d^c_{rl}, & & r = 2, 3, \ldots, n; \quad l = 1, 2, \ldots, K \end{aligned}$$

Combining the above transformations, the ILP can be rewritten as:

$$
\text{Maximize} \quad \sum_{i=1}^{n}\sum_{j=1}^{K} a_i\,(K+1-j)\,z_{ij}
$$

s.t.:

$$
\begin{array}{lcll}
z_{r1} + x_{r1} & = & e_r & r = 1,2,\ldots,n \\
z_{rl} - x_{r,l-1} + x_{rl} & = & d^c_{r,l-1} + d^p_{r,l-1} & l = 2,3,\ldots,K \\
 & & & r = 1,2,\ldots,n \\
 & & & \\
-z_{1l} - y_{1l} & = & -C_{\underline{d}_l} - d^c_{1l} & l = 1,2,\ldots,K \\
-z_{rl} + y_{r-1,l} - y_{rl} & = & -d^c_{rl} & r = 2,3,\ldots,n \\
 & & & l = 1,2,\ldots,K \\
 & & & \\
-\sum_{r=1}^{n} x_{rK} + \sum_{l=1}^{K} y_{nl} & = & \multicolumn{2}{l}{-\sum_{r=1}^{n} e_r - \sum_{r=1}^{n}\sum_{j=1}^{K-1}\left(d^c_{rj} + d^p_{rj}\right) + \sum_{l=1}^{K} C_{\underline{d}_l} + \sum_{l=1}^{K}\sum_{i=1}^{n} d^c_{il}} \\
z_{rl} & \geq & \multicolumn{2}{l}{0 \text{ and integers}, \quad r = 1,2,\ldots,n, \quad l = 1,2,\ldots,K}
\end{array}
$$

It can be seen that in the above transformed model, each variable appears in exactly two constraints with coefficients of $+1$ and -1. Hence this model represents a transshipment problem with $2Kn+1$ nodes and $3Kn$ arcs. This allows solving larger instances of this problem more efficiently than using a general purpose IP solver. The results of this section are summarized in the following theorem.

Theorem 3.3 *Let n be the number of customers and K be the number of periods in the planning horizon. For a fixed route, the deterministic and finite horizon case can be modeled as the Transshipment Problem with $2Kn+1$ nodes and $3Kn$ arcs.*

4 General Returns Problem

A study of the value function of MDP (cf. Section 2.1) for the general case where customers have different service priorities showed that this function is neither convex nor concave. Hence, it is not clear that a simple optimal pickup policy exists. Instead, in Section 4.1, a simple and easy to implement heuristic procedure is proposed. Computational results that evaluate the quality of this heuristic procedure are presented in Section 4.2.

4.1 The Weighted Leveling Heuristic:

The basic idea behind the weighted leveling heuristic procedure is to assign each stop i an appropriate weight w'_i and then in each period, determine the pickup vector $\underline{z}$ so as to minimize $\sum_{i=1}^{n} w'_i(e_i - z_i)$. Based on intuition as well as some numerical experiments the values of weights w'_i depend on:

1. Penalty cost a_i
2. Position of the stop along the route
3. Demand distributions
4. Number of returning units left at stop i

Item (4) suggests that rather than using a fixed value of w'_i in each period, its value should depend on the number of returning units left at each stop, where the weight is higher for more returning units. Use $w'_i = w_i(e_i - z_i)$, where w_i is a fixed number that captures the effect of list items (1) to (3) and the multiplier $(e_i - z_i)$ captures the effect of list item (4). Thus at each period, if $\underline{e}$ is the vector of returning units, and $\underline{d}$ is the demand vector, determine the pickup vector $\underline{z} \in Z^{(\underline{e},\underline{d})}$ so as to

$$\text{Minimize} \sum_{i=1}^{n} w_i(e_i - z_i)^2$$

Hence, the following one period problem (P1) needs to be solved.

$$\begin{array}{llrcll}
\text{(P1)} & \text{Minimize} & \displaystyle\sum_{i=1}^{n} w_i(e_i - z_i)^2 & & & \\
& \text{subject to:} & z_i & \leq & e_i & i = 1, 2, \ldots, n \\
& & \displaystyle\sum_{j=1}^{i} z_j & \leq & C_{\underline{d}} + \displaystyle\sum_{j=1}^{i} d_j & i = 1, 2, \ldots, n \\
& & z_i & \geq & 0 \text{ and integer} &
\end{array}$$

A greedy heuristic approach to solving this problem is to start with $\underline{z} = \underline{0}$ and iteratively increment by one the value of z_{i^*}. Among all stops where it is feasible to increment by one its pickup value z_i, stop i^* is the one that provides the maximum improvement in the objective function value. That is, this is the one that has maximum value of $w_i(e_i - z_i)^2 - w_i(e_i - z_i - 1)^2$ (or equivallently a maximum value of $2w_i(e_i - z_i) - w_i$). The greedy approach results in the following algorithm.

Algorithm GREEDY

Step 1: Let $\underline{w} = (w_1, w_2, \ldots, w_n)$ be a specified vector of weights.

Step 2: For any period and a given state vector $(\underline{e}, \underline{d}) \in S$, start with $\underline{z} = (z_1, z_2, \ldots, z_n) = (0, \ldots, 0)$ and perform the following steps.

Step 2(a): Let J be the set of stops where it is feasible to pick up an additional returning unit. If $J = \phi$, stop. Otherwise, find i^*, such that

$$2w_{i^*}(e_i - z_i) - w_{i^*} = \max_{j \in J} \{2w_j(e_j - z_j) - w_j\}$$

Break ties in favor of stops visited first.

Step 2(b): Set $z_{i^*} = z_{i^*} + 1$, and go to Step 2(a).

It should be pointed out that this greedy algorithm in fact solves the one period problem (P1) optimally. This claim is stated as Theorem 4.1. A detailed proof is in Appendix C.

