

Technical Memorandum Number 767

**Joint Pricing-Production Decisions in Supply Chains of
Complimentary Products with Uncertain Demand**

by

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January 2004

Revised September 2004 and May 2005

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Joint Pricing-Production Decisions in Supply Chains of Complementary Products with Uncertain Demand

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Abstract: Consider n manufacturers, each producing a different product and selling it to a market, either directly or through a common retailer. The n products are perfectly complementary, in the sense that they are always sold and consumed jointly or in sets of one unit of each. Demand for the products during a selling season is both price-sensitive and uncertain. Each of the n manufacturers faces the problem of choosing a production quantity and a selling price for his product. Two settings are considered, regarding the decision sequence of the n manufacturers: they are either simultaneous or sequential. The retailer, when present, employs a consignment-sales contract with revenue-sharing to bind her relationship with the manufacturers and to extract profit for herself. Using a multiplicative demand model in this paper, we fully characterize individual firms' decisions in equilibria, under each of the two game settings, and derive closed-form performance measures, both for the channel and for individual channel members. These closed-form solutions allow us to explore the effects of channel structure and parameters on firms' decisions and performance that lead to conclusions of managerial interest.

¹ The author wishes to thank an anonymous Associate Editor and two referees for numerous suggestions that greatly improved the paper. He is grateful to Li Jiang, Fernando Bernstein and seminar participants at Fuqua School of Business of Duke University, School of Business and Economics of Wilfrid Laurier University and Graduate School of Management of University of California at Irvine, for useful comments.

1. Introduction

We study the production and pricing decisions of multiple manufacturers/suppliers, who produce and sell to a market a set of complementary products. (We will use manufacturers and suppliers interchangeably, throughout the paper.) There are numerous examples that can motivate such a model setting. For example, Amazon.com has an online marketplace where anyone can list for sale a variety of items (books, CDs, electronics, tools & hardware, kitchen & house-ware, etc.). There, one often finds complementary products (e.g., monitors and keyboards needed to assemble personal computers) listed by different sellers. In retail stores, some products are almost always sold and consumed jointly according to some proportion (e.g., solder and flux for plumbing; mascarpone cream and savoiardi biscuit used to make Tiramisú; bricks, wood and other building materials for home-building, etc.), and are produced and delivered by different manufacturers. In assembly systems (e.g., computers, automobiles and aircrafts, etc.), complete sets of components or modules, supplied by various manufacturers, are needed to put units together. See Gerchak and Wang (2004) and Granot and Yin (2004c) for more industry examples of decentralized assembly supply chains. Under these situations, demand for sets of the complementary products is influenced by the total price of all products, and their sales are constrained by the product(s) with the least stocking quantity. Consequently, in choosing his own production quantity and price, one manufacturer will have to contemplate what other manufacturers do. In addition, when a retailer is involved in the channel, the retailer will design a contract to maximize her own profit, in anticipating manufacturers' behavior.

The purpose of this paper is to analyze the strategic decisions of firms providing complementary products, and their implications to supply chain/channel performance and to individual firms' performance. (We use channel and supply chain interchangeably.) We consider the following model settings. There are n manufacturers, each producing a different product at a constant marginal cost and selling it to a market, either directly or through a common retailer. The n products are *perfectly* complementary to each other, in the sense that they are always purchased and consumed jointly or in set of, without loss of generality, one unit of each. (Of course, complementarity of products in reality is rarely perfect; we make this assumption, together with several others to be made later, so as to have an analytically tractable model and to gain sharper insights into the problem. We will discuss limitations and applicability of our insights gained under these assumptions and their possible extensions.) Demand for sets of the

products during a selling season is price-sensitive, and is subject to uncertainty. The manufacturers each face the problem of choosing a production quantity and a selling price for their individual products. These decisions have to be made before the start of the selling season or before observing the realized demand. We explore and compare two settings with respect to the sequence of decisions of different manufacturers. In the first setting, all the n manufacturers make their decisions simultaneously; and in the second, they make decisions sequentially: without loss of generality, manufacturer #1 goes first, manufacturer #2 second, and so on, manufacturer # n goes last.

When the channel involves a retailer, the retailer offers the manufacturers a consignment-sales contract with revenue sharing. Under such a contract, the retailer provides the manufacturers with a market medium (e.g., the physical shelf-space in a retail store or the Internet marketplace of Amazon.com, etc.) for selling their products, and allows the manufacturers to choose delivery quantities and retail prices for their products. She then charges the manufacturers a pre-determined percentage of the selling price on each unit of their products actually sold. Consignment-sales contract of this type is used for e-business on the Internet (e.g., Amazon.com's marketplace), as well as under traditional retail store settings (c.f., Bolen 1978). Wang et al. (2004) study this contract in the context of a single supplier and retailer channel.

To capture the price-sensitivity and uncertainty of demand, we employ a deterministic, iso-price-elastic demand model multiplied by a random factor with general probability distribution. This demand function is one of a few models that have been adopted in the literature on studying joint pricing-production decisions for centralized systems (e.g., Karlin and Carr 1962, Petruzzi and Dada 1999 and references therein) and for decentralized supply chains (e.g., Emmons and Gilbert 1998 and Wang et al. 2004).

Our key contributions in this paper include to show that there exists a unique, Pareto-optimal equilibrium solution for each of the two n -manufacturer production-pricing games (i.e., the simultaneous-decision game and the sequential-decision game), and there exists a unique, closed-form solution for the retailer's optimal contract for each of the two games. Second, we derive closed-form performance measures for the decentralized channels and for individual firms in the channels. These closed-form solutions and performance measures then allow us to gain insights into how channel structure and parameters affect firms' decisions and performances. Specifically, we show that under each of the two settings, the equilibrium product price

(production quantity) is always higher (lower) than the centralized optimal price (production quantity), resulting in a channel profit that is lower than the centralized profit. The performance of decentralized channels improves as the retailer's share of channel cost increases and/or as the number of manufacturers n decreases. On the performance of individual firms, we find that in each of the two channels, retailer's net profit improves as her share of channel cost increases and/or as the number of manufacturers n decreases. Furthermore, retailer's profit measured as a share of the total channel profit can never be below $1/(n+1)$. When the channel does not involve a retailer, every manufacturer's profit always decreases as the number of manufacturers increases. If the channel involves a retailer, however, a manufacturer's profit can either increase or decrease as the number of manufacturers increases or as the retailer's cost share increases.

Concerning the relative profitability of different manufacturers, the results vary with the game settings: Under the simultaneous-decision setting, the n manufacturers each always make the same amount of profit, even though they may have different production costs. Under the sequential-decision setting, on the other hand, each manufacturer in general makes a different profit that depends on nothing but the 'position' a manufacturer takes in the overall sequence by which all manufacturers make their decisions. We find that keeping the total production cost as a constant, how to allocate it among the manufactures has no effect either on the channel performance or on individual firms' performance.

Comparing the performance of the two game settings, we show that when switching manufacturers' decision sequence from simultaneous to sequential, overall channel profit always improves, with or without a retailer's involvement; retailer's profit, when relevant, always improves as well. For manufacturers, it depends on whether the channel involves a retailer: Without a retailer, each and every manufacturer's profit improves; with a retailer, however, any of the n manufacturers can be either better off or worse off, depending on system parameters. The price-elasticity of demand plays a major role when comparing performances of the two game settings.

This research contributes to the literature on joint production-pricing decision problems under uncertainty. In the classic newsvendor setting of centralized decision-making, Whitin (1955), Mills (1959, 1962) and Karlin and Carr (1962) are the earliest researchers who formulate and solve such problems. Petruzzi and Dada (1999) provide an excellent review and extensions to problems under newsvendor settings. Federgruen and Heching (1999), Chen and Simchi-Levi

(2004a, b) study production-pricing problems for multi-period settings. Yano and Gilbert (2004) provide a comprehensive review that covers a much broader range of production-pricing decision problems.

Extending the newsvendor framework to decentralized supply chains, Emmons and Gilbert (1998) and Granot and Yin (2003) consider a setting where a supplier wholesales a product to a retailer who makes pricing-procurement decisions. Both papers explore how the supplier can improve channel performance by using an inventory-return policy for items overstocked by the retailer. Taylor (2003), Granot and Yin (2004a,b) study the effect of postponement of retail price or order quantity decisions on channel performance. Other recent papers dealing with joint production-pricing decisions of supply chain settings include Bernstein and Federgruen (2003, 2005), Ray et al. (2004), etc.

Wang et al. (2004) consider a supply chain structure where a downstream retailer offers a consignment-sales contract with revenue sharing to a supplier who then makes production-pricing decisions. Using a multiplicative and iso-price-elastic demand model, they derive equilibrium solution for the channel and closed-form performance measures. Research in the current paper extends the analyses of Wang et al. to systems with multiple suppliers of complementary products. In particular, while in Wang et al. (2004) the retailer plays a game against a single supplier who chooses his production quantity and product price, in this paper the retailer plays the game against multiple suppliers who in turn play an *imbedded* game, either simultaneously or sequentially, against each other in choosing their individual production quantities and prices.

The complementary products in our model can be viewed as a set of different components from which a final product is assembled. As such, our supply chain setting relates directly to decentralized assembly systems. Research on contracting and coordination of such systems include Gerchak and Wang (2004), Granot and Yin (2004c), Bernstein and DeCroix (2004a, b), Tomlin (2003), Wang and Gerchak (2003), Gurnani and Gerchak (1998) and Zhang (2004). A key difference from the current paper is that all models in those papers assume that the product's market demand, while uncertain, is not influenced by the selling price. Thus, all those models do not consider product pricing as part of the decision. Carr and Karmarkar (2005) consider decentralized assembly systems with price-sensitive, but deterministic demand.

Finally, this research complements the literature of Marketing and Economics that studies channel structure, competition and performance for *substitutable* products, e.g., Seade (1980), Choi (1991), Tyagi (1999) and references therein. Some of our findings here for complementary products naturally mirror those for substitutables. For example, competition leads to high price and low demand for complementary products vs. to low price and high demand for substitutables. Some other properties derived here, however, are not comparable with those for substitutables, as one will see from our discussions in Section 5.

The paper proceeds as follows. Section 2 details the model assumptions and derives the centralized decisions. Sections 3 and 4 analyze equilibrium decisions for the simultaneous-decision model and for the sequential-decision model, respectively. Section 5 characterizes the effects of channel structure and parameters on performance, and compares the two model settings. Section 6 concludes the paper with discussions about model limitations and possible extensions. All mathematical proofs are placed in an Appendix.

2. Model assumptions and Centralized Decision

Consider n suppliers each producing a different product and selling it through a common retailer to the market. The n products are perfectly complementary to each other, in the sense that they are always sold and consumed jointly or in set of, without loss of generality, one unit of each. One can also think of the products as n different components or sub-modules from which a final product is then assembled and sold at the retailer.

