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On Credit Risk in Supply Chains : Is Negative Default Correlation Among Suppliers Desirable?

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Abstract

We study the effects of credit risk in a supply chain where one retailer deals with competing risky suppliers who may default during their production lead-times. The suppliers, who compete for business with the retailer by establishing wholesale prices, are leaders in a Stackelberg game with the retailer. The retailer, facing uncertain future demand, chooses order quantities while weighing the benefits of procuring from the cheapest supplier against the advantages of reducing credit risk through diversification. If the wholesale prices were exogenous, the retailer would benefit from decreasing correlation of defaults. However, when prices are endogenous, decreasing the correlation dampens competition among the suppliers, increasing the equilibrium wholesale prices. We show that the retailer prefers suppliers with positively correlated default events. In contrast, the suppliers and the channel prefer defaults that are negatively correlated.

1 Introduction

Credit rating firms report that in 2002 over 240 firms defaulted on 160 billion dollars of debt, the largest amount ever over any one year period. The default rate for high yield bonds in 2002 was also at a record level of near 10%, and recovery rates were hovering at record low levels of just over 20% of par. The combined volume of defaults in 2001 and 2002 exceeded the total volume of defaults in the US over the previous twenty years. What is especially striking about the current trends is the surge in the defaults of large, well-established companies. Since 2000 almost 50 firms with assets or liabilities exceeding one billion dollars have filed for bankruptcy.[1] The consequences of these defaults are widespread and the ripple effects extend beyond the direct claimants, to suppliers and customers as well as competitors throughout their respective supply chains.

Recognition of credit risk among counterparties in a supply chain is now more important than ever before. As banks have tightened their credit policies, firms have found it more difficult to raise funds, and this has created a need for retailers and suppliers to work closely together to better bundle products with loans. As a result, it is not surprising that this form of trade credit is now by far the largest source of short-term debt financing for firms, representing over one-third of the current liabilities of all non-financial corporations. As a result of deteriorating credits, retailers are now more inclined to split orders among several suppliers and to diversify their customer base. In addition, retailers may enter into third party insurance contracts that provide financial protection against an important supplier defaulting.[2]

This paper explores the effects of credit risk in a supply chain consisting of a retailer and one or two risky suppliers. The suppliers, who compete for business with the retailer, establish wholesale pricing policies, and collectively act as leaders in a Stackelberg game with the retailer. The retailer, facing uncertain future demand, chooses order quantities. Without credit risk, the retailer would place the entire order with the supplier who charges the lowest price. In the presence of credit risk, the response of the retailer is more complex. If the benefits of diversification offset the increased costs of ordering from the higher priced supplier, the retailer may choose to split the order. The

[1] Examples of such firms include Adelphia, Century, Conseco, Global Crossing, Kmart, Nextel International, Williams Communications, and WorldCom.

[2] Indeed, a mushrooming market for credit default swaps has emerged where the financial ramifications of defaults can be mitigated. The estimated notional value of credit derivatives contract at year-end 2000 stood at $1000 billion, and is expected to grow very rapidly over this decade.

benefits, of course, will only arise if the default correlation is less than one, and ceteris paribus, will increase as correlation decreases. However, as default correlation decreases, it reduces the level of competition among the suppliers, and this in turn tends to raise the equilibrium wholesale prices. Therefore, the effects of decreasing correlation on the retailer's, suppliers', and channel profits are not clear a priori.

Our paper contributes to the literature by fully investigating how default risk affects the various firms in a supply chain. In our setting we find that increasing default intensities hurt all firms, but in different ways. Specifically, as default intensities increase, the rate of decline in profits for firms in different echelons of the supply chain depend on the shape of the demand distribution. If the wholesale prices were fixed, the retailer would benefit from the decreasing correlation of the defaults. However, if prices are endogenous to the model, the decreasing default correlation alters the nature of the competition among the suppliers and increases the equilibrium wholesale prices. We show that, in equilibrium, the retailer prefers suppliers with positively correlated default events. In contrast, the suppliers and the supply chain prefer defaults that are negatively correlated.