Theorem 4.1 *Algorithm GREEDY finds an optimal solution to the quadratic one period problem (P1).*

This procedure is termed the *Weighted Leveling Heuristic* since it determines the pickup vector $\underline{z}$ by attempting to level the value of $2w_i(e_i - z_i) - w_i$ across all stops. The question is how to determine a weight vector $\underline{w}$. One possible choice is to define $w_i = a_i$. However, as stated earlier, the best value for $\underline{w}$ depends not only on penalty costs a_i, but also on the route sequence as well as the stop volume distribution. An appropriate weight vector $\underline{w}$ was obtained through a simulation based search.

An obvious issue is how to evaluate the performance of this heuristic. Comparing its performance against an optimal solution is computationally prohibitive even for problems with only a few customers. One commonly used approach is to compare the heuristic against some known lower bound on the optimal cost. Based on the special cases discussed in Section 3, for each instance of the problem, two lower bounds can be computed. The first is the expected cost when customers can be visited in any desired sequence each period. Another is the cost of the associated deterministic problem where the instance is assumed to be known a priori. As discussed in Section

3.1, this deterministic problem can be solved as a Transshipment Problem, which allows solving larger instances more efficiently. A preliminary computational study shows that among the two lower bounds, the one obtained by solving the deterministic instance is consistently tighter. In what follows, the weighted leveling heuristic is compared against the lower bound derived using a deterministic analog.

4.2 Computational Results

As mentioned above, the weight vector $\underline{w}$ needed in the weighted leveling heuristic is obtained through simulation. Two different search methods were used to find $\underline{w}$, the Simplex Search Method (Humphrey and Wilson, 2000) and the Pattern Search Method (Hooke and Jeeves, 1961). Barton and Ivey (1996) studied these two methods and reported that the simplex search method tends to perform better for problems with dimensions of 10 or less. Based on this observation, the simplex search method is used for problems with 10 or fewer stops and the pattern search method for problems with greater than 10 stops.

4.2.1 Problem Generation

Creating test problems for this computational study, various parameters were generated as described below.

Number of stops (n) and number of periods (K)

To study the influence of the n and K on the quality of the solution procedures, several values were used. These values were chosen based on what is generally common in practice as well as the limitation imposed by the computational effort needed.

Penalty cost (a_i)

The penalty cost a_i of postponing the pick up of a returning materials unit at stop i for one period was a uniform random number generated in the interval $[1, n]$, where n is the number of stops.

Stop volume distribution for each customer

Stop volume was assumed to follow a triangular distribution as it allowed approximating different distribution patterns by varying its three parameters (minimum, maximum, and mode). Integer stop volume for a customer was generated by discretizing the triangular distribution. For stop i, $min\text{-}dc_i$, $max\text{-}dc_i$, and $mode\text{-}dc_i$ were the minimum, maximum and the mode, respectively, of the triangular distribution for the quantity of products delivered by the company-owned vehicle. Likewise, $min\text{-}dp_i$, $max\text{-}dp_i$, and $mode\text{-}dp_i$ were the respective parameters of the triangular distribution for the stop volume delivered by for-hire carriers. These distribution parameters were generated as follows for three classes of stop volume patterns considered.

(a) **Random Pattern** The triangular distribution parameters were generated as follows:

$min\text{-}dc_i = a$ $\qquad$ $min\text{-}dp_i = a$
$max\text{-}dc_i = \text{Uniform[a, b]}$ $\qquad$ $max\text{-}dp_i = \alpha * \text{Uniform[a, b]}$
$mode\text{-}dc_i = \text{Uniform}[a, max\text{-}dc_i]$ $\qquad$ $mode\text{-}dp_i = \text{Uniform}[a, max\text{-}dp_i]$

Unless otherwise stated, the values of a and b were fixed to 0 and 21 respectively. This choice of a and b was reasonable for the kind of problems investigated here. The delivered items were of large sizes, as small items were usually moved in storage boxes, containers, pallets, etc. For example, the American Red Cross Blood Services organization delivered blood items in special insulated boxes where hospitals' blood demand varied from 0 to 14 boxes per day.

Parameter α controls the ratio of the units delivered by for-hire carriers to the units delivered by a company-owned vehicle. Several values of α were used to investigate the effect of this factor on the performance of the algorithm.

(b) **Symmetric Random Pattern** Here the stop volume distribution parameters were generated as follows:
$min\text{-}dc_i = 0$,
$max\text{-}dc_i = 21$, and
$mode\text{-}dc_i = \text{Uniform}[5.25, 15.75]$.
Values for the parameters $min\text{-}dp_i$, $max\text{-}dp_i$, and $mode\text{-}dp_i$ were generated in a similar way.

(c) **Right-Skewed Random Pattern**

The stop volume distribution parameters were generated as follows:
$min\text{-}dc_i = 0$,

$max\text{-}dc_i = 21$, and
$mode\text{-}dc_i = \text{Uniform}[15.75, 21]$.

Values for the parameters $min\text{-}dp_i$, $max\text{-}dp_i$, and $mode\text{-}dp_i$ were generated in a similar way.

Vehicle capacity (Q)

For each problem situation, three increasing values of vehicle capacities Q^L, Q^M, and Q^H were considered. Relative to a given stop volume distribution, they represented low, medium, and high vehicle capacities respectively. First, Q^L was set to be the lowest possible value, which is the sum of the maximum stop volume values that might be delivered to stops by the company-owned vehicle. That is,

$$Q^L = \sum_{i=1}^{n} max\text{-}dc_i$$

Based on computational experiments, Q^H (high) was chosen so that there were, on average, two returning units left per period per stop when the policy of picking up as much as possible was used. Value for Q^M (medium) was chosen to be the average of low and high capacities rounded to nearest integer.

Initial returning materials units ($\underline{e}$) located at stops

At the beginning of the planning horizon, rather than starting with zero or an arbitrary positive number of returning units at each stop, a value that equals one-days random delivery quantity was used. These values were randomly generated from each stop's volume distribution.