Let p_i be the selling price of product i , $i = 1, \dots, n$, and define $P \equiv \sum_{i=1}^n p_i$ as the total price for one set of the n products. Since the products are always consumed jointly, demand for sets of them during a selling season, denoted by D , depends on the total price P , and furthermore, can be uncertain. We use the following multiplicative functional-form to capture price-sensitivity and uncertainty of the demand:

$$D(P) = y(P) \cdot \varepsilon, \quad (1)$$

where $y(P)$ is a deterministic and decreasing function of price P , and ε is a random factor with general CDF $F(\cdot)$, PDF $f(\cdot)$ and a mean value of μ . Assume that $f(\cdot)$ has a support on $[A, B]$ with $B > A > 0$ and so $\mu > 0$. Define $h(x) \equiv f(x)/[1 - F(x)]$ as the failure rate of the distribution function. We further let $y(P)$ take the form of

$$y(P) = aP^{-b} \text{ where } a > 0, b > 1. \quad (2)$$

The above demand function form is one of a few models that have often been adopted by the literature studying joint production-pricing decisions; see Petruzzi and Dada (1999) for a review and extensions. In this formulation, the parameter b is the price-elasticity index of (expected) demand. The larger the b value, the more sensitive the demand is to a change in price. Products with a price-elasticity index greater than 1 are defined as being price-elastic, and as inelastic otherwise. We focus on price-elastic products and thus assume initially that $b > 1$. In our later-on analyses of decentralized decisions, we may put additional restrictions on the range for b in order to guarantee the existence of equilibria.

Product i , $i = 1, \dots, n$, is produced at a constant marginal-cost of $\$c_i$, and there is a unit cost of $\$c_0$ incurred at the retail stage for handling one set of the n products. Define $C \equiv \sum_{i=1}^n c_i$ as the total *production* cost of the n products, and $\alpha \equiv \frac{c_0}{C + c_0}$, $0 \leq \alpha \leq 1$, as the share of the total *channel* cost $C + c_0$ that is incurred at the retail stage. For simplicity, we assume that any unsold product at the end of the season neither has any salvage value nor bears any disposal cost. Similarly, in case of shortages, unsatisfied demand carries no penalty beyond the loss of revenue.

The problem is to choose a production quantity q_i and a selling price p_i for each product i , $i = 1, \dots, n$. These decisions have to be made before observing the realized demand. In Sections 3 and 4, we will specify the contract that binds the relationships of the independent firms and their individual decisions. In the following, however, we consider the scenario where these decisions are made in a centralized fashion, which will serve as a benchmark for the decentralized decision-making cases.

For a centralized system with a single decision-maker, it is obvious that one should choose to produce the same quantity for all the n products, i.e., $q_1 = q_2 = \dots = q_n \equiv q$. This is simply due to the perfect complementarity of the products or due to the fact that unmatched products cannot be sold. On decisions for individual prices $\{p_i\}$, since the final demand depend on nothing but the total price $P = \sum_{i=1}^n p_i$, it suffices to choose a value for P itself, instead of for each individual p_i . Let $\Pi_c(P, q)$ denote the expected profit of the system. We have

$$\Pi_c(P, q) = -(C + c_0)q + PE[\min\{q, D\}] = -(C + c_0)q + PE[\min\{q, y(P)\varepsilon\}]. \quad (3)$$

The rest of the analyses of this section can be found in Wang et al. (2004); we include them here for easy reference later in the paper. Following Petruzzi and Dada (1999), define $z \equiv q/y(P)$, and call it the ‘stocking factor’ of production. Then, the problem of choosing a price P and a production quantity q is equivalent to choosing a price P and a stocking factor z . Substituting $q = zy(P)$ into (3), the objective function can be rewritten as

$$\Pi_c(P, z) = y(P)\{P[z - \Lambda(z)] - (C + c_0)z\}, \quad (4)$$

where

$$\Lambda(z) = \int_A^z (z - x)f(x)dx > 0 \text{ for } A \leq z \leq B. \quad (5)$$

The optimal solution maximizing $\Pi_c(P, z)$, denoted by (P_c^*, z_c^*) , satisfies the following two first-order conditions, which are derived by substituting $y(P) = aP^{-b}$ into (4) and then taking derivatives of $\Pi_c(P, z)$ with respect to P and z , respectively:

$$P_c^* = \frac{b(C + c_0)}{b - 1} \cdot \frac{z_c^*}{z_c^* - \Lambda(z_c^*)} \quad (6)$$

and

$$F(z_c^*) = \frac{z_c^* + (b - 1)\Lambda(z_c^*)}{bz_c^*}. \quad (7)$$

Wang et al. (2004) show that when the distribution of ε in the demand function satisfies the Increasing Generalized Failure Rate (IGFR) condition (Lariviere and Porteus 2001), i.e., $d[xh(x)]/dx = h(x) + xdh(x)/dx > 0$, the first order conditions of (6)-(7) provide a unique solution to the problem of maximizing $\Pi_c(P, z)$. IGFR is a rather weak condition; it is obviously implied by the Increasing Failure Rate (IFR) condition, which is known to be satisfied by distributions like Normal and Uniform distributions, as well as the Gamma and Weibull families subject to parameter restrictions (Barlow and Proschan 1965). Lariviere (2004) provides excellent discussions on properties and applications of IGFR distributions.

Substituting (P_c^*, z_c^*) of (6)-(7) into (4), we obtain the optimal system profit as

$$\Pi_c^* = \frac{a(C + c_0)}{b - 1} (P_c^*)^{-b} z_c^*. \quad (8)$$

3. Decentralized Channel I – Simultaneous Decisions of Suppliers

In a decentralized channel, there are n independent suppliers, each producing one of the n products. The suppliers sell their products to the market through a common retailer under a consignment contract with revenue sharing. Under such a contract, each supplier i , $i = 1, \dots, n$, chooses a production quantity q_i and a selling price p_i for his own product; for each unit sold, the retailer keeps r share of the sales revenue p_i for herself and remits the rest, i.e., $(1-r)p_i$, to supplier i . The retailer is more powerful in the channel and decides unilaterally on the terms of the contract, i.e., the value of r , $0 \leq r \leq 1$. We thus have a Stackelberg leader-followers' game: The retailer, acting as the leader, moves first to offer the contract, and the n suppliers, as followers, then simultaneously choose their individual production quantities and selling prices.

Notice that for a given revenue share r chosen by the retailer, the n suppliers' decisions constitute a gaming problem as well. This game, called the suppliers' sub-game, is imbedded in the overall retailer-suppliers' game. Furthermore, since each supplier has two decisions to make, namely, production quantity and retail price, the n suppliers' sub-game here could be played in three prescriptive settings: 1) they simultaneously choose their production quantities in first stage, and then simultaneously choose their prices in second stage; 2) they simultaneously choose their prices first, and then simultaneously choose their production quantities in second stage; and 3) they simultaneously choose their production quantities *and* prices all in one stage. It turns out that the suppliers' sub-game may or may not have the same equilibrium outcome(s) when played under each of the above three settings, and that it involves a different set of analyses for each setting. In this paper, we will assume that the first game setting is in place, namely, the suppliers simultaneously choose their production quantities in one stage, and then simultaneously choose their prices in second stage. Analyses for the other two settings and a comparison of the three shall be reported in the near future. See also Bernstein and Federgruen (2004) and Allon and Federgruen (2003), for related game setups and technical issues.

We will also examine the channel when the suppliers sell their products directly to the market, i.e., without the involvement of a retailer. The solution to such a channel is a special case of the general retailer-suppliers' model, specializing on $r = 0$ and $\alpha = 0$, and, hence, will be derived accordingly.

3.1. Suppliers' Sub-Gaming Problem

For a given revenue share r , $0 \leq r \leq 1$, chosen by the retailer, the n suppliers make decisions in two stages: In stage 1, they simultaneously choose their individual production quantities $\{q_i\}$ and in stage 2, they then simultaneously choose their individual prices $\{p_i\}$. The following observation helps to streamline the analyses for this gaming problem:

Key Observation: *In any Nash equilibrium, the suppliers' production quantities are identical, that is, $q_1 = q_2 = \dots = q_n = q$.*

The reason here is simple: Since the n products are perfectly complementary, one supplier can have a chance to sell a unit of his product *only if* that unit can be matched by all other suppliers' products. As a consequence, no supplier will produce more than any other supplier, and, hence, it has to be the case that all suppliers produce the same quantity in the end.

Define $z_i \equiv q_i/y(P)$ as supplier i 's stocking factor. Then, for any supplier i , choosing a production quantity q_i in the first stage is equivalent to choosing a stocking factor z_i . Furthermore, choosing an identical quantity q for all suppliers is equivalent to choosing an identical stocking factor z , i.e., $z_1 = z_2 = \dots = z_n = z$. Thus, the overall two-stage sub-game of suppliers can be solved naturally by following a backward induction procedure as follows: For *any* stocking factor z chosen in the first stage, we find the equilibrium prices $\{p_i^*(z)\}$ of individual suppliers in the second stage; knowing $\{p_i^*(z)\}$ as the response functions of second stage, we then identify the equilibrium stocking factor z^* for first stage of the game.

3.1.1. The Equilibrium Prices $\{p_i^*(z)\}$ under any Given Stocking Factor z

We first derive supplier i 's price response to any prices $\{p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n\}$ of all other suppliers. Define $P_{-i} \equiv \sum_{j=1}^n p_j - p_i = P - p_i$. Supplier i 's expected profit can be written as

$$\begin{aligned} \Pi_i(p_i | P_{-i}, z) &= -c_i z y(p_i + P_{-i}) + (1-r)p_i y(p_i + P_{-i}) E[\min\{z, \varepsilon\}] \\ &= y(p_i + P_{-i}) \{(1-r)p_i [z - \Lambda(z)] - c_i z\}. \end{aligned} \quad (9)$$

Lemma 1. *For any prices $\{p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n\}$ chosen by all other suppliers, supplier i 's profit function $\Pi_i(p_i | P_{-i}, z)$ is quasi-concave in his price p_i and has the unique maximizer:*

$$p_i^*(P_{-i}, z) = \frac{bc_i z}{(b-1)(1-r)[z - \Lambda(z)]} + \frac{P_{-i}}{b-1}, \quad i = 1, \dots, n. \quad (10)$$

The pure-strategy equilibrium prices $\{p_i^*(z)\}$ of all suppliers, if any, are found by solving the n simultaneous equations given in (10). We have the following Theorem:

Theorem 1. *If $b > n$, then for any given stocking factor z , the n suppliers' pricing game has a unique Nash equilibrium that is given by*

$$p_i^*(z) = \frac{z}{[z - \Lambda(z)]} \cdot \frac{1}{(b-n)(1-r)} [(b-n)c_i + C], \quad i = 1, \dots, n. \quad (11)$$

We see from (11) that for $b > n$, each and every supplier's equilibrium price increases as the product price elasticity b decreases, which is rather intuitive. As $b \rightarrow n$, the equilibrium price of every supplier goes to infinity, as seen from (11) as well. If $b < n$, however, the pricing game, on the strategy space $p_i(z) \geq c_i \geq 0$ for $i = 1, \dots, n$, does not have a pure-strategy equilibrium. To see this, we know that any pure-strategy equilibrium must satisfy (11); if $b < n$, however, the total price, given in (12) below, becomes negative, which implies that $p_i^*(z) < 0$ at least for some i .