The correlation effect is particularly profound when the probability of a default over the fixed time horizon is small. To see this, let ρ be the default correlation over a finite horizon. Then:

$$\rho = \frac{\pi_{12} - \pi_1 \pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

where π_i is the probability that supplier i defaults in the given time period, and π_{12} is the probability that both suppliers default. Then, for $\pi_1 = \pi_2 = \pi$, and π small, we have:

$$\pi_{12} = \rho\pi + (1-\rho)\pi^2 \approx \rho\pi.$$

Further, given supplier 1 has defaulted, the probability that supplier 2 defaults is given by

$$\pi_{2|1} = \frac{\pi_{12}}{\pi_1} \approx \rho.$$

These results show that when events are rare, default probability dependence is largely determined by the correlation coefficient.

In addition to determining the effect of default correlation on performance of firms in a supply chain, this paper also examines the consequences of the suppliers offering different payment policies, ranging from up-front payments to on-delivery payments. In the presence of default risk the timing of the payments is an important consideration. Orders can no longer be viewed as binding forward contracts, but rather as risky contractual arrangements. The retailer may be reluctant to pay up-front because if a supplier defaults the goods will not be delivered and retailer's payment will be lost. At the same time, the suppliers may be hesitant to prepare goods for shipment without some kind of collateral or good faith money that signals the retailer's true commitment. Consequently, the pricing policies adopted might involve some combination of up-front and on-delivery payments. Often, the supplier may create incentives, in the form of large per unit discounts, to entice the retailer to pay up-front, rather than on-delivery. Since the timing of payments demanded by the suppliers can induce different responses from the retailer, the nature of pricing policies looms larger when credit risk is present. Our paper contributes to the literature by showing that as long as pricing policies of the suppliers are restricted to a general linear pricing family, which includes up-front and on-delivery payments as special cases, then, in equilibrium, the retailer and suppliers will be indifferent to the timing of payments.

The paper proceeds as follows. Section 2 reviews the random yield literature as well as the financial literature on defaults, defaults' correlations, and pricing defaultable claims under conditions of no arbitrage. Section 3 introduces the model, describes the default processes for the suppliers, and the nature of the competition. Section 4 investigates the effects of timing of the retailer-to-supplier payments and shows that for linear pricing policies, in equilibrium, the suppliers and the retailer are indifferent over the timing of the payments. This result is helpful for the subsequent analysis, because we no longer need to concern ourselves with details of particular pricing policies. In section 5, we examine the model with only one supplier and identify conditions on the demand distribution function that ensure the existence and the uniqueness of the equilibrium. We describe the equilibrium solution and investigate behavior of the supplier's, the retailer's, and the channel profits and several measures of system coordination. Section 6 extends the analysis to the two supplier model. We derive analytical solutions for the equilibrium in three important cases: deterministic demand and stochastic demand with correlation of either $+1$ or -1. These cases lead us to several interesting and, apparently, counterintuitive results. Although, for the general case of stochastic demand with arbitrary correlation we cannot obtain an analytical solution, we can prove the existence of an equilibrium and compute the solution numerically.

2 Literature Review

Our problem relates to the random yield research, the majority of which is dedicated to finding optimal inventory and procurement decisions for a single firm whose supply in not certain. Yano and Lee (1995) offer an excellent review of this literature and propose the following taxonomy of the random yield research for lot sizing problems: general papers; single-stage, continuous time models with constant demand; single-stage continuous time models with random demand; single-period discrete time models; multiperiod discrete time models; facilities in series; assembly system; multiple products and multiple periods; models with rework; multiple suppliers of the same item.

Our research can be linked to several of the above categories. The retailer's problem in our model with one supplier is a "single period discrete time" random yield model with a stochastically proportional yield. Therefore, the retailer's ordering policies that we derive are similar to the policies obtained by Gerchak, Parlar and Vickson (1986). The retailer's problem in our model with two suppliers is similar to a single-period model by Anupindi and Akella (1993) and falls into the "multiple suppliers of the same item" category of Yano and Lee (1995). Anupindi and Akella (1993) study one- and multi- period discrete-time problems of a retailer who can order from one or two unreliable suppliers. The authors consider various stochastic yield assumptions (all or nothing yield, partial recovery, delayed delivery) and derive optimal ordering policies. Unlike traditional random yield research, our analysis tackles not just the retailer's problem but also the suppliers' problems in the context of the Stackelberg game between the suppliers and the retailer. In addition, we employ risk-neutral valuation and specify financial defaults as the source of supply uncertainty. Consequently, we can use credit risk data from financial markets to determine probability distribution of random yield. Finally, unlike Anupindi and Akella (1993), who assume defaults are uncorrelated events, we allow defaults to be correlated.