Experimental Design

To know how the proposed heuristic procedure performs relative to the lower bound, following steps were used:

(1) Generate a random instance of the problem, that is, the value of n, K, a_i, and the parameters of the stop volume distributions.

(2) Use simplex or pattern search to determine the appropriate weight vector $\underline{w}$ for the leveling heuristic.

(3) Based on the demand parameters in (1), simulate 100 replications. For each replication, compute the relative difference of the total cost using the weighted leveling policy and the lower bound obtained by assuming that the stop volume is known a priori. The average of these 100 relative differences is used as a measure of the quality of the proposed heuristic. In this study, relative difference is defined as:

$$\text{Relative Difference} = \frac{\left\{\begin{array}{c}\text{Total Cost of}\\ \text{Heuristic}\end{array}\right\} - \left\{\begin{array}{c}\text{Total Cost of Deterministic}\\ \text{Solution}\end{array}\right\}}{\left\{\begin{array}{c}\text{Total Cost of Deterministic}\\ \text{Solution}\end{array}\right\}}$$

For a given class of the problem, Steps (1)-(3) are repeated 100 times to estimate the average relative cost difference and its 95% confidence interval.

4.2.2 Computational Results

In the proposed heuristic, the most time consuming aspect is to determine the weight vector $\underline{w}$ using the simplex or pattern search algorithm. This time grows exponentially as the number of stops increases. However, determining vector $\underline{w}$ is a one time strategic decision and potential savings over K periods will more than justify spending a few minutes (or even an hour) of computer time to find its best value. Therefore in this study, the focus is on the quality of the heuristic solution rather than the computational efficiency. In this section, results of the computational study are summarized.

Effect of n and Q

For various values of n and Q, Table 1 shows the confidence intervals for the average relative cost difference based on 100 test problems generated using the random demand pattern with $\alpha = 1$. It shows that the expected cost obtained using the weighted leveling policy is, on average, within 10% of the lower bound. The number of customers seems to have no affect on the performance of the heuristic. Increasing the vehicle capacity resulted in a solution farther from the lower bound; however, this should not be a concern as in such cases the total penalty cost is relatively small.

Number of Stops (n)	Number of Periods (K)	Vehicle Capacity (Q)	Avg. Penalty Cost ($)	Relative Difference	
				Mean	95% CI*
10	365	Q^L	418.57	0.03	± 0.01
		Q^M	134.38	0.07	± 0.02
		Q^H	2.72	0.07	± 0.02
20	365	Q^L	388.47	0.03	± 0.02
		Q^M	124.32	0.06	± 0.02
		Q^H	2.93	0.07	± 0.02
30	365	Q^L	337.67	0.04	± 0.02
		Q^M	97.78	0.06	± 0.02
		Q^H	3.47	0.09	± 0.03

* Confidence Interval

Table 1: Relative Cost Performance of the Weighted Leveling Heuristic as a Function of n and Q

Effect of Planning Horizon (K)

For various values of K, Table 2 shows the confidence intervals for the average relative difference based on 100 test problems generated with $n = 10$ customers, each having the random demand pattern with $\alpha = 1$. *The table shows that the length of the planning horizon seems to have no affect on the performance of the heuristic.* As before, increasing the vehicle capacity causes a solution farther from the lower bound.

Effect of Stop Volume Patterns

Table 3 shows the confidence intervals for the average relative cost difference based on 100 test problems generated with $n = 10$, $K = 365$, and three different stop volume patterns (random with $\alpha = 1$, symmetric, and right skewed). The algorithm is seen to perform quite well with costs exceeding the deterministic lower bound by less than 10%. A closer look at the individual problem instances, where the heuristic solution was not close to the lower bound, revealed a common characteristic that the stop volume distributions towards the end of the vehicle route has higher variability. Hence, when designing a route, *stops with higher volume variability should be assigned*

Number of Stops (n)	Number of Periods (K)	Vehicle Capacity (Q)	Avg. Penalty Cost ($)	Relative Difference	
				Mean	95% CI*
10	90	Q^L	105.32	0.04	± 0.01
		Q^M	39.51	0.06	± 0.02
		Q^H	3.13	0.06	± 0.01
	180	Q^L	197.65	0.05	± 0.02
		Q^M	66.62	0.08	± 0.02
		Q^H	3.12	0.08	± 0.02
	365	Q^L	418.57	0.03	± 0.01
		Q^M	134.38	0.07	± 0.02
		Q^H	2.72	0.07	± 0.02

* Confidence Interval

Table 2: Relative Cost Performance of the Weighted Leveling Heuristic as a Function of K and Q

to the beginning of the route.

Effect of Ratio α of Stop Volume Delivered by For-hire Carriers and the Company-owned Vehicle

Recall in generating random parameters of the triangular distribution, α controls the ratio of stop volume delivered by for-hire carriers to that delivered by the company-owned vehicle. For example, $\alpha = 2$ implies that, on average, for-hire carriers deliver twice as many units than company-owned vehicles. That is, on average 67% of a stop's volume is delivered by for-hire carriers and 33% by the company-owned vehicle. In contrast, $\alpha = 0.3$ means that, on average, the amount delivered by for-hire carriers is about 30% of units delivered by company-owned vehicle. Table 4 shows the confidence intervals for the average relative difference based on 100 test problems generated with $n = 10$, $K = 365$, and a random stop volume pattern with different values for α. It shows that the algorithm performs quite well when α exceeds 1. However, for α less than 1, the gap is as much as 15%. However, this must not be of much concern as in such cases the costs are lower.