From (11), a supplier's price also depends on, and increases with, the total number n of suppliers in the channel. This reflects the effect of competition among suppliers: Given that no one can sell more units of his product than anyone else, they each strive to increase their individual product prices so as to gain an advantage on sales revenue; the more the number of suppliers are in the channel, the more severe such a competition will be. As a consequence, when there are more suppliers competing in the channel, it requires a higher critical price elasticity $b > n$ to contain their prices from exploding into infinity. The requirement of $b > n$ here for an n -supplier's problem seems to be a natural generalization of the single Newsvendor's problem where it is required to have $b > 1$; see Petruzzi and Dada (1999) and Wang et al. (2004).

The above model insights suggest that for complementary products exhibiting low price elasticity relative to the number of suppliers, decentralized or uncoordinated pricing decisions can be detrimental to the supply channel. In reality, price elasticity varies widely across different types of goods and services. For most consumer goods (e.g., food, tobacco, electricity, etc.), price elasticity tends to be between 0.5 and 1.5 (Gwartney and Stroup 1997). Goods that are

luxuries (or non-essentials) or that have many competitive substitutes tend to have much higher elasticity values. For example, it is estimated that the elasticity for Chevrolet automobiles, which have many competing or substitutable vehicles of other brands, is 4.0 (Gwartney and Stroup 1997) and that for music products is 6.3 (Gast 2002).

The following additional properties of equilibrium prices follow directly from (11) and are rather intuitive:

Corollary 1. *In Equilibrium,*

- 1) *supplier i 's optimal price $p_i^*(z)$ increases both in his own production cost c_i and in all other suppliers' costs c_j , $j \neq i$;*
- 2) *supplier i chooses a price higher than supplier j , i.e., $p_i^*(z) > p_j^*(z)$, if and only if supplier i has a higher production cost than supplier j , i.e., iff $c_i > c_j$;*
- 3) *each and every supplier i 's price $p_i^*(z)$ and, hence, the total price P are decreasing in their revenue share $(1-r)$ allocated by the retailer.*

In equilibrium, the total price for a set of the n products can be calculated from (11) as

$$P^*(z) = \sum_{i=1}^n p_i^*(z) = \frac{z}{[z - \Lambda(z)]} \cdot \frac{bC}{(b-n)(1-r)}. \quad (12)$$

It is interesting to notice here that for any given total production cost C of all suppliers, the total products price $P^*(z)$ is not affected by the allocation of C among the suppliers. This is due to the fact that the price differential of any two suppliers is proportional to the difference in their production cost, as seen from (11).

3.1.2. The Equilibrium Stocking Factor z^*

We know that in equilibrium the suppliers will equalize their individual stocking factors z_i on some common value z in the first stage of their game; and for any such z , we also know that in the second stage, each supplier will choose his own corresponding price $p_i^*(z)$ according to (11). Towards finding the specific value(s) of z that the suppliers collectively will end up with in the first stage of their decisions, we characterize the optimal z each supplier i would prefer *individually*, so as to maximize his profit $\Pi_i[p_i^*(z) | P_{-i}^*(z), z]$ of (9). The result is presented as follows:

Lemma 2. *If $d[xh(x)]/dx = h(x) + xdh(x)/dx > 0$, then each supplier i 's profit function $\Pi_i[p_i^*(z) | P_{-i}^*(z), z]$, $i = 1, \dots, n$, is quasi-concave in z and reaches its maximum at $z = z_c^*$ -- the centralized stocking factor of (7).*

Since each supplier individually prefers $z = z_c^*$, it forms a Nash equilibrium for all suppliers each choosing $z^* = z_c^*$ in the first stage and then choosing their individual, optimal prices $p_i^*(z_c^*)$, $i = 1, \dots, n$, in the second stage. However, there are other equilibria: for *any* z in the range of $A \leq z < z_c^*$, all suppliers each choosing $z^* = z$ in the first stage and then their individual, optimal prices $p_i^*(z)$, $i = 1, \dots, n$, in the second stage, form a Nash equilibrium. To see this, first notice that suppose suppliers choose different stocking factors $\{z_i\}$ in the first stage, then in the second stage all suppliers will base their pricing decisions on the lowest stocking factor of all suppliers, namely, $z = \min_i \{z_i\}$. Thus, when all other suppliers choose a common z such that $A \leq z < z_c^*$, any supplier i would make himself worse-off by unilaterally choosing either a lower $z_i < z$ value (due to the quasi-concavity of profit function) or a higher $z_i > z$ value (due to the waste of his product units un-matched by other suppliers). On the other hand, for *any* $z > z_c^*$, $z^* = z$ cannot be an equilibrium, since any supplier i can benefit by unilaterally moving to z_c^* . We summarize these properties as follows:

Theorem 2. *For the n suppliers' simultaneous production-pricing game, if $d[xh(x)]/dx = h(x) + xdh(x)/dx > 0$ and $b > n$, then for any value of z such that $A \leq z \leq z_c^*$, it forms a Nash equilibrium for all suppliers each to choose the stocking factor z in the first stage and then their corresponding individual prices $p_i^*(z)$, $i = 1, \dots, n$, in the second stage. When $z = z_c^*$, the corresponding equilibrium is Pareto optimal.*

Notice from Lemma 2 that the Pareto optimality of $z^* = z_c^*$ here is in a strict sense: among all the Nash equilibria, *each and every* supplier strictly prefers $z^* = z_c^*$. Due to such a strict and Pareto dominance, it is reasonable to predict $z^* = z_c^*$ as the unique outcome of supplier's sub-game. For this reason, we will focus on this equilibrium for the rest of our analyses.

Substituting $z = z_c^*$ into (12), we have that in equilibrium the total price of the n products, as a function of r , is given by

$$P^*(r) = \frac{z_c^*}{[z_c^* - \Lambda(z_c^*)]} \cdot \frac{bC}{(b-n)(1-r)} = \frac{(b-1)(1-\alpha)}{(b-n)(1-r)} \cdot P_c^*, \quad (13)$$

where P_c^* is the centralized price of (6).

Remarks: The two-step procedure works well for solving the suppliers' production-pricing gaming problem. The key here is to introduce the stocking factor z . The stocking factor effectively scales out the effect of pricing on production decision(s). Consequently, while the equilibrium/optimal price(s) depend heavily on the channel structure and competition parameters, the optimal stocking factor(s) do not. Since the stocking factor here is chosen or defined based on the *specific*, multiplicative demand function form of (1) – (2), however, whether such a 'methodology' can be usefully applied to, or generalized for, other settings remains to be explored.

3.2. Retailer's Problem

The retailer in the channel has the profit function of

$$\Pi_0(r) = -c_0q + rPE[\min\{q, D\}] = y(P)\{rP[z - \Lambda(z)] - c_0z\}, \quad (14)$$

where for given r , z and P are chosen by the n suppliers according to $z = z_c^*$ and $P = P^*(r)$ of (13). Substituting $z = z_c^*$ and $P = P^*(r)$ into (14), we can show that

$$\Pi_0(r) = \frac{a(b-n)^{b-1}[z_c^* - \Lambda(z_c^*)]^b}{[b(1-\alpha)]^b(C+c_0)^{b-1}(z_c^*)^{b-1}} \cdot g(r), \quad (15)$$

where

$$g(r) \equiv \{(1-\alpha)b + (b-n)\alpha\}r - (b-n)\alpha\}(1-r)^{b-1}. \quad (16)$$

Theorem 3. *For $b > n$, the retailer's profit function $\Pi_0(r)$ is quasi-concave in r , and has the unique maximizer:*

$$r^* = \frac{(b-n-1)\alpha + 1}{b-n\alpha}. \quad (17)$$

As expected, if the retailer incurs a bigger share α of the channel cost, she allocates a bigger share r^* of the channel revenue to herself, so as to cover her increased cost. Second,

when there are more suppliers involved in the channel (i.e., if n gets larger), the retailer leaves more share $(1-r^*)$ of the revenue to suppliers, so as to damp the negative effect of more severe price competition of suppliers on channel revenue. The effect of price elasticity b on r^* , however, is more complicated: An increase in b leads to a lower optimal product price which results in a lower demand that are determined by self-interested suppliers. In such a situation, it is not immediately obvious as to how the retailer should optimally adjust the revenue share allocation r^* to influence suppliers' behavior. Indeed, as can be shown from (17), r^* can be either increasing in b , when $n\alpha > 1$, or decreasing in b , when $n\alpha < 1$.

Theorems 1-3 completely characterize the decentralized decisions in equilibrium when the suppliers make their decisions simultaneously. We next formulate and solve the problem when suppliers make decisions sequentially.

4. Decentralized Channel II – Sequential Decisions of Suppliers

As before, the retailer offers the suppliers a revenue sharing contract stipulating that for each unit of their products sold, the retailer keeps r share of the sales revenue and remits the rest, i.e., $1-r$, back to suppliers, and the suppliers then choose their individual production quantities and selling prices. Also as before, the suppliers make their decisions in two steps: step 1 - choosing their production quantities, and then step 2 – determining their prices. The difference here is that the suppliers make their decisions *sequentially* within each of the two steps: without loss of generality, supplier 1 goes first, supplier 2 second, ..., and supplier n last.

4.1. Suppliers' Sub-Gaming Problem

Although the suppliers make their decisions sequentially, the key observation that in equilibrium all suppliers choose the same production quantity q or equivalently the same stocking factor z in the first stage still applies. (As a matter of fact, the sequencing among suppliers in the first stage when choosing their production quantities has no impact at all on the outcome of the game analyzed here or in Section 3.) Thus, we again solve the suppliers' two-stage sub-game following a backward induction procedure: For any given stocking factor z , find the equilibrium prices $\{p_i^*(z)\}$, and then identify the equilibrium stocking factor z^* .

4.1.1. The Equilibrium Prices $\{p_i^*(z)\}$

For given prices $\{p_1, p_2, \dots, p_{i-1}\}$ chosen by suppliers 1 through $i-1$, supplier i faces the problem of choosing his price p_i . In doing so, supplier i knows that suppliers $i+1$ through n will respond sequentially in choosing their prices $\{p_{i+1}, p_{i+2}, \dots, p_n\}$. We introduce the following notation: $P_{\{1, i-1\}} \equiv \sum_{j=1}^{i-1} p_j$ and $P_{\{i+1, n\}}(p_i) \equiv \sum_{j=i+1}^n p_j$, where in $P_{\{i+1, n\}}(p_i)$ we stress the fact that dynamically, $\{p_{i+1}, p_{i+2}, \dots, p_n\}$ depend on p_i , though they depend on $\{p_1, p_2, \dots, p_{i-1}\}$ as well. Then, we can write the n -supplier's pricing problem as the following dynamic programming:

$$\begin{aligned} \max_{p_i} \Pi_i(p_i | P_{\{1, i-1\}}, z) &= -c_i z y [P_{\{1, i-1\}} + p_i + P_{\{i+1, n\}}(p_i)] \\ &\quad + (1-r) p_i y [P_{\{1, i-1\}} + p_i + P_{\{i+1, n\}}(p_i)] E[\min\{z, \varepsilon\}] \\ &= y [P_{\{1, i-1\}} + p_i + P_{\{i+1, n\}}(p_i)] \{(1-r) p_i [z - \Lambda(z)] - c_i z\}, \end{aligned} \quad (18)$$

for $i = 1, 2, \dots, n$.