The problem of a single supplier selling to a newsvendor has been addressed by Lariviere and Porteus (2001). The authors consider a one-period Stackelberg game with a single supplier, who announces wholesale price, and a single retailer, who responds by choosing an order quantity. Under mild assumptions on the demand distribution, they prove the existence and the uniqueness of the solution to this game and provide conditions that the equilibrium order quantity must satisfy. The authors also study how market size and demand variability affect the equilibrium solution, the firms' profits, and the overall supply-chain performance. In our paper we add a possibility of supplier's default to the problem in Lariviere and Porteus (2001) and focus on the effects of the credit risk on supply-chain performance. We further generalize the problem in Lariviere and Porteus (2001) by considering a game with more than one supplier.

In the analysis that follows there are two fundamental sources of uncertainty. The first relates to the demand distribution for the good sold by the retailer. The second relates to the joint default process for the two suppliers and the recovery rates for the orders, should default occur. If we assume all agents are risk neutral, then the true demand distribution has to be given exogenously and the true joint default process has to be estimated, typically from historical default data. Usually such data is limited and one has to use average values obtained from firms in similar industries. Rating agencies, for example, provide defaults' correlations by industry. Examples of such studies include Carty (1997) and Erturk (2000).

Rather than estimate actual default probabilities, it may be more appropriate, and perhaps simpler, to estimate risk-neutralized probabilities. Indeed, pricing models for defaultable claims, as developed by Merton (1974), Jarrow and Turnbull (1995), Duffie and Singleton (1999a), Lando (1998) and others, all require risk-neutralized processes rather than the true data-generating processes. If the suppliers are large firms that have traded equity, debt and perhaps other claims on the assets of their respective firms, then these prices contain information on the parameters of the default processes. For example, if the price of a supplier's debt falls, then this is a signal that default is more likely. The idea, then, is to use traded prices to infer parameter estimates for processes that control the well being of the firm.

The first family of models for defaultable claims, dating back to Merton (1974), are based on the structural notion that default occurs at the moment when the firm's assets drops below its liabilities. Extensions of these models to handle multiple defaults, primarily through modeling correlation among the equity values, has been considered by Hull and White (2001) and Zhou (1997).

An alternative reduced form approach treats defaults as a jump process with an exogenous intensity process. Models in this family include Jarrow and Turnbull (1995), Duffie and Singleton (1999a) and many others. Such models are now routinely used to price credit derivatives on single firms. These models can be extended to incorporate defaults' correlation in several ways. The first approach is to allow the default intensities to follow stochastic correlated processes. However, such approaches produce defaults' correlations that are too small. Jarrow and Yu (2001) develop infection models, where the intensity of surviving firms are heavily influenced by recent defaults. Duffie and Singleton (1999b) present an alternative approach where point processes are used to trigger simultaneous defaults. More recently, Schönbucher and Schubert (2001) permit individual firms to have arbitrary marginals, and then they build in a dependency structure via a copula function.

In our models we assume the parameter values for default likelihoods over the given time horizon, as well as default correlations, have been extracted using a reduced form model.

3 Model Assumptions

Consider a model of a simple supply chain with one retailer and two suppliers, who produce perfectly substitutable products using technologies with identical production lead-times. Without loss of generality, assume that the lead time is 1 and that production begins at date 0 and ends at date 1. At date 0, the suppliers determine their pricing policies. The retailer responds by choosing order quantities. Thus, the suppliers compete with each other for the retailer's business, and collectively, they are Stackelberg leaders in a game with the retailer. As soon as the suppliers receive orders, they commence production. The per unit production cost for supplier i is c_i and the bulk of production costs is incurred up-front (at date 0).