Demand Pattern	Number of Periods (K)	Vehicle Capacity (Q)	Avg. Penalty Cost ($)	Relative Difference	
				Mean	95% CI*
Random	365	Q^L	418.57	0.03	± 0.01
		Q^M	134.38	0.07	± 0.02
		Q^H	2.72	0.07	± 0.02
Symmetric	365	Q^L	85.52	0.08	± 0.01
		Q^M	16.62	0.09	± 0.01
		Q^H	2.74	0.03	± 0.00
Right Skewed	365	Q^L	1817.81	0.01	± 0.00
		Q^M	614.00	0.01	± 0.00
		Q^H	2.71	0.02	± 0.00

* Confidence Interval

Table 3: Relative Cost Performance of the Weighted Leveling Heuristic for Different Demand Patterns

α	Number of Periods (K)	Vehicle Capacity (Q)	Avg. Penalty Cost ($)	Relative Difference	
				Mean	95% CI*
0.3	365	Q^L	187.11	0.10	± 0.03
		Q^M	65.72	0.15	± 0.03
		Q^H	2.86	0.12	± 0.03
1.0	365	Q^L	418.57	0.03	± 0.01
		Q^M	134.38	0.07	± 0.02
		Q^H	2.72	0.07	± 0.02
2.0	365	Q^L	2769.47	0.01	± 0.00
		Q^M	825.13	0.01	± 0.00
		Q^H	2.96	0.03	± 0.01

* Confidence Interval

Table 4: Relative Cost Performance of the Weighted Leveling Heuristic for Different Values for α

Observations

The computational results summarized in this section show that the proposed heuristic algorithm performs quite well with a gap of less than 8% between its solution and the associated deterministic lower bound. For certain classes of problems (for example, high α or high vehicle capacity Q), the gap was higher than 10%. However in these cases, the costs are much lower and the solution quality is less of a concern.

5 Conclusions

In this paper, a dynamic reverse logistics problem is considered where the forward and reverse distribution of products are integrated at the routing level. Instead of sending vehicles exclusively for collecting the returning materials, utilizing the capacity of the vehicle delivering products as well as collecting returning materials simultaneously is investigated. In particular, the situation is that the recovered materials from the delivered units are available for return the following day. This paper focuses on the Returns Problem for a given vehicle route that requires determining the pickup policy for returning materials. The problem can be formulated as a Markov Decision Process (MDP), which in general is difficult to solve. However, three special cases are identified where the optimal pickup policies can be easily derived. For the general returns problem, it is not clear that a simple optimal pickup policy exists since solutions to small problems show that *the value functions are, in general, neither convex nor concave*. Therefore, an easily implementable heuristic myopic policy, the *Weighted Leveling Heuristic* is proposed. The key results and observations of this research are summarized as follows:

1. If the stops are placed in a non-increasing order of the penalty costs a_i, it is shown that the optimal myopic policy is: *to pick as many returning units at each stop along the route as available vehicle space allows.*

2. If the route travel cost is insignificant compared to the penalty cost of not picking up the returning units and a different route can be used in each delivery period, then a myopic policy is optimal, where this easily computable policy simultaneously determines the route and pickup amounts.

3. If all stop future volumes are known in advance, the returns problem requires solving an Integer Linear Program (ILP); however, *this problem*

can be transformed into a Transshipment Problem, which allows solving large problems efficiently.

4. For the general problem, an extensive computational study with the *Weighted Leveling Heuristic* showed that the total cost of the heuristic solution was, on average, within 8% of the lower bound and that the total cost generally is higher when the stops toward the end of the route have a volume patterns with high variance; hence, *when designing the vehicle route, place stops with high volume variance toward the beginning of the route.*

A more challenging and comprehensive problem is to see how the returns policies developed in this paper can be used to find a solution to the joint vehicle routing and product returns problems. This question is addressed in another paper (Alshamrani et al., 2004).

Appendix A
Proof of Theorem 3.1

In the proof of this theorem, recall the following definition.

Definition For any two non-negative integer n-vectors $\underline{a}$ and $\underline{b}$, define

$$\underline{a} \succ \underline{b} \quad \text{if} \quad \sum_{i=1}^{j} a_i \geq \sum_{i=1}^{j} b_i, \quad j = 1, 2, \ldots, n$$

Using this definition, the following lemma is proved.

Lemma A.1: Let π' represent the policy of picking up as much as possible at each stop and π be any other policy. Then,

1. For a given state $(\underline{e}, \underline{d}) \in S$,

$$\sum_{i=1}^{j} \left(e_i - z_i^{\pi'}\right) \leq \sum_{i=1}^{j} \left(e_i - z_i^{\pi}\right) \quad \text{for} \quad j = 1, 2, \ldots, n$$

2. If $(\underline{e}^s, \underline{d}) \in S$ and $(\underline{e}^l, \underline{d}) \in S$, such that $\underline{e}^s \prec \underline{e}^l$, then

   (a) $\sum_{i=1}^{j} \left(e_i^s - z_i^{s,\pi'}\right) \leq \sum_{i=1}^{j} \left(e_i^l - z_i^{l,\pi}\right)$ for $j = 1, 2, \ldots, n$

   (b) $\sum_{i=1}^{j} a_i \left(e_i^s - z_i^{s,\pi'}\right) \leq \sum_{i=1}^{j} a_i \left(e_i^l - z_i^{l,\pi}\right)$ for $j = 1, 2, \ldots, n$

   where $\underline{z}^{s,\pi'}$ $[\underline{z}^{l,\pi}]$ is the decision vector when following the policy π' $[\pi]$ at the state $(\underline{e}^s, \underline{d})$ $[(\underline{e}^l, \underline{d})]$.

Proof:
(1) This result is a direct implication of the fact that $z^{\pi'} \succ z^{\pi}$.