Lemma 3. *For any prices $\{p_1, \dots, p_{i-1}\}$ chosen by suppliers 1 through $i-1$, supplier i 's profit function $\Pi_i(p_i | P_{\{1, i-1\}}, z)$ is quasi-concave in its own price p_i and has the unique maximizer:*

$$p_i^*(P_{\{1, i-1\}}, z) = \frac{(bc_i + C_{\{i+1, n\}})z}{(b-1)(1-r)[z - \Lambda(z)]} + \frac{P_{\{1, i-1\}}}{b-1}, \quad i = 1, \dots, n, \quad (19)$$

where, $C_{\{i+1, n\}} \equiv \sum_{j=i+1}^n c_j$.

From (19), we can find the equilibrium prices $\{p_i^*(z)\}$ through a series of substitutions: We first obtain $p_1^*(z)$ directly from (19), since by definition $P_{\{1, 0\}} \equiv 0$; substituting $P_{\{1, 1\}} = p_1^*(z)$ into (19), we then obtain $p_2^*(z)$; substituting $P_{\{1, 2\}} = p_1^*(z) + p_2^*(z)$ into (19), we subsequently get $p_3^*(z)$; and so forth, until we find $p_n^*(z)$. The outcome is described as the following Theorem:

Theorem 4. *If $b > 1$, then for any given z , the n suppliers' sequential pricing game has a unique equilibrium that is given by*

$$p_i^*(z) = \left[c_i + \left(\frac{b}{b-1} \right)^i \frac{C}{b} \right] \frac{z}{(1-r)[z - \Lambda(z)]}, \quad i = 1, \dots, n. \quad (20)$$

It may not be a surprise to see from (20) that like in the simultaneous game setting, each supplier i 's price increases both in his own cost and in all other suppliers' cost, and decreases in his revenue share $(1-r)$. In addition to cost parameters and revenue share, however, an individual supplier's price here also heavily depends on his position in the sequence of decisions: a supplier who makes decision later in the sequence tends to choose a higher price. For example, suppose two suppliers i and j have the same cost, i.e., $c_i = c_j$. Then, in equilibrium supplier j chooses a higher price than supplier i , if and only if supplier j follows supplier i in their decision making, i.e., if and only if $i < j$. This implies that when $c_i = c_j$, supplier j makes a larger profit than supplier i , since in equilibrium they all produce the same quantity and sell the same quantity. As a matter of fact, we will show later in Proposition 1 that supplier j makes a larger profit than supplier i , *even if* $c_i < c_j$.

From (20), the total price of the n products is calculated as

$$P^*(z) = \sum_{i=1}^n p_i^*(z) = \left(\frac{b}{b-1} \right)^n \frac{Cz}{(1-r)[z - \Lambda(z)]}. \quad (21)$$

While a later supplier in the decision sequence 'over-cuts' earlier suppliers' prices as discussed above, an earlier supplier does have the ability to proactively influence or control all later suppliers' prices, through an early commitment of his own. Intuitively, such a vertical-control-like relationship among the suppliers would lead to a lesser competitive environment, compared with under the simultaneous-decisions setting. Indeed, due to the lesser competitive relationship among suppliers, a sequential decisions channel always results in a lower product price than a simultaneous setting, as we will show later in Section 5. Also note that the simultaneous decision setting requires a minimum price elasticity of at least $b > n$ to prevent an explosion of suppliers' competitive prices, while here the sequential setting only requires $b > 1$, which is essentially the same as when there is only one supplier in the channel.

4.1.2. The Equilibrium Stocking Factor z^*

Substituting (20) and (21) into supplier i 's profit function, we can show that

$$\Pi_i(z) = y[P^*(z)]\{(1-r)p_i^*(z)[z - \Lambda(z)] - c_i z\} = a \frac{(b-1)^{bn-i} (1-r)^b}{b^{bn-i+1} C^{b-1}} \cdot \frac{[z - \Lambda(z)]^b}{z^{b-1}}. \quad (22)$$

Lemma 4. *If $d[xh(x)]/dx = h(x) + xdh(x)/dx > 0$ and $b > 1$, then each supplier i 's profit function $\Pi_i(z)$, $i = 1, \dots, n$, is quasi-concave in z and reaches its maximum at $z = z_c^*$, the centralized stocking factor given by (7).*

With Lemma 4 and following the same arguments as those for Theorem 2 for the simultaneous suppliers' decision model, we can reach the following conclusion:

Theorem 5. *For the n suppliers' sub-game of first choosing their individual production quantities and then sequentially choosing their prices, if $d[xh(x)]/dx = h(x) + xdh(x)/dx > 0$ and $b > 1$, for any value of z such that $A \leq z \leq z_c^*$, it forms a Nash equilibrium for all suppliers each to choose the stocking factor z and their corresponding prices $p_i^*(z)$ of (20), $i = 1, \dots, n$. When $z = z_c^*$, the corresponding equilibrium is Pareto optimal.*

Like in the simultaneous setting, the Pareto optimality of $z = z_c^*$ is in a strict sense: each and very supplier is strictly better off when choosing $z = z_c^*$, and so we will again focus on this unique equilibrium point for the rest of the analyses.

4.2. Retailer's Decision

Substituting $P^*(z_c^*)$ of (21), $C = (1 - \alpha)(C + c_0)$ and $c_0 = \alpha(C + c_0)$ into retailer's profit function of (14), we can show that

$$\Pi_0(r) = y[P^*(z_c^*)]\{rP^*(z_c^*)[z_c^* - \Lambda(z_c^*)] - c_0z_c^*\} = \frac{a(b-1)^{n(b-1)}[z_c^* - \Lambda(z_c^*)]^b}{(1-\alpha)^b(C+c_0)^{b-1}b^{bn}(z_c^*)^{b-1}} \cdot \hat{g}(r), \quad (23)$$

where,
$$\hat{g}(r) \equiv \{(1-\alpha)b^n + \alpha(b-1)^n\}r - \alpha(b-1)^n\} (1-r)^{b-1}. \quad (24)$$

Theorem 6. *The retailer's profit function $\Pi_0(r)$ is quasi-concave in r , and has the unique maximizer:*

$$r^* = \frac{(1-\alpha)b^{n-1} + \alpha(b-1)^n}{(1-\alpha)b^n + \alpha(b-1)^n}. \quad (25)$$

Similar to under the setting of simultaneous suppliers' decisions, here the retailer again is able to allocate optimally to herself a bigger share r^* of the sales revenue as she incurs a bigger share α of the total channel cost and/or as the channel involves a larger number n of suppliers.

5. Channel Performance and Managerial Implications

Sections 3 and 4 fully characterize how self-interested firms interact to arrive at their individual decisions in the two decentralized channels, i.e., the simultaneous-suppliers-decision channel and the sequential-suppliers-decision channel, respectively. In each case, these decisions collectively result in a specific channel performance or profit and a specific allocation of the channel profit among the firms. In this section, we study these performances.

For convenience, we will use subscripts I and II on notation to denote channel I (simultaneous-suppliers-decision) and channel II (sequential-suppliers-decisions), respectively. Second, without further specification, all results concerning channel I and channel II are valid for $b > n \geq 1$ and for $b > 1$, respectively.

We first present the following Proposition that characterizes the relative performance of the n suppliers in each of the two channels.

Proposition 1.

- 1) *In a decentralized channel with simultaneous suppliers' decisions, all suppliers each earn the same amount of profit, even though they may incur different costs. That is, we have*

$$\frac{\Pi_{i+1,I}}{\Pi_{i,I}} = 1, \quad \text{for } i = 1, 2, \dots, n-1. \quad (26)$$

- 2) *In a decentralized channel with sequential suppliers' decisions, supplier $i+1$ earns $b/(b-1)$ times of supplier i 's profit, irrespective of the difference in their individual costs. That is,*

$$\frac{\Pi_{i+1,II}}{\Pi_{i,II}} = \frac{b}{b-1} > 1, \quad \text{for } i = 1, 2, \dots, n-1. \quad (27)$$

The first interesting fact we observe here is that the relative profitability of different suppliers in both channels does not depend on their individual production costs. We know that a high-unit-cost supplier always incurs a bigger total production cost, since all suppliers produce the same quantity in equilibrium. On the other hand, we also know from (11) and (20) that everything else being equal, a high-cost supplier always chooses a higher price for his product. The ending result is that a high-cost supplier compensates his cost disadvantage by taking on a more aggressive pricing strategy.

Part 2) of Proposition 1 implies that under the sequential game setting, the profitability of one supplier relative to that of others depends heavily on the 'position' a supplier takes in the overall decision sequence. In particular, a supplier who moves later in the sequence earns more

than a supplier who moves earlier. In that sense, ‘moving early’ renders a supplier a disadvantage in payoff.

Most of the results of Proposition 1 appear to be rather robust with respect to model settings and assumptions. For example, consider an alternative setting where the n suppliers move first, either simultaneously or sequentially, to set their individual wholesale prices charged to the retailer, and the retailer then chooses an order quantity together with a retail price for the final product and bears all the overstocking risk. Under such a setting, using the same demand function as in the current paper, Li and Wang (2005) show that all properties described in Proposition 1 hold fully. Considering the case where suppliers choose their wholesale prices simultaneously while assuming that demand is uncertain but price-insensitive (or with a constant retail price determined exogenously), Granot and Yin (2004c) show that part 1) of Proposition 1 hold.

Return to the original model setting where retailer sets the revenue-share scheme and suppliers then choose their individual production quantities and retail prices. If we assume now that each supplier i has a unit salvage value, denoted by s_i with $0 \leq s_i < c_i$, for overstocked items, we can show that all properties of Proposition 1 hold fully. Such a model generalization is of particular interest since it mimics a situation of imperfect complements: Consider two suppliers with $s_1 > 0$ and $s_2 = 0$. Then, while the sales of product 2 are completely tied to the availability of product 1, product 1 can always be sold, perhaps less profitably, independent of the availability of product 2.

How about the impact of the choice of demand function $y(P)$? Under the simultaneous-suppliers-decision setting, it can be easily verified that part 1) of Proposition 1 always holds, *regardless of the specific form of $y(P)$* . (Of course, $y(P)$ needs to obey some desirable properties of a proper demand function, e.g., decreasing in P .) For sequential suppliers’ decisions, on the other hand, part 2) of Proposition 1 seems to be quite sensitive to the choice of $y(P)$. For example, using a two-supplier system, one can verify that if we instead choose $y(P) = a \cdot \exp(-bP)$ with $a, b > 0$, we get $\Pi_{2,II}/\Pi_{1,II} = 1$, and if $y(P) = a - bP$ with $a, b > 0$, we have $\Pi_{2,II}/\Pi_{1,II} = 1/2$. This illustrates that under sequential decisions, the *relative profitability* of different suppliers actually depends heavily on the specific form of the demand function. In particular, moving early may render a supplier either an advantage or a

disadvantage, depending on the shape of the demand function. That said, however, the property that the relative profitability of different suppliers does not depend on their individual costs appear to be rather robust even under the sequential-decisions setting: with limited efforts we have yet to find an example that violate this property. Finally, we point out that within the multiplicative demand function family of $D(P) = y(P) \cdot \varepsilon$, the choice of distribution function for ε does not play much of a role.