At date 0 the retailer is faced with ordering decisions to satisfy uncertain demand, D, that is realized at date 1. The cumulative distribution function for demand, $G(\cdot)$, is continuous with probability density function $g(\cdot)$. If a supplier defaults during the production cycle, the exact quantity delivered will depend on the timing of the default and on the nature of the creditors that have claims on the assets of the defaulted firm. In general, let β_i represent the proportional random recovery rate for the supplier i, with $0 \leq \beta_i \leq 1$. We assume that the default process for supplier i is a random stopping time which is unaffected by the pricing and payment policies and, in particular, by the order quantities. This assumption is justified if the default risk is attributed to exogenous events, or if the business that the retailer brings to the supplier is a small part of the supplier's full line of business activities. There is no asymmetric information and the joint default distribution is known by all agents.

The default and demand random variables are independent and the per unit retail sales price, s, is predetermined. One can think of s as the expected value of the future random price, $S(T)$, where $S(T)$ is independent from other random variables in the model. We assume, for simplicity, that any unsatisfied demand is lost and any unsold goods are costlessly discarded. Holding and shortage costs could be easily added to our model, however, because they do not alter the nature of our findings we omit them for the ease of exposition.

We assume that there is no arbitrage and markets are complete. Therefore, standard finance arguments guarantee the existence and the uniqueness of a pricing measure, also called risk-neutral measure [see, for example, Harrison and Kreps (1979), Harrison and Pliska (1981)] under which asset prices, normalized by the money fund, are martingales. The money fund grows at the riskless rate r. Each firm in a supply chain maximizes its expected discounted profit, where expectation is taken with respect to the risk-neutral pricing measure[3].

4 The Timing of Payments for the Retailer's Orders

In the presence of credit risk, the timing of retailer-to-supplier payments is important. To reduce credit risk exposure, the retailer would prefer to pay at date 1 after the product has been delivered, whereas the suppliers would prefer to receive payments at date 0, before production has begun. This section establishes that, if pricing policies are restricted to $\mathcal{F} = \{$policies for which the discounted expected retailer's cost is linear in order quantity$\}$, then, in *equilibrium*, both the retailer and the supplier are indifferent to the timing of payments. Consider the following examples:

Example 1.
Supplier i announces her policy $\phi_i = \{\alpha_i, w_i^F, w_i^D\}$ where w_i^F is the per unit *up-front* wholesale price, w_i^D is the per unit *on-delivery* price, and $0 \leq \alpha_i \leq 1$ is the proportion of the units for which the retailer must pay up-front. If the retailer orders z_i units from suppler i, then the retailer makes an immediate up-front payment of $\alpha_i z_i w_i^F$. Payment for the remaining units will be made on delivery at a price w_i^D. The amount of goods delivered to the retailer at time 1 is random variable $\beta_i z_i$ and the additional payment due on receipt is $(\beta_i - \alpha_i)^+ z_i w_i^D$. Denote the family of policies generated by this rule by $\mathcal{F}_0$.

For any policy $\phi_i = (\alpha_i, w_i^F, w_i^D) \in \mathcal{F}_0$, the expected discounted cost to the retailer is

$$K_i(\phi_i)z_i = \left\{e^{-r}E[(\beta_i - \alpha_i)^+]w_i^D + \alpha_i w_i^F\right\} z_i, \tag{1}$$

[3] If one assumed, alternatively, that firms are risk-neutral and used the data-generating measure, the analysis in this paper would not change.

This is a linear policy since the expected cost is linear in the number of units ordered, that is $\mathcal{F}_0 \subset \mathcal{F}$. When $\alpha_i = 0$ we obtain an on-delivery payment policy, and when $\alpha_i = 1$ we obtain an up-front payment policy.

There exist linear policies that are not in $\mathcal{F}_0$. For example, a policy that calls for the up-front payment of a certain percent of the total expected cost with the actual balance due on-delivery. ■

Let $P(z_1, z_2)$ be the retailer's discounted expected revenue obtained from selling the product after orders of size z_1 and z_2 are placed with the suppliers.

$$P(z_1, z_2) = e^{-r} s E[\min(D, z_1\beta_1 + z_2\beta_2)], \tag{2}$$

The retailer's discounted expected profit, $R(z_1, z_2)$, given the suppliers' linear pricing policies $\phi_i \in \mathcal{F}, i = 1, 2$ is:

$$R(z_1, z_2) = P(z_1, z_2) - K_1(\phi_1)z_1 - K_2(\phi_2)z_2. \tag{3}$$

Note that the retailer's discounted expected profit depends on suppliers' policies $\phi_i, i = 1, 2$ only through $K_i = K_i(\phi_i), i = 1, 2$. Therefore, the retailer responds with the same order quantity to an equivalence class of policies $\mathcal{C}_K \equiv \{\phi \in \mathcal{F} : K_i(\phi) = K\}$ from supplier i. For example, if there are two distinct pricing policies ϕ' and ϕ'', one stipulating that a supplier be paid up-front and the other policy requiring a payment to a supplier to be made on-delivery, with the property that $K(\phi') = K(\phi'') = K$, then the retailer is indifferent between these policies, and, therefore, between the timing of payments.