(2a) This result is proved by induction.
For $j = 1$, we have

$$\begin{aligned} z_1^{s,\pi'} &= \min(e_1^s, d_1 + C_{\underline{d}}) \\ z_1^{l,\pi} &\leq \min(e_1^l, d_1 + C_{\underline{d}}) \end{aligned}$$

If $z_1^{s,\pi'} = e_1^s$, then

$$e_1^s - z_1^{s,\pi'} = 0 \leq e_1^l - z_1^{l,\pi}$$

Otherwise, if $z_1^{s,\pi'} = d_1 + C_{\underline{d}}$ and since $e_1^s \leq e_1^l$ and $z_1^{l,\pi} \leq d_1 + C_{\underline{d}}$, then

$$e_1^s - z_1^{s,\pi'} \leq e_1^l - z_1^{l,\pi}$$

Now, assume that

$$\sum_{i=1}^{j-1} \left(e_i^s - z_i^{s,\pi'} \right) \leq \sum_{i=1}^{j-1} \left(e_i^l - z_i^{l,\pi} \right)$$

At the j^{th} customer,

$$\begin{aligned} z_j^{s,\pi'} &= \min\left(e_j^s, d_1 + d_2 + \ldots + d_j + C_{\underline{d}} - z_1^{s,\pi'} - z_2^{s,\pi'} - \ldots - z_{j-1}^{s,\pi'} \right) \\ z_j^{l,\pi} &\leq \min\left(e_j^l, d_1 + d_2 + \ldots + d_j + C_{\underline{d}} - z_1^{l,\pi} - z_2^{l,\pi} - \ldots - z_{j-1}^{l,\pi} \right) \end{aligned}$$

Case 1: $z_j^{s,\pi'} = e_j^s$. Then,

$$e_j^s - z_j^{s,\pi'} = 0 \leq e_j^l - z_j^{l,\pi}$$

and hence

$$\sum_{i=1}^{j} \left(e_i^s - z_i^{s,\pi'} \right) \leq \sum_{i=1}^{j} \left(e_i^l - z_i^{l,\pi} \right)$$

Case 2: $z_j^{s,\pi'} = d_1 + d_2 + \ldots + d_j + C_{\underline{d}} - z_1^{s,\pi'} - z_2^{s,\pi'} - \ldots - z_{j-1}^{s,\pi'}$.

In this case, $\sum_{i=1}^{j} z_i^{s,\pi'} = C_{\underline{d}} + \sum_{i=1}^{j} d_i$. Then,

$$\begin{aligned} \sum_{i=1}^{j} \left(e_i^s - z_i^{s,\pi'} \right) &= \sum_{i=1}^{j} e_i^s - \sum_{i=1}^{j} z_i^{s,\pi'} \\ &= \sum_{i=1}^{j} e_i^s - \left(d_1 + d_2 + \ldots + d_j + C_{\underline{d}} \right) \\ &\leq \sum_{i=1}^{j} e_i^l - \left(d_1 + d_2 + \ldots + d_j + C_{\underline{d}} \right) \\ &\leq \sum_{i=1}^{j} e_i^l - \left(z_1^{l,\pi'} + z_2^{l,\pi'} + \ldots - z_j^{l,\pi'} \right) \\ &= \sum_{i=1}^{j} \left(e_i^l - z_i^{l,\pi} \right) \end{aligned}$$

(2b) Since $a_1 \geq a_2 \geq \ldots \geq a_n$, multiplying the ith inequality of Lemma 2(a) by a nonnegative number $a_i - a_{i+1}$ (where $a_{n+1} = 0$) and adding first j inequalities results in:

$$\sum_{i=1}^{j} a_i \left(e_i^s - z_i^{s,\pi'}\right) \;\leq\; \sum_{i=1}^{j} a_i \left(e_i^l - z_i^{l,\pi}\right) \quad \text{for} \quad j = 1, 2, \ldots, n$$

□

Proof of Theorem 3.1 Let $(\underline{e}_1, \underline{d}_1) \in S$ is the state at the beginning of the planning horizon. Consider a demand instance $(d_{j,l}^c, d_{j,l}^p)$, $j = 1, 2, \ldots, n; l = 1, 2, \ldots, K$. Let $\underline{z}_l^{\pi'}, l = 1, 2, \ldots, K$ is the pickup vector in period l if policy "pickup as much as possibe" is used in each period. $\underline{z}_l^{\pi}, l = 1, 2, \ldots, K$ are the associated vectors if some other policy π is followed.

It is proved by induction that if $\underline{e}_l^{\pi'}$ and $\underline{e}_l^{\pi}$ represents the vectors of returning units available to be picked up in period l under policy π' and π respectively, then

$$\underline{e}_l^{\pi'} \prec \underline{e}_l^{\pi}; \quad l = 1, 2, \ldots, K$$

It is clear that $\underline{e}_1^{\pi'} \prec \underline{e}_1^{\pi}$. Assume that

$$\underline{e}_l^{\pi'} \prec \underline{e}_l^{\pi} \quad \text{for} \quad l = 1, 2, \ldots, k$$

For period $k+1$, since $\underline{e}_k^{\pi'} \prec \underline{e}_k^{\pi}$, then from Lemma 1,

$$\sum_{i=1}^{j} \left(e_{i,k}^{\pi'} - z_{i,k}^{\pi'}\right) \leq \sum_{i=1}^{j} \left(e_{i,k}^{\pi} - z_{i,k}^{\pi}\right) \quad \text{for} \quad j = 1, 2, \ldots, n$$

Hence,

$$\sum_{i=1}^{j} \left(e_{i,k}^{\pi'} - z_{i,k}^{\pi'} + d_{i,k}^c + d_{i,k}^p\right) \;\leq\; \sum_{i=1}^{j} \left(e_{i,k}^{\pi} - z_{i,k}^{\pi} + d_{i,k}^c + d_{i,k}^p\right) \qquad \text{for} \quad j = 1, 2, \ldots, n,$$

or,

$$\underline{e}_{k+1}^{\pi'} \prec \underline{e}_{k+1}^{\pi}$$

Then,

$$\underline{e}_k^{\pi'} \prec \underline{e}_k^{\pi} \quad \text{for} \quad k = 1, 2, \ldots, K \leq \infty$$

From Lemma 1,

$$\sum_{i=1}^{n} a_i \left(e_{i,k}^{\pi'} - z_{i,k}^{\pi'}\right) \leq \sum_{i=1}^{n} a_i \left(e_{i,k}^{\pi} - z_{i,k}^{\pi}\right) \quad \text{for} \quad k = 1, 2, \ldots, K \leq \infty$$

Hence in each period, the cost of delaying pick up under policy π' never exceeds the cost of any other policy π. □

Appendix B
Proof of Theorem 3.2

The following two lemmas are needed to prove this theorem.