For the rest of this section, in Subsection 5.1 we will characterize a few important properties, as to how system structure and parameters affect channel and individual firms' performance, that are common for both simultaneous and sequential suppliers-decisions channels. In Subsection 5.2, we then compare the two channels in terms of their performances.

5.1. Effects of Channel Structure and Parameters on Performance

We consider two different channel structures, one without the retailer and the other with the retailer. For each case, we benchmark the channel performance against that of a corresponding centralized channel. For the latter case, we also characterize how the total channel profit is distributed between the retailer and the suppliers. Notice, however, that Proposition 1 above regarding the relative profitability of suppliers applies to both cases, i.e., with or without a retailer in the channels.

5.1.1. Channel Performance without a Retailer

Assume that the n suppliers sell their products to the market directly, without the involvement of a retailer. A corresponding centralized channel would be one with a zero retail cost, i.e., $c_0 = 0$, and so $\alpha = 0$. Specializing on $r = 0$ and $\alpha = 0$ in (13) and (21), we obtain the equilibrium total price of the n products as

$$P_I^* = \frac{b-1}{b-n} \cdot P_c^* \quad (28)$$

and

$$P_{II}^* = \left(\frac{b}{b-1} \right)^{n-1} \cdot P_c^* \quad (29)$$

for the simultaneous-decisions channel and for the sequential-decisions channel, respectively, where, P_c^* is the corresponding centralized price of (6).

Notice from (28) and (29) that $P_I^*, P_{II}^* > P_c^*$ as long as $n \geq 2$. That is, as long as the channel involves two or more suppliers, the product price in a decentralized channel is higher than the centralized product price, and, hence, the channel profit is lower than the centralized profit. This is nothing but due to the curse of price competition ramifying itself in the setting of complementary products: A price increase by any one supplier decreases the demand of *all* suppliers, and yet, in deciding on their individual prices, each supplier focuses on maximizing his *own* profit, resulting in a price that is too high from the channel's point of view. Such an outcome is in a predictable contrast with that under a substitutable products setting, where competition of multiple suppliers/retailers usually leads to too low products prices, e.g., Seade (1980), Choi (1991) and references therein.

Substituting each of the two prices of (28) and (29) together with $z = z_c^*$ and $c_0 = 0$ into (4), we can write the decentralized channel profits as $\Pi_k^* = \mu_k(b, n) \cdot \Pi_c^*$, for $k = I, II$, where Π_c^* is the centralized channel profit of (8), specializing on $c_0 = 0$, and

$$\mu_I(b, n) \equiv \frac{n(b-n)^{b-1}}{(b-1)^{b-1}} \quad (30)$$

and

$$\mu_{II}(b, n) \equiv (b-1) \left[\left(\frac{b}{b-1} \right)^n - 1 \right] / \left(\frac{b}{b-1} \right)^{b(n-1)} \quad (31)$$

for the simultaneous-decisions channel and for the sequential-decisions channel, respectively. $\mu_I(b, n), \mu_{II}(b, n) < 1$, and they each measure the performance of one of the two channels.

Proposition 2.

- 1) $\mu_I(b, n)$ and $\mu_{II}(b, n)$ are each decreasing in n ;
- 2) $\lim_{b \rightarrow \infty} \mu_I = \lim_{b \rightarrow \infty} \mu_{II} = ne^{-(n-1)}$.

Part 1) of the Proposition 2 implies that keeping the total production cost at a constant, more suppliers and, hence, more competition lead to lower channel efficiency or performance, which is as expected. Part 2) of Proposition 2 shows that when facing a product that is extremely price elastic, the two decentralized channels converges to each other, in terms of channel performance. We will use this result and those to be presented in Propositions 3 and 4 when we compare the two channels' performances in Subsection 5.2.

Total channel profit is distributed among the suppliers according to the schemes specified by (26) for channel *I* and (27) for channel *II*, respectively. For a given total production cost of all suppliers, as the number of suppliers increases, the total channel profit in each channel decreases and, with a smaller total profit being distributed among more suppliers, the profit of each and every supplier decreases even more dramatically.

5.1.2. Channel Performance with a Retailer

When the channel involves a retailer, by substituting r^* of (17) and (25) into (13) and (21) respectively, we can show that the equilibrium product prices in the two channels are given by

$$P_I^* = \frac{b - n\alpha}{b - n} \cdot P_c^* \quad (32)$$

and

$$P_{II}^* = \left[(1 - \alpha) \left(\frac{b}{b - 1} \right)^n + \alpha \right] \cdot P_c^*, \quad (33)$$

respectively, where, P_c^* is the corresponding centralized price. $P_I^*, P_{II}^* > P_c^*$ as long as $\alpha < 1$. Substituting each of the two prices of (32) and (33) together with $z = z_c^*$ into the channel profit function of (4), we write the channel profits as $\Pi_k^* = \gamma_k(b, n) \cdot \Pi_c^*$, for $k = I, II$, where Π_c^* is the centralized channel profit given in (8) and

$$\gamma_I(\alpha, b, n) \equiv \frac{(b - n)^{b-1} [b(n + 1 - n\alpha) - n]}{(b - n\alpha)^b} \quad (34)$$

and

$$\gamma_{II}(\alpha, b, n) \equiv \frac{(1 - \alpha)b \left[\left(\frac{b}{b - 1} \right)^n - 1 \right] + 1}{\left[(1 - \alpha) \left(\frac{b}{b - 1} \right)^n + \alpha \right]^b}. \quad (35)$$

Proposition 3.

- 1) γ_I and γ_{II} are each increasing in α , and $\lim_{\alpha \rightarrow 1} \gamma_I = \lim_{\alpha \rightarrow 1} \gamma_{II} = 1$;
- 2) γ_I and γ_{II} are each decreasing in n ;
- 3) $\lim_{b \rightarrow \infty} \gamma_I = \lim_{b \rightarrow \infty} \gamma_{II} = (n + 1 - n\alpha)e^{-n(1-\alpha)}$, that is increasing in α and decreasing in n .

Part 1) of the Proposition implies that the performance of each of the two decentralized channels improves as the retailer's portion of the total channel cost increases; and in the limiting

case, as the retailer incurs almost all the channel cost, each decentralized channel performs almost as good as the centralized channel. Part 2) of Proposition 3 indicates that with the addition of a strategic, downstream retailer, the negative effect of suppliers-competition on channel performance persists. That is, more suppliers and, hence, more upstream competition lead to lower channel performance. The opposite, however, was found to hold in distribution channels of substitutable products: With a strategic upstream manufacturer, adding more downstream competing retailers improve channel performance; see Tyagi (1999).

We next study how the channel profit is distributed between the retailer and the suppliers and how individual firms' performances are affected by system parameters. Substituting $r = r^*$ of (17) and $P = P_I^*$ of (32) for channel I , and $r = r^*$ of (25) and $P = P_{II}^*$ of (33) for channel II , into the retailer's profit function of (14), we have

$$\Pi_{0,I}^* = \frac{(b-n)^{b-1}}{(b-n\alpha)^{b-1}} \cdot \Pi_c^*, \quad (36)$$

and

$$\Pi_{0,II}^* = \frac{1}{[(1-\alpha)\left(\frac{b}{b-1}\right)^n + \alpha]^{b-1}} \cdot \Pi_c^*. \quad (37)$$

Retailer's *shares* of the channel profit are calculated as

$$\beta_I(\alpha, b, n) \equiv \frac{\Pi_{0,I}^*}{\Pi_I^*} = \frac{b-n\alpha}{b-n+nb(1-\alpha)}, \quad (38)$$

and

$$\beta_{II}(\alpha, b, n) \equiv \frac{\Pi_{0,II}^*}{\Pi_{II}^*} = \frac{(1-\alpha)\left(\frac{b}{b-1}\right)^n + \alpha}{(1-\alpha)b\left[\left(\frac{b}{b-1}\right)^n - 1\right] + 1}. \quad (39)$$

Proposition 4.

- 1) β_I and β_{II} are each increasing in α , converging to 1 as $\alpha \rightarrow 1$, and bounded below by $1/(n+1)$;
- 2) β_I and β_{II} are each decreasing in n ;
- 3) $\lim_{b \rightarrow \infty} \beta_I = \lim_{b \rightarrow \infty} \beta_{II} = 1/[n(1-\alpha) + 1]$.

The above Proposition shows that as the retailer incurs a bigger portion of the channel cost, she is able to extract a bigger share of the channel profit; when the retailer takes on almost all the channel cost, she is capable to capture almost the entire portion of the channel profit for herself. Second, as the number of suppliers n increases, the retailer is forced to sacrifice on her share of profit. The retailer, however, is always able to secure herself a share that is above the average share $1/(n+1)$ of all firms involved in the channel, *even if* she is to incur a zero cost.

From Propositions 3 and 4, we see that both the total channel profit and the retailer's share of it in each of the two channels increase with retailer's portion of channel cost, namely, α . Consequently, the retailer's actual profits, i.e., $\Pi_{0,I}^*$ and $\Pi_{0,II}^*$, each increase as she takes on more of the channel cost. On the other hand, also from Propositions 3 and 4, the retailer's actual profits each decrease as more competing suppliers are involved in the channel. The latter observation is again in sharp contrast with distribution channels, where Tyagi (1999) shows that the profitability of the strategic upstream manufacturer improves as the number of competing downstream retailers increases.

How about suppliers' profits? Propositions 3 and 4 do not provide an answer to how retailer's cost share α or the number of suppliers n affect suppliers' profits. For example, as α increases, while the size of the total channel profit pie gets bigger, a smaller share of it goes to suppliers. Indeed, these effects are more complicated and not monotone in nature: a few simple numerical examples illustrate that each supplier's profit or their total can be either increasing or decreasing in α and in n , depending on the values of parameters.

5.1.3. Summary of Key Findings

For a quick comparison, we summarize our findings about the effects of channel structure and parameters on channel and individual firms' performances in a table format, see Table 1. In the Table, a notation of ' \uparrow ' (' \downarrow ') indicates an increase (decrease) of a parameter or quantity.

Included in Table 1 are also the effects of total channel cost ($c_0 + C$) and the allocation of the total production cost C among the suppliers, i.e., specific values of c_1, c_2, \dots, c_n . These results, though not specifically stated in our earlier analyses, can be seen as follows: The centralized channel profit Π_c^* of (8) decreases in $(c_0 + C)$ and does not depend on the allocation

of C among the suppliers; each of the channel profits Π_k^* and individual firms' profits $\Pi_{i,k}^*$, for $k = I, II$ and $i = 0, 1, \dots, n$, is an expression of the centralized profit Π_c^* multiplied by a factor that does not depend on $(c_0 + C)$ or the allocation of C .