Example 2.

As can be seen from equation (1), the retailer makes the same profit and orders the same amount from supplier i regardless of whether the payment is made up-front ($\alpha = 1$) or on-delivery ($\alpha = 0$) provided that w_i^F and w_i^D satisfy the following equation:

$$e^{-r} E[\beta_i] w_i^D = w_i^F. \tag{4}$$

Because suppliers can choose arbitrary values for w_i^F and w_i^D, equation (4) need not hold. Therefore, in general, the retailer may favor either the up-front or the on-delivery payment policy. ■

Let $S_i(\phi_i, \phi_{-i})$ denote the discounted expected profit of the supplier i given that the other supplier selects pricing policy $\phi_{-i} \in \mathcal{F}$. The suppliers are engaged in a Bertrand competition with each other, trying to maximize

$$S_i(\phi_i, \phi_{-i}) = [K_i(\phi_i) - c_i]\, z_i[K_1(\phi_1), K_2(\phi_2)], \tag{5}$$

where $z_i[K_1(\phi_1), K_2(\phi_2)]$ is the optimal order quantity placed by the retailer to supplier i, given pricing policies $\phi_i, i = 1, 2$. Observe that the supplier i's problem is also a function of $[K_1 = K_1(\phi_1), K_2 = K_2(\phi_2)]$ only. Therefore, we can rewrite the suppliers' profit functions:

$$S_i(K_i, K_{-i}) = (K_i - c_i)\, z_i(K_1, K_2). \tag{6}$$

Proposition 1. *Given values (K_1, K_2), the suppliers' profits, the retailer's order quantity, and the retailer's profit are the same for any $\phi_1 \in \mathcal{C}_{K_1}$ and $\phi_2 \in \mathcal{C}_{K_2}$.*

An immediate consequence of this proposition is

Corollary 1. *If there exists an equilibrium solution (ϕ_1^*, ϕ_2^*) of the suppliers' game where $K_1(\phi_1^*) = K_1^*$ and $K_2(\phi_2^*) = K_2^*$ then for all $\phi_1 \in \mathcal{C}_{K_1^*}$ and for all $\phi_2 \in \mathcal{C}_{K_2^*}$, ϕ_1, ϕ_2 is also an equilibrium solution. Further, the suppliers' profit, the retailer's order quantity, the retailer's profit and the system profit are the same.*

In particular, in *equilibrium*, the retailer, the suppliers, and the supply chain are indifferent between up-front and on-delivery payments.

Example 3.

For pricing policies in $\mathcal{F}_0$, the above results indicate that once the optimal K_i^* values are obtained, both suppliers are indifferent among the set of pricing policies $\{w_i^F, w_i^D, \alpha_i\}, i = 1, 2$, that satisfy:

$$e^{-r}E[(\beta_i - \alpha_i)^+]w_i^D + \alpha_i w_i^F = K_i^*.$$

In particular, in equilibrium, the system is indifferent between payment up-front and payment on-delivery and the wholesale prices satisfy

$$e^{-r}E[\beta_i]w_i^D = w_i^F = K_i^*.$$

That is, if the supplier i offers a on-delivery–payment price of w_i^D, then the equivalent price for an up-front payment, w_i^F, is lower by a factor that reflects the survival probability and the time value of money. ■

Proposition 1 can be extended to more than two competing suppliers. However, if the payment policies, $\phi \notin \mathcal{F}$, then simple sufficient statistics may no longer be found and the structure of payment policies will affect the analysis in complex ways. For this general case, the retailers profit is given by

$$R(z_1, z_2) = P(z_1, z_2) - K_1(\phi_1, z_1) - K_2(\phi_2, z_2).$$

This paper will consider only linear pricing policies, $\mathcal{F}$. Therefore, we can ignore the differences between particular policies and focus on suppliers' problem of identifying optimal K values. Note that K can be thought of as an up-front wholesale price. We will refer to K as wholesale price from now on.