Lemma B.1: If $\underline{0} \prec \underline{e} \prec \underline{e}'$ and the stops are numbered such that $a_1 \geq a_2 \geq \ldots \geq a_n > 0$, then

$$\sum_{i=1}^{j} a_i(e_i' - e_i) \geq \sum_{i=1}^{j} a_{j+1}(e_i' - e_i) \geq 0, \quad j = 1, 2, \ldots, n.$$

where $a_{n+1} = 0$.

Proof: Since $\underline{e} \prec \underline{e}'$,

$$\sum_{i=1}^{l}(e_i' - e_i) \geq 0 \quad l = 1, 2, \ldots, j, \ldots, n$$

Multiplying the l^{th} inequality by the nonnegative term $(a_l - a_{l+1})$ and combining first j inequalities results in:

$$\sum_{l=1}^{j}(a_l - a_{l+1}) \sum_{i=1}^{l}(e_i' - e_i) \geq 0$$

or

$$\sum_{i=1}^{j} a_i(e_i' - e_i) - a_{j+1} \sum_{i=1}^{j}(e_i' - e_i) \geq 0$$

□

Lemma B.2: For $\underline{0} \prec \underline{e} \prec \underline{e}'$, if $z^{\pi''}$ is the pickup vector when the policy DYNAMIC (π'') is used at state $(\underline{e}, \underline{d})$, and z^{π} is the pickup vector when any policy π is used at state $(\underline{e}', \underline{d})$, then:

1. If $z_j^{\pi''} > 0,$ then $\quad z_i^{\pi''} = e_i \quad$ for $\quad i = 1, 2, \ldots, j-1.$
2. If $z_j^{\pi''} < e_j,$ then $\quad z_1^{\pi''} + z_2^{\pi''} + \ldots + z_j^{\pi''} = Q.$

3. (a) $\sum_{i=1}^{j}(e_i - z_i^{\pi''}) \;\leq\; \sum_{i=1}^{j}(e'_i - z_i^{\pi}), \quad j = 1, 2, \ldots, n$

(b) $\sum_{i=1}^{j} a_i(e_i - z_i^{\pi''}) \;\leq\; \sum_{i=1}^{j} a_i(e'_i - z_i^{\pi}), \quad j = 1, 2, \ldots, n$

Proof:
(1) Assume that $z_j^{\pi''} > 0$ and $\exists\, i < j : z_i^{\pi''} < e_i$.
Case 1: $i \in S_2$.

In this case, stop i is visited after stop j. Hence, any vehicle capacity given to stop j could instead be used to pick up returning units from stop i. Allowing pickups from stop j while stop i has returning materials contradicts the myopic policy of giving priorities to costly stops.

Case 2: $i \in S_1$
Stop j is visited after stop i. Since $d_i > e_i$, some vehicle capacity is reserved from stop i for stops with higher priorities in set S_2. But $\forall k < i$, with $k \in S_2$, stop k is visited after stop j. This contradicts the myopic policy that reserves the vehicle capacity for k from stop j before stop i.

(2) Let

C_1 be the set of stops visited by the vehicle before arriving at stop j,

C_2 be the set of stops visited by the vehicle after visiting stop j, and

R be the unused vehicle capacity after visiting stop j.

From Part (1) of this lemma, $\forall k > j$, $z_k^{\pi''} = 0$; thus,

$$z_j^{\pi''} = Q - \sum_{\substack{i \in C_1 \\ i < j}} z_i^{\pi''} - \left(\sum_{i \in C_2} d_i + R \right) \tag{15}$$

Case 1: $j \in S_1$. In this case, since $d_j > e_j, R > 0$. Now, for the stops in set C_2, the vehicle can pick up returning materials only from stops in set $C_2 \cap S_2$ that have higher priority than stop j. Moreover, since $d_i \leq e_i$ for $i \in (C_2 \cap S_2)$, thus

$$\sum_{\substack{i \in C_2 \\ i < j}} z_i^{\pi''} = \left(\sum_{i \in C_2} d_i + R \right)$$

Otherwise, some available vehicle capacity is wasted.

Case 2: $j \in S_2$. In this case, $C_2 \subset S_2$. It is clear that

$$\sum_{\substack{i \in C_2 \\ i < j}} z_i^{\pi''} = \left(\sum_{i \in C_2} d_i + R \right)$$

Otherwise, some available vehicle capacity is wasted.

In either case, substituting the value of $(\sum_{i \in C_2} d_i + R)$ in Equation (15) proves the lemma.

(3) First 3(a) is proved by induction. Note that, for the first stop, $e_1 \leq e'_1$, $z_1^{\pi''} = \min(e_1, Q)$, and $z_1^{\pi} \leq \min(e'_1, Q)$. Hence,

$$\begin{aligned} (e_1 - z_1^{\pi''}) &= \max(0, e_1 - Q) \\ &\leq \max(0, e'_1 - z_1^{\pi}) \\ &= (e'_1 - z_1^{\pi}) \end{aligned}$$

Now, assume that $\sum_{i=1}^{j-1}(e_i - z_i^{\pi''}) \leq \sum_{i=1}^{j-1}(e'_i - z_i^{\pi})$. Consider stop j.