Table 1: Summary of Effects of Channel Structure and Parameters on Performances

Channel Structure	Profit of Channel Members	Channel Cost $(c_0 + C) \uparrow$	Allocation of C among Suppliers	Retailer's Cost Share $\alpha \uparrow$	Number of Suppliers $n \uparrow$
Without Retailer	Channel	↓	No Effect	(N/A)	↓
	Individual Suppliers	↓	No Effect	(N/A)	↓
With Retailer	Channel	↓	No Effect	↑	↓
	Retailer	↓	No Effect	↑	↓
	Individual Suppliers	↓	No Effect	↑↓	↑↓

5.2. Performance Comparison of the Two Channels

The analyses of Subsection 5.1 show that *qualitatively*, the two decentralized channels are identical in terms of how channel structure and parameters affect performances. In this Subsection, we focus on the differences of the two in terms of their performances. Intuitively, as we have discussed earlier, sequential decisions of suppliers render the channel a less competitive environment and, hence, a better channel performance, compared with simultaneous decisions. We will formally confirm such a result. In addition, we will also compare the performances of individual firms under the two environments and show that when switching suppliers' decision sequence from simultaneous to sequential, while the retailer is always better off, suppliers themselves may or may not.

We first note that a simultaneous-suppliers-decision channel, i.e., decentralized channel I , with or without the involvement of a retailer, requires $b > n$ in order to have a finite product price together with a positive production quantity, so as to generate a positive profit for any party. On the other hand, a sequential-suppliers-decision channel, i.e., decentralized channel II , only requires $b > 1$. Thus, for any $n \geq b > 1$, channel II dominates channel I in absolute sense.

(Note, since we don't know whether mixed-strategy equilibria exist for the suppliers' games, the comparison here is restricted to considering pure-strategy equilibria.)

As $b \rightarrow \infty$, the two channels converge to each other, which can be seen as follows: First, their total channel profits converge to each other, as seen from Part 2) of Proposition 2 for the case of without a retailer, and from Part 3) of Proposition 3 for the case with a retailer. Second, when relevant, the retailer's shares of the channel profits in the two channels converge to each other, as seen from Part 3) of Proposition 4. And, third, as $b \rightarrow \infty$, all suppliers in channel *II* each earn the same amount of profit as seen from (27), as they always do in channel *I* – Part 1 of Proposition 1.

For the rest of this Subsection, we compare the two channels when $\infty > b > n > 1$. (When $n = 1$, the two systems are physically identical.)

5.2.1. Channels without a Retailer

From (28) and (29), we have that

$$\frac{P_I^*}{P_{II}^*} = \frac{(b-1)^n}{(b-n)b^{n-1}} \equiv J(b, n). \quad (40)$$

Lemma 5. *For any $\infty > b > n > 1$, $J(b, n) > 1$, which implies that without a retailer, the price of decentralized channel *I* is higher than that of decentralized channel *II*, i.e., $P_I^* > P_{II}^*$.*

Since we know that for $n > 1$, the prices of both decentralized channels are higher than the centralized price P_c^* and that the channel profit function is quasi-concave in price, Lemma 5 then leads to the conclusion that without a retailer, the channel profit of decentralized channel *II* is higher than that of decentralized channel *I*. This again reflects the fact that sequential decisions render a less competitive environment and, hence, a better channel performance.

In the following we compare individual suppliers' profits of the two channels. In channel *I*, we know from Proposition 1 that the channel profit of (30) is evenly distributed among the n suppliers, that is, each earning the same profit of

$$\Pi_{iI}^* = \frac{(b-n)^{b-1}}{(b-1)^{b-1}} \cdot \Pi_c^*, \text{ for } i = 1, \dots, n. \quad (41)$$

In channel *II*, the channel profit of (31) is distributed among the n suppliers according to Equation (27), from which we can show that

$$\Pi_{iII}^* = \left(\frac{b}{b-1}\right)^{-b(n-1)} \left(\frac{b}{b-1}\right)^{i-1} \cdot \Pi_c^*, \text{ for } i = 1, \dots, n. \quad (42)$$

Proposition 5. *Without a retailer, for $\infty > b > n > 1$, each and every supplier earns a higher profit in channel II than in channel I, i.e., $\Pi_{iII}^* > \Pi_{iI}^*$ for $i = 1, \dots, n$.*

Thus, without a retailer, the less competitive environment of sequential decisions benefits every member in the channel.

5.2.2. Channels with a Retailer

From (32) and (33) we have that

$$\frac{P_I^*}{P_{II}^*} = \frac{(b-n\alpha)(b-1)^n}{(b-n)[(1-\alpha)b^n + \alpha(b-1)^n]} \equiv K(\alpha, b, n). \quad (43)$$

Lemma 6. *For any $\infty > b > n > 1$ and $1 > \alpha \geq 0$, $K(\alpha, b, n) > 1$, which implies that with a retailer, the product price in channel I is always higher than that in channel II, i.e., $P_I^* > P_{II}^*$.*

The above Lemma implies that with a retailer, the channel profit of channel II is still always higher than that of channel I.

We next investigate the difference of retailer's profits in the two channels. From (36) and (37) we can show that the ratio of the retailer's profits in the two channels can be expressed as

$$\frac{\Pi_{0II}^*}{\Pi_{0I}^*} = [K(\alpha, b, n)]^{b-1} > 1,$$

where $K(\alpha, b, n)$ is defined in (43), and the inequality follows from Lemma 6. Thus, we have that the retailer in channel II earns a higher profit than in channel I. Such a result is not surprising: With a less competitive environment and, hence, a more efficient channel being created when switching suppliers' decision from simultaneous to sequential, the 'powerful' retailer surely takes advantage of the situation and acts strategically to improve her own profit.

The fact that the channel profit and the retailer's profit improve simultaneously leaves it in doubt whether suppliers can improve *their* individual benefits, when switching their decision sequence from simultaneous to sequential. Indeed, it turns out that one channel cannot always dominate the other from any supplier's point of view. Rather, it depends on system parameters. Let $R(i) \equiv \Pi_{iII}^* / \Pi_{iI}^*$, for $i = 1, \dots, n$, be the ratio of supplier i 's profits in the two channels.

Obviously, $R(i)$ is increasing in i , i.e., $R(1) < R(2) < \dots < R(n)$. Simple numerical examples show that when switching from channel I to channel II , supplier 1 (the least profitable one among all the n suppliers in channel II) can be better off under some parameter combinations, i.e., one can have $R(1) > 1$. On the other hand, even supplier n can be worse off under some other parameter combinations, i.e., one can also have $R(n) < 1$. Our numerical studies suggest that $R(i)$ is increasing in n and is decreasing in α , which are rather intuitive. Table 2 summarizes the results:

Table 2: Performance Changes when Switching from Channel I to Channel II

Without Retailer	Channel	↑
	Individual Suppliers	↑
With Retailer	Channel	↑
	Retailer	↑
	Individual Suppliers	↑↓

6. Concluding Remarks

In this paper we studied decentralized production-pricing decisions of complementary products and their implications to supply chain performance. Many of the qualitative properties and insights obtained here for complementary products ‘naturally’ mirror, and are just the opposite of, those for substitutable products that are well known in the literature. For example, decentralized decisions and competition lead to higher product prices and lower production quantities in our complementary products setting, while they would generally result in lower prices and higher quantities in a substitutable products setting. Some other properties obtained in this paper, however, are rather unique. For example, suppliers’ relative profitability does not depend on their individual costs.

The closed-form solutions and performance measures derived in the paper are elegant, and greatly enhance our understanding of system behaviors. In obtaining these analytical solutions and insights, however, we have made several major model assumptions. In the

following, we discuss further the applicability and limitations of these assumptions and comment on to what degree our model solution and insights may hold if one were to relax these assumptions. In doing so, we also point out a few future research directions.

The assumption of *perfect* complementarity of products in our model approximates best assembly systems, where a final product like a car is assembled from a set of unique components. The degree of complementarity of products in many other settings, however, varies and is in general not perfect. For example, mascarpone cream and savoiardi biscuit, mentioned in the Introduction, are not always sold and consumed together or not in some constant proportion. That is, they each have their individual demands, in addition to their joint demand. For these situations, we believe that some of our general model insights should still hold. For example, competition should always lead to too high product prices and to lower demand or production quantities. Some other more specific insights, like individual suppliers' costs having no effect on individual profits, may disappear.

The exact type of consignment-sales contract with revenue-sharing adopted in our model is used both by online retailers (e.g., Amazon.com) and by traditional retailers (c.f., Bolen 1978); see Wang et al. (2004) for discussions on the pros and cons of such a contract from the administrative point of view. Within the domain of consignment-sales, this contract can be shown to be equivalent to one where the retailer moves first to determine the retail price as a markup percentage of suppliers' wholesale prices and suppliers then choose their individual wholesale prices charged to the retailer. Li and Wang (2005) recently consider a setting where suppliers move first to announce their individual wholesale prices and the retailer then chooses an order quantity and a retail price. Under such a setting, they show that while some properties, like those described in Proposition 1, still hold, others, like the effect of retailer's cost share on performance, are violated. There are also many other types of contracts, which, when applied to our model setting, will lead to different equilibrium outcomes. For example, within the consignment-sales setting, the retailer may move first to determine the retail price as a net profit margin above suppliers' wholesale price and her own cost. It would be interesting to explore how the insights gained based on the consignment contract with revenue-sharing will change when different contracts are considered.

In this research, we analyze and compare two *given* channel structures regarding suppliers' decision sequence: they are simultaneous or sequential. We show that when switching

from the simultaneous decision structure to the sequential decision structure, the retailer and the channel are always better off, while individual suppliers can be either better off or worse off. Furthermore, even if all suppliers are better off, their individual gains can be different. In practice, recognizing the benefits of sequential decisions to the channel and to the retailer, a channel manager or the retailer should strive to push towards such a channel structure, through central or negotiation power or through contractual arrangements, depending on the specific situations. These findings, as interesting and relevant to practice as they are, also lend us a fundamentally important and interesting new question, which is: how do firms resolve their conflicts of interest so as to reach an equilibrium point in terms of channel structure? From a theoretical point of view, answering such a question amounts to formulating and solving a gaming problem that includes suppliers' decision sequencing as part of firms' (suppliers and retailer) overall decisions. Economists, e.g., Gal-Or (1985), has sensed such a fundamental problem as well. To the best of our knowledge, however, a usable modeling framework or solution concept has yet to be built.

In order to obtain the closed-form solutions in this paper, we adopted the specific iso-price-elastic and multiplicative demand function. Wang et al. (2004) show that for other demand function forms, like the linear and *additive* model, the solution would be almost intractable, even for their simpler setting with one supplier. On the other hand, they also show, through extensive numerical examples, the properties and insights generated based on the iso-price-elastic and multiplicative model do hold strongly for general demand models. It needs to be explored whether this observation also holds in our multi-supplier settings studied in this paper.

Finally, the retailer in our model theoretically is able to improve her profit by setting a different revenue share r_i for each supplier. It would be interesting to see how our current model solutions and properties would be affected or whether new insights can be generated when using different r_i 's in the contract. Unfortunately, this seems to result in a model that is analytically intractable, even for the simplest case with two suppliers.

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Appendix Mathematical Proofs

Proof of Lemma 1. It follows from (9) that

$$\frac{\partial \Pi_i(p_i | P_{-i}, z)}{\partial p_i} = a(p_i + P_{-i})^{-(b+1)} \{bc_i z - p_i(b-1)(1-r)[z - \Lambda(z)] + P_{-i}(1-r)[z - \Lambda(z)]\}.$$

Since $a(p_i + P_{-i})^{-(b+1)} > 0$, solving the first-order condition for p_i we get (10). Furthermore, $\frac{\partial \Pi_i(p_i | P_{-i}, z)}{\partial p_i} > 0$ for all $p_i < p_i^*(P_{-i}, z)$ and $\frac{\partial \Pi_i(p_i | P_{-i}, z)}{\partial p_i} < 0$ for all $p_i > p_i^*(P_{-i}, z)$. So $\Pi_i(p_i | P_{-i}, z)$ is quasi-concave and, hence, $p_i^*(P_{-i}, z)$ is the unique maximizer.

Proof of Lemma 2. Using the chain rule, it follows from (9) that

$$\begin{aligned} \frac{d\Pi_i[p_i^*(z) | P_{-i}^*(z), z]}{dz} &= \frac{\partial \Pi_i[p_i^*(z) | P_{-i}^*(z), z]}{\partial z} + \frac{\partial \Pi_i[p_i^*(z) | P_{-i}^*(z), z]}{\partial P_{-i}} \frac{dP_{-i}^*(z)}{dz} \\ &\quad + \frac{\partial \Pi_i[p_i^*(z) | P_{-i}^*(z), z]}{\partial p_i} \frac{dp_i^*(z)}{dz} \\ &= a[p_i^*(z) + P_{-i}^*(z)]^{-b} \{(1-r)p_i^*(z)[1 - F(z)] - c_i\} \\ &\quad - ab[p_i^*(z) + P_{-i}^*(z)]^{-(b+1)} \{(1-r)p_i^*(z)[z - \Lambda(z)] - c_i z\} \frac{dP_{-i}^*(z)}{dz}, \end{aligned}$$

where in deriving the second equation we have used the fact that $\partial \Pi_i[p_i^*(z) | P_{-i}^*(z), z] / \partial p_i = 0$ due to the optimality of $p_i^*(z)$.

From (11) and (12), we have $P_{-i}^*(z) = P^*(z) - p_i^*(z) = \frac{(n-1)c_i + (b-1)C_{-i}}{(b-n)(1-r)} \cdot \frac{z}{z - \Lambda(z)}$, and $\frac{dP_{-i}^*(z)}{dz} = \frac{(n-1)c_i + (b-1)C_{-i}}{(b-n)(1-r)} \cdot \frac{zF(z) - \Lambda(z)}{[z - \Lambda(z)]^2}$. Substituting $p_i^*(z)$ from (11), and $P_{-i}^*(z)$

and $dP_{-i}^*(z)/dz$ from the above into the last expression of $d\Pi_i[p_i^*(z)|P_{-i}^*(z),z]/dz$, we have, after some algebra, that
$$\frac{d\Pi_i[p_i^*(z)|P_{-i}^*(z),z]}{dz} = \frac{a[p_i^*(z) + P_{-i}^*(z)]^{-b} C}{(b-n)[z - \Lambda(z)]} \cdot [z - bzF(z) + (b-1)\Lambda(z)].$$

Since the first factor in the above expression is positive for any $A < z < B$, the first-order condition amounts to solving $G(z) \equiv z - bzF(z) + (b-1)\Lambda(z) = 0$ for z , which leads to $z^* = z_c^*$, as in (7), for supplier i . Now, $G'(z) = [1 - F(z)][1 - bzh(z)]$ and $G''(z) = -h(z)G'(z) - b[1 - F(z)][h(z) + zh'(z)]$. So, if $h(z) + zh'(z) > 0$, then $G''(z) < 0$ at $G'(z) = 0$, which implies that $G(z)$ is either quasi-concave or monotonically decreasing. In conjunction with the facts that $G(z)$ is continuous, $G(A) = A > 0$ and $G(B) = -(b-1)\mu < 0$, we have that $G(z) > 0$ for $z < z_c^*$ and $G(z) < 0$ for $z > z_c^*$. Thus, $d\Pi_i[p_i^*(z)|P_{-i}^*(z),z]/dz > 0$ for $z < z_c^*$ and $d\Pi_i[p_i^*(z)|P_{-i}^*(z),z]/dz < 0$ for $z > z_c^*$. That is, $\Pi_i[p_i^*(z)|P_{-i}^*(z),z]$ is quasi-concave and reaches its maximum at $z = z_c^*$.

The above argument applies to any supplier i 's individual decision on z . Thus, each and every supplier's individual preference on z is the same, namely, $z^* = z_c^*$. This completes the proof of Lemma 2.

Proof of Theorem 3. Since the first factor in (15) is a constant, maximizing $\Pi_0(r)$ is equivalent to maximizing $g(r)$. It follows from (16) that $g'(r) = b(1-r)^{b-2}[(b-n-1)\alpha + 1 - (b-n\alpha)r]$. Now, $g'(r) = 0$ if $r = r^* \equiv [(b-n-1)\alpha + 1]/(b-n\alpha)$, $g'(r) > 0$ for all $0 \leq r < r^*$ and $g'(r) < 0$ for $r^* < r \leq 1$. Thus, $\Pi_0(r)$ is quasi-concave and has the unique maximizer r^* .

Proof of Lemma 3. The proof is by induction. First, consider the case of $i = n$. Notice that by definition, $P_{\{n+1,n\}}(p_{n+1}) = 0$, $C_{\{n+1,n\}} = 0$ and $P_{\{1,n-1\}} = P_{-n}$. With $i = n$, the problem of (18) is exactly the same as that of (9), and, hence, by Lemma 1, the conclusions of Lemma 3 hold for this case.

Second, we consider the case of $i = n-1$. Substituting $P_{\{n,n\}}(p_{n-1}) = P_n^*(P_{\{1,n-1\}}, z)$ of (19) into (18), we can show that the profit function of supplier $n-1$ is given by

$$\Pi_{n-1}(p_{n-1} | P_{\{1,n-2\}}, z) = a \left(\frac{b}{b-1} \right)^{-b} \left(\frac{c_n z}{(1-r)[z - \Lambda(z)]} + P_{\{1,n-1\}} \right)^{-b} \{ (1-r)p_{n-1}[z - \Lambda(z)] - c_{n-1}z \}.$$

We have, after some algebra, that

$$\begin{aligned} \frac{\partial \Pi_{n-1}(p_{n-1} | P_{\{1,n-2\}}, z)}{\partial p_{n-1}} &= a \left(\frac{b}{b-1} \right)^{-b} \left(\frac{c_n z}{(1-r)[z - \Lambda(z)]} + P_{\{1,n-1\}} \right)^{-b-1} \times \\ &\quad \{ (bc_{n-1} + c_n)z + (1-r)[z - \Lambda(z)]P_{\{1,n-2\}} - (b-1)(1-r)[z - \Lambda(z)]p_{n-1} \}. \end{aligned}$$

It then follows that $\Pi_{n-1}(p_{n-1} | P_{\{1,n-2\}}, z)$ is quasi-concave in p_{n-1} and reaches its maximum at

$$p_{n-1}^*(P_{\{1,n-2\}}, z) = \frac{(bc_{n-1} + C_{\{n,n\}})z}{(b-1)(1-r)[z - \Lambda(z)]} + \frac{P_{\{1,n-2\}}}{b-1},$$

which is (19) specializing on $i = n-1$. Thus, Lemma 3 holds for $i = n-1$.

To complete the induction, we now assume that Lemma 3 holds for any arbitrary $i = k$ and then show that it holds for $i = k - 1$ as well. From the assumption that (19) holds for $i = k, k + 1, \dots, n$, we can show after a series of substitution that

$$P_{\{k,n\}}(p_{k-1}) = \left(\frac{b}{b-1}\right)^{n-k+1} \frac{C_{\{k,n\}}z}{(1-r)[z-\Lambda(z)]} + \frac{b^{n-k+1} - (b-1)^{n-k+1}}{(b-1)^{n-k+1}} P_{\{1,k-1\}}.$$

Substituting the above expression into (18), we get

$$\begin{aligned} \Pi_{k-1}(p_{k-1} | P_{\{1,k-2\}}, z) &= y[P_{\{1,k-1\}} + P_{\{k,n\}}(p_{k-1})] \{(1-r)p_{k-1}[z-\Lambda(z)] - c_{k-1}z\} \\ &= a \left(\frac{b}{b-1}\right)^{-b(n-k+1)} \left(\frac{C_{\{k,n\}}z}{(1-r)[z-\Lambda(z)]} + P_{\{1,k-1\}} \right)^{-b} \{(1-r)p_{k-1}[z-\Lambda(z)] - c_{k-1}z\}. \end{aligned}$$

It follows that

$$\begin{aligned} \frac{\partial \Pi_{k-1}(p_{k-1} | P_{\{1,k-2\}}, z)}{\partial p_{k-1}} &= a \left(\frac{b}{b-1}\right)^{-b(n-k+1)} \left(\frac{C_{\{k,n\}}z}{(1-r)[z-\Lambda(z)]} + P_{\{1,k-1\}} \right)^{-b-1} \times \\ &\quad \{(bc_{k-1} + C_{\{k,n\}})z + (1-r)[z-\Lambda(z)]P_{\{1,k-2\}} - (b-1)(1-r)[z-\Lambda(z)]p_{k-1}\}. \end{aligned}$$

Thus, we see again that $\Pi_{k-1}(p_{k-1} | P_{\{1,k-2\}}, z)$ is quasi-concave in p_{k-1} and reaches its maximum at $p_{k-1}^*(P_{\{1,k-2\}}, z) = \frac{(bc_{k-1} + C_{\{k,n\}})z}{(b-1)(1-r)[z-\Lambda(z)]} + \frac{P_{\{1,k-2\}}}{b-1}$, which is (19) specializing on $i = k - 1$. We thus complete the induction proof of Lemma 3.

Proof of Lemma 4. It follows from (22) that $\frac{d\Pi_i(z)}{dz} = a \frac{(b-1)^{bn-i} (1-r)^b [z-\Lambda(z)]^{b-1}}{b^{bn-i+1} C^{b-1} z^{b+1}} \cdot G(z)$ with $G(z) \equiv z - bzF(z) + (b-1)\Lambda(z)$. The first factor of this expression is positive, and the rest of the proof follows the exact arguments as those in the proof of Lemma 2.

Proof of Theorem 6. Working on the profit function of (23), one can develop a proof by following the same procedure as that for Theorem 3.

Proof of Proposition 1. It follows from (9) that $\frac{\Pi_{i+1}}{\Pi_i} = \frac{(1-r)p_{i+1}[z-\Lambda(z)] - c_{i+1}z}{(1-r)p_i[z-\Lambda(z)] - c_i z}$, which applies to both the simultaneous suppliers' decision channel and the sequential suppliers decision channel. Now, for Part 1), substituting the equilibrium prices (11) into the above expression, we can show after some algebra that $\Pi_{i+1,I} / \Pi_{i,I} = 1$; and for Part 2), substituting the equilibrium prices of (20), we get $\Pi_{i+1,I} / \Pi_{i,I} = b/(b-1)$.