Case 1: $z_j^{\pi''} = e_j$. Then,

$$\sum_{i=1}^{j}(e_i - z_i^{\pi''}) = \sum_{i=1}^{j-1}(e_i - z_i^{\pi''}) \leq \sum_{i=1}^{j-1}(e'_i - z_i^{\pi}) \leq \sum_{i=1}^{j}(e'_i - z_i^{\pi})$$

Case 2: $z_j^{\pi''} < e_j$. From Part (2) of this lemma,

$$z_1^{\pi''} + z_2^{\pi''} + \ldots + z_j^{\pi''} = Q \geq z_1^{\pi} + z_2^{\pi} + \ldots + z_j^{\pi}$$

This inequality combined with the fact that $\underline{e} \prec \underline{e}'$, gives

$$\sum_{i=1}^{j}(e_i - z_i^{\pi''}) \leq \sum_{i=1}^{j}(e_i' - z_i^{\pi})$$

To prove 3(b), note that 3(a) implies that $0 \prec (\underline{e} - z^{\pi''}) \prec (\underline{e}' - z^{\pi})$. Using results of Lemma B.1 gives

$$\sum_{i=1}^{j} a_i \left((\underline{e}_i' - z_i^{\pi}) - (\underline{e}_i - z_i^{\pi''}) \right) \geq 0; \quad j = 1, 2, \ldots, n$$

or

$$\sum_{i=1}^{j} a_i \left(\underline{e}_i - z_i^{\pi''} \right) \leq \sum_{i=1}^{j} a_i \left(\underline{e}_i' - z_i^{\pi} \right); \quad j = 1, 2, \ldots, n$$

□

Proof: [Proof of Theorem 3.2]
Let $(\underline{e}_1, \underline{d}_1) \in S$ be the state at the beginning of the planning horizon. Consider a demand instance $(d_{j,l}^c, d_{j,l}^p)$, $j = 1, 2, \ldots, n; l = 1, 2, \ldots, K$. Let $\underline{z}_l^{\pi''}, l = 1, 2, \ldots, K$ be the pickup vector in period l if the myopic policy π'' is used in each period. $\underline{z}_l^{\pi}, l = 1, 2, \ldots, K$ are the associated vectors if some other policy π is followed.

It is first proved by induction that if $\underline{e}_l^{\pi''}$ and $\underline{e}_l^{\pi}$ represents the vectors of returning units available to be picked up in period l under policy π'' and π respectively, then

$$\underline{e}_l^{\pi''} \prec \underline{e}_l^{\pi}; \quad l = 1, 2, \ldots, K$$

It is clear that $\underline{e}_1^{\pi''} \prec \underline{e}_1^{\pi}$. Assume that

$$\underline{e}_l^{\pi''} \prec \underline{e}_l^{\pi} \quad \text{for} \quad l = 1, 2, \ldots, k$$

For period $k+1$, since $\underline{e}_k^{\pi''} \prec \underline{e}_k^{\pi}$, then from Lemma B.1,

$$\sum_{i=1}^{j} \left(e_{i,k}^{\pi''} - z_{i,k}^{\pi''} \right) \leq \sum_{i=1}^{j} \left(e_{i,k}^{\pi} - z_{i,k}^{\pi} \right) \quad \text{for} \quad j = 1, 2, \ldots, n$$

Hence,

$$\sum_{i=1}^{j}\left(e_{i,k}^{\pi''}-z_{i,k}^{\pi''}+d_{i,k}^{c}+d_{i,k}^{p}\right) \;\le\; \sum_{i=1}^{j}\left(e_{i,k}^{\pi}-z_{i,k}^{\pi}+d_{i,k}^{c}+d_{i,k}^{p}\right) \qquad \text{for} \quad j=1,2,\ldots,n,$$

or,

$$\underline{e}_{k+1}^{\pi''} \prec \underline{e}_{k+1}^{\pi}$$

Then,

$$\underline{e}_{k}^{\pi''} \prec \underline{e}_{k}^{\pi} \quad \text{for} \quad k=1,2,\ldots,K\le\infty$$

From Part (3b) of Lemma B.2,

$$\sum_{i=1}^{n} a_i\left(e_i^{(k,\pi'')}-z_i^{(k,\pi'')}\right) \le \sum_{i=1}^{n} a_i\left(e_i^{(k,\pi)}-z_i^{(k,\pi)}\right), \quad k=1,2,\ldots,K$$

Hence in each period, the cost of delaying pick up under policy π' never exceeds the cost if any other policy π is followed. □

Appendix C
Proof of Theorem 4.1

Taking the constant term $\sum_{i=1}^{n} w_i e_i^2$ out of the objective function and adding slack variables x_i, problem (P1) is equivallent to the following problem:

$$
\begin{array}{lllll}
\text{(P2)} & \text{Maximize} & \sum_{i=1}^{n} w_i(2e_i z_i - z_i^2) & & \\
& \text{subject to:} & \sum_{j=1}^{i} z_j + x_i & = & C_{\underline{d}} + \sum_{j=1}^{i} d_j \qquad i = 1, 2, \ldots, n \\
& & z_i & \leq & e_i \qquad\qquad\qquad i = 1, 2, \ldots, n \\
& & z_i & \geq & 0 \text{ and integer}
\end{array}
$$

Since z_i is integer, the non-decreasing term $w_i(2e_i z_i - z_i^2)$ can be replaced by a piecewise linear function $h_i(z_i)$ with break points at $z_i = 0, 1, 2, \ldots, e_i$. The slope S_{ik} of the piece between $z_i = k$ and $z_i = k + 1$ is $w_i[2e_i(k+1) - (k+1)^2] - w_i[2e_i k - k^2] = 2w_i(e_i - k) - w_i$. Note that for $k < e_i$, slopes $S_{ik} \geq 0$ and are decreasing in k. Hence, $h_i(z_i)$ is a piecewise concave function. Using this observation and the elementary row operations analogous to one used in Section 3.3, problem (P2) can be written as:

Maximize $h_1(z_1) + h_2(z_2) + \ldots + h_i(z_i) + \ldots + h_n(z_n)$
subject to:

$$
\begin{array}{cccccccccccc}
z_1 & & & & & +x_1 & & & & & = & C_{\underline{d}} + d_1 \\
& z_2 & & & & -x_1 & +x_2 & & & & = & d_2 \\
& & \ddots & & & & & \ddots & & & \vdots & \vdots \\
& & & z_i & & & & -x_{i-1} & +x_i & & = & d_i \\
& & & & \ddots & & & & & \ddots & \vdots & \vdots \\
& & & & z_n & & & & -x_{n-1} & +x_n & = & d_n \\
-z_1 & -z_2 & \ldots & -z_i & \ldots \; -z_n & & & & & -x_n & = & -(C_{\underline{d}} + \sum_{i=1}^{n} d_i)
\end{array}
$$

$0 \leq z_i \leq e_i$ and $x_i \geq 0; i = 1, 2, \ldots, n$
All variables are integer.