Proof of Proposition 2. Part 1). It follows from (30) that $\frac{\partial \mu_I(b,n)}{\partial n} = -\frac{b(b-n)^{b-2}(n-1)}{(b-1)^{b-1}} < 0$, and from (31) that $\frac{\partial \mu_{II}(b,n)}{\partial n} = -(b-1)^2 \left[\left(\frac{b}{b-1}\right)^{n-1} - 1 \right] \log\left(\frac{b}{b-1}\right) / \left(\frac{b}{b-1}\right)^{b(n-1)-1} < 0$. Thus, $\mu_I(b,n)$ and $\mu_{II}(b,n)$ are each decreasing in n .

Part 2). It is straight forward to show that $\lim_{b \rightarrow \infty} \mu_I(b,n) = ne^{-(n-1)}$. For $\mu_{II}(b,n)$, the limit of its numerator can be derived as

$\lim_{b \rightarrow \infty} (b-1) \left[\left(\frac{b}{b-1} \right)^n - 1 \right] = \lim_{b \rightarrow \infty} \left[\left(\frac{b}{b-1} \right)^n - 1 \right] / \left(\frac{1}{b-1} \right) = \lim_{b \rightarrow \infty} n \left(\frac{b}{b-1} \right)^{n-1} = n$, and that of its denominator is,

$$\lim_{b \rightarrow \infty} \left(\frac{b}{b-1} \right)^{b(n-1)} = e^{\lim_{b \rightarrow \infty} b(n-1) \log \left(\frac{b}{b-1} \right)} = e^{\lim_{b \rightarrow \infty} (n-1) \log \left(\frac{b}{b-1} \right) / \left(\frac{1}{b} \right)} = e^{\lim_{b \rightarrow \infty} (n-1) \left(\frac{b}{b-1} \right)} = e^{n-1}.$$

Thus, $\lim_{b \rightarrow \infty} \mu_{II}(b, n) = \frac{n}{e^{n-1}} = ne^{-(n-1)}$.

Proof of Proposition 3. Part 1): It follows from (34) and (35) that

$$\frac{\partial \gamma_I(\alpha, b, n)}{\partial \alpha} = \frac{(b-n)^{b-1} n^2 b (1-\alpha)(b-1)}{(b-n\alpha)^{b+1}} > 0$$

$$\text{and } \frac{\partial \gamma_{II}(\alpha, b, n)}{\partial \alpha} = \frac{(1-\alpha)b(b-1) \left[\left(\frac{b}{b-1} \right)^n - 1 \right]^2}{\left[(1-\alpha) \left(\frac{b}{b-1} \right)^n + \alpha \right]^{b+1}} > 0.$$

So, $\gamma_I(\alpha, b, n)$ and $\gamma_{II}(\alpha, b, n)$ are each increasing in α . That $\gamma_I(\alpha, b, n) = \gamma_{II}(\alpha, b, n) = 1$ as $\alpha \rightarrow 1$ can each be verified directly by plugging in $\alpha = 1$.

Part 2): We have $\frac{\partial \gamma_I(\alpha, b, n)}{\partial n} = -\frac{(b-n)^{b-2} n b^2 (1-\alpha) [b(1-\alpha) + \alpha]}{(b-n\alpha)^{b+1}} < 0$ and

$$\frac{\partial \gamma_{II}(\alpha, b, n)}{\partial n} = -\frac{(1-\alpha)^2 b(b-1) \left(\frac{b}{b-1} \right)^n \left[\left(\frac{b}{b-1} \right)^n - 1 \right]^2 \log \left(\frac{b}{b-1} \right)}{\left[(1-\alpha) \left(\frac{b}{b-1} \right)^n + \alpha \right]^{b+1}} < 0. \text{ So, they are each}$$

decreasing in n .

Part 3): For $\gamma_I(\alpha, b, n)$, we have

$$\begin{aligned} \lim_{b \rightarrow \infty} \gamma_I(\alpha, b, n) &= \lim_{b \rightarrow \infty} e^{b \log \left[\frac{(b-n)}{(b-n\alpha)} \right]} \cdot \lim_{b \rightarrow \infty} [b(n+1-n\alpha) - n] / (b-n) \\ &= e^{-n(1-\alpha)} \cdot (n+1-n\alpha), \end{aligned}$$

For $\gamma_{II}(\alpha, b, n)$, the limit of its numerator can be derived as

$$\begin{aligned} \lim_{b \rightarrow \infty} \left\{ (1-\alpha)b \left[\left(\frac{b}{b-1} \right)^n - 1 \right] + 1 \right\} &= \lim_{b \rightarrow \infty} (1-\alpha)b \left[\left(\frac{b}{b-1} \right)^n - 1 \right] + 1 \\ &= \lim_{b \rightarrow \infty} (1-\alpha) \left[\left(\frac{b}{b-1} \right)^n - 1 \right] / \left(\frac{1}{b} \right) + 1 = \lim_{b \rightarrow \infty} (1-\alpha)n \left(\frac{b}{b-1} \right)^{n+1} + 1 = (1-\alpha)n + 1, \end{aligned}$$

and that of its denominator is

$$\begin{aligned} \lim_{b \rightarrow \infty} [(1-\alpha) \left(\frac{b}{b-1} \right)^n + \alpha]^b &= e^{\lim_{b \rightarrow \infty} \log[(1-\alpha) \left(\frac{b}{b-1} \right)^n + \alpha] / \left(\frac{1}{b} \right)} \\ &= e^{\lim_{b \rightarrow \infty} (1-\alpha)n \left(\frac{b}{b-1} \right)^{n+1} / [(1-\alpha) \left(\frac{b}{b-1} \right)^n + \alpha]} = e^{(1-\alpha)n / [(1-\alpha) + \alpha]} = e^{(1-\alpha)n}. \end{aligned}$$

Thus, $\lim_{b \rightarrow \infty} \gamma_{II}(\alpha, b, n) = \frac{(1-\alpha)n+1}{e^{(1-\alpha)n}} = [(1-\alpha)n+1]e^{-(1-\alpha)n}$. The fact that this limit, namely, $(n+1-n\alpha)e^{-n(1-\alpha)}$, is increasing in α follows directly from Part 1) and that it is decreasing in n follows from Part 2).

Proof of Proposition 4. Part 1): It follows from (38) and (39) that

$$\frac{\partial \beta_I(\alpha, b, n)}{\partial \alpha} = \frac{n(b-1)(b-n)}{[b-n+nb(1-\alpha)]^2} > 0$$

and $\frac{\partial \beta_{II}}{\partial \alpha} = (b-1) \left[\left(\frac{b}{b-1} \right)^n - 1 \right] / \{ (1-\alpha)b \left[\left(\frac{b}{b-1} \right)^n - 1 \right] + 1 \}^2 > 0$, so β_I and β_{II} are each increasing in α . That they each converge to 1 can be verified by substituting $\alpha = 1$ into (38) and (39). For the lower bound, we have that

$$\beta_I(\alpha, b, n) > \beta_I(\alpha = 0, b, n) > \lim_{b \rightarrow \infty} \beta_I(\alpha = 0, b, n) = \lim_{b \rightarrow \infty} \frac{b}{b(n+1) - n} = \frac{1}{n+1},$$

where, the first “>” follows from the fact that β_I is increasing in α , and the second “>” follows from the fact that $\beta_I(\alpha = 0, b, n)$ is decreasing in b . Similarly, we can show that $\beta_{II} > 1/(n+1)$.

Part 2): We have that

$$\frac{\partial \beta_I(\alpha, b, n)}{\partial n} = -\frac{b(b-1)(1-\alpha)}{[b-n+nb(1-\alpha)]^2} < 0 \quad \text{and} \quad \frac{\partial \beta_{II}}{\partial n} = -\frac{(1-\alpha)(b-1) \left(\frac{b}{b-1} \right)^n \log \frac{b}{b-1}}{\{ (1-\alpha)b \left[\left(\frac{b}{b-1} \right)^n - 1 \right] + 1 \}^2} < 0.$$

Part 3): All the results here can be verified directly from (38) and (39).

Proof of Lemma 5. From (40), it can be verified that $J(b, n = 1) = 1$. Furthermore, we can show that $J(b, n+1) - J(b, n) = \frac{n(b-1)^n}{(b-n-1)(b-n)b^n} > 0$, and so $J(b, n)$ is strictly increasing in n . We thus prove that $J(b, n) > 1$ for any $b > n > 1$.

Proof of Proposition 5. Since supplier 1 earns the least profit in channel II and all suppliers each earn the same profit in channel I, we only need to show that $\Pi_{1II}^* > \Pi_{iI}^*$. Or, from (41) and

(42), we need to show that $R(b, n) \equiv \frac{\Pi_{1II}^*}{\Pi_{iI}^*} = \frac{b-n}{b-1} [J(b, n)]^b > 1$ for $b > n > 1$, where $J(b, n)$ is

defined in (40). To this end, since it can be easily verified that $\lim_{b \rightarrow \infty} R(b, n) = 1$, then we only need to show that $R(b, n)$ is decreasing in b , or $\partial R(b, n) / \partial b < 0$. It follows after some algebra

that $\frac{\partial R(b, n)}{\partial b} = \frac{[J(b, n)]^b}{b-1} \cdot [(b-n) \log J(b, n) - \frac{(n-1)^2}{b-1}] < 0$, where the inequality follows since

$T(b, n) \equiv (b-n) \log J(b, n) - \frac{(n-1)^2}{b-1} < 0$, which is true since $\lim_{b \rightarrow \infty} T(b, n) = 0$ and

$\frac{\partial T(b, n)}{\partial b} = \log J(b, n) - \frac{(n-1)(b-n)}{b(b-1)^2} > 0$, where the inequality follows since

$\lim_{b \rightarrow \infty} \partial T(b, n) / \partial b = 0$ and $\frac{\partial^2 T(b, n)}{\partial b^2} = -\frac{(n-1)[b^3(n-2) + 3bn(b-n) + n^2]}{b^2(b-1)^3(b-n)} < 0$ for $b > n \geq 2$.

Proof of Lemma 6. From (43), we need to show that $K(\alpha, b, n) > 1$ for any $b > n > 1$ and $1 > \alpha \geq 0$. To that end, we show in the following that for any $b > n > 1$, $K(\alpha, b, n) = 1$ for $\alpha = 1$ and $K(\alpha, b, n)$ is strictly decreasing in α .

Now, $K(\alpha, b, n) = 1$ for $\alpha = 1$ can be verified directly by plugging $\alpha = 1$ into (43). It follows after some algebra that

$\frac{\partial K(\alpha, b, n)}{\partial \alpha} = \frac{(b-1)^{2n}}{(b-n)[(1-\alpha)b^n + \alpha(b-1)^n]^2} \left[\left(\frac{b}{b-1} \right)^n (b-n) - b \right] < 0$, where the inequality

follows if and only if $L(b, n) \equiv \left(\frac{b}{b-1} \right)^n (b-n) - b < 0$ for all $b > n > 1$. That $L(b, n) < 0$ for all

$b > n > 1$ is true, since it can be verified that $L(b, n=1) = 0$ and

$L(b, n+1) - L(b, n) = -\frac{n}{b-1} \left(\frac{b}{b-1} \right)^n < 0$.

<End of Proofs>