As illustrated in Figure C1, this problem is a minimum cost network flow problem with $n+1$ nodes and $2n$ arcs. Node 1 is a supply node with a supply of $C_{\underline{d}} + d_1$, Node i is also a supply node with supply of d_i, $i = 2, 3, \ldots, n$. Node $n+1$ is a demand node with demand of $C_{\underline{d}} + \sum_{i=1}^{n} d_i$. The arc $(i, i+1)$ is associated with the slack variable

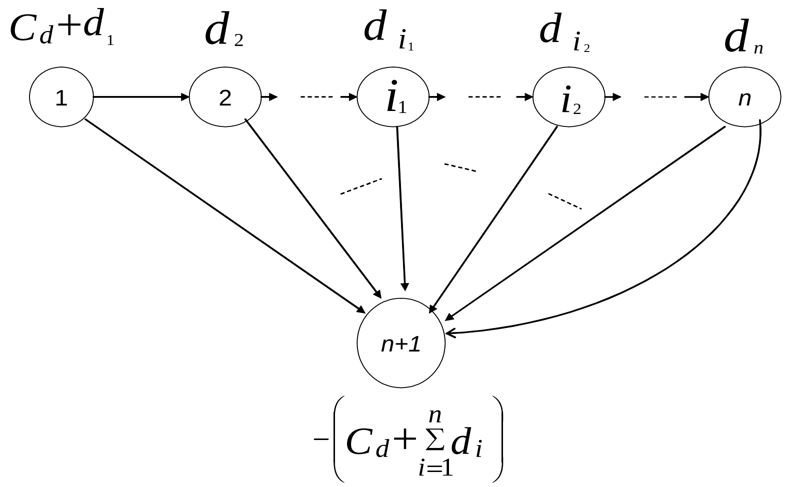

Figure C1: Network Representation of Problem (P2)

x_i, $i = 1, 2, \ldots, n$. This arc has *profit* of 0 and is uncapacitated. Arc $(i, n+1)$ is associated with the variable z_i and has a capacity of e_i and a concave piecewise linear profit function $h_i(z_i)$.

Let $z^* = \{z_i^*\}$ be the pickup vector generated by the GREEDY algorithm. Define $x_i^* = C_{\underline{d}} + \sum_{j=1}^{i} d_j - \sum_{j=1}^{i} z_j^*$. In the network given in Figure C1, consider any cycle $(n+1) \to i_1 \to i_1+1 \to \ldots \to i_2 \to (n+1)$. If $z_{i_1}^* > 0$ and $z_{i_2}^* < e_{i_2}$, this cycle allows additional flow in a clockwise direction. For such a case, the increamental profit of sending one unit of flow around the cycle is computed as follows.

$$\left\{\begin{array}{l}\text{Incremental}\\ \text{Profit in}\\ \text{Clockwise}\\ \text{Direction}\end{array}\right\} = \left\{\begin{array}{l}\text{Additional Profit}\\ \text{from Increasing Flow}\\ \text{in Arc } (i_2, n+1) \text{ by } 1\end{array}\right\} - \left\{\begin{array}{l}\text{Loss of Profit from}\\ \text{Decreasing Flow in}\\ \text{Arc } (i_1, n+1) \text{ by } 1\end{array}\right\}$$
$$= \left\{2w_{i_2}(e_{i_2} - z_{i_2}^*) - w_{i_2}\right\} - \left\{2w_{i_1}(e_{i_1} - z_{i_1}^* - 1) - w_{i_1}\right\}$$

This incremental profit is nonpositive, otherwise the greedy algorithm would have incremented z_{i_2} from $z^*_{i_2}$ to $z^*_{i_2}+1$ rather than incrementing z_{i_1} from $z^*_{i_1}-1$ to $z^*_{i_1}$.

Similarly, if $z^*_{i_1} < e_{i_1}0$, $z^*_{i_2} > 0$, and $x_i > 0, i = 1_1, \ldots, (i_2-1)$, this cycle allows additional flow in a counter-clockwise direction . For such a case, the incremental profit of sending one unit of flow around the cycle is computed as follows.

$$\left\{\begin{array}{l}\text{Incremental}\\ \text{Profit in}\\ \text{Counter-}\\ \text{Clockwise Di-}\\ \text{rection}\end{array}\right\} = \left\{\begin{array}{l}\text{Additional Profit}\\ \text{from Increasing Flow}\\ \text{in Arc } (i_1, n+1) \text{ by } 1\end{array}\right\} - \left\{\begin{array}{l}\text{Loss of Profit from}\\ \text{Decreasing Flow in}\\ \text{Arc } (i_2, n+1) \text{ by } 1\end{array}\right\}$$

$$= \left\{2w_{i_1}(e_{i_1} - z^*_{i_1}) - w_{i_1}\right\} - \left\{2w_{i_2}(e_{i_2} - z^*_{i_2} - 1) - w_{i_2}\right\}$$

This incremental profit is nonpositive, otherwise, the greedy algorithm would have increamented z_{i_1} from $z^*_{i_1}$ to $z^*_{i_1}+1$ rather than increamenting z_{i_2} from $z^*_{i_2}-1$ to $z^*_{i_2}$. This is because when the greedy algorithm incremented z_{i_2} from $z^*_{i_2}-1$ to $z^*_{i_2}$, condition $x_i > 0, i = 1_1, \ldots, (i_2-1)$ implies that it is also possible to increment z_{i_1} from $z^*_{i_1}$ to $z^*_{i_1}+1$.

In summary, corresponding to the solution given by the greedy algorithm, all flow augmenting cycles have a nonpositive incremental profit, hence, the solution is optimum (see Ahuja et al., 1993).

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