5 Ramifications of Credit Risk in the One Supplier Case

To focus on the effects of credit risk on supply chains, consider a model with one supplier first.

5.1 The Retailer's Problem

With one risky supplier, the retailer's discounted expected revenue, given by equation (2), reduces to

$$P(z) = e^{-r} sE[\min(D, z\beta)].$$

The retailer's expected profit, $R(z)$, given a supplier's wholesale price K, is

$$R(z) = P(z) - Kz.$$

Note that $R(z)$ is concave in z, with

$$R'(z) = P'(z) - K = e^{-r} sE[\beta\overline{G}(z\beta)] - K,$$

where $\overline{G}(x) = 1 - G(x)$. The optimal order quantity z satisfies the following first order condition

$$P'(z) \equiv e^{-r} sE[\beta\overline{G}(z\beta)] = K. \tag{7}$$

When $K = c$, the retailer's problem coincides with the problem of a central planner.

Consider two random recovery rates β_1 and β_2. By definition[4], β_2 is stochastically smaller than β_1 (notation: $\beta_2 <_{st} \beta_1$) iff $Pr(\beta_1 > a) \geq Pr(\beta_2 > a)$ for all a. We will equate the increasing credit risk with the recovery rate becoming stochastically smaller, which will be denoted by $\beta \downarrow_{st}$. Because $\min(D, z\beta)$ is an increasing function of β for every D and z, it follows that the profit of the centralized system

$$C(z) \equiv e^{-r} sE[\min(D, z\beta)] - cz$$

is decreasing as $\beta \downarrow_{st}$. Hence,

[4]For discussion on stochastic order relations, see Shaked and Shanthikumar (1994)

Proposition 2. *The optimal profit of the centralized system, $C^* = C(z^*)$, decreases as credit risk increases.*

For the centralized system we are interested in identifying conditions under which we can characterize the dependence of the optimal order quantity and service level on the credit risk. Towards this goal define $A(z, \beta) = \beta\overline{G}(z\beta)$. Then, differentiating with respect to β, we obtain $A_\beta(z, \beta) = \overline{G}(z\beta)[1 - h(z\beta)]$, where $h(z) = z\frac{g(z)}{\overline{G}(z)}$ is the generalized failure rate, as defined by Lariviere and Porteus (2001). We would like to identify z's for which $A(z, \cdot)$ is increasing. Assume that $h(\cdot)$ is increasing [Increasing Generalized Failure Rate (IGFR) property]. Many common distributions have the IGFR property. For example, any IFR (Increasing Failure Rate) distribution is also IGFR. Define $\overline{z} = \sup\{z : h(z) \leq 1\}$. Note that $A(\overline{z}, \cdot)$ is increasing. Therefore, as $\beta \downarrow_{st}$, $E[A(\overline{z}, \beta)]$ decreases. Because $E[A(\overline{z}, 0)] = 0$ and $\frac{c}{e^{-r}s} > 0$, there exists a random variable β_{max} for which $e^{-r}sE[A(\overline{z}, \beta_{max})] < c$ and a solution to equation (7), $z^* \leq \overline{z}$, for $\beta <_{st} \beta_{max}$. The following proposition describes the effect of credit risk on the optimal order quantity and the service level of the centralized system.

Proposition 3. *Suppose that $G(\cdot)$ is IGFR and for some random variable β_{max}, $e^{-r}sE[A(\overline{z}, \beta_{max})] < c$. Then for all $\beta <_{st} \beta_{max}$ as credit risk increases (as $\beta \downarrow_{st}$)*

(i) *The optimal order quantity, $z^{central}$, decreases.*

(ii) *The service level, $Pr(D < z^{central}\beta)$, decreases.*

Proof. See Appendix. ■

A stronger assumption on the distribution of the recovery rate β can make the IGFR requirement unnecessary. For example, if the recovery rate, β, follows a Bernoulli distribution with the probability of default π:

$$\beta = \begin{cases} 0 & \text{with probability } \pi \\ 1 & \text{with probability } 1 - \pi, \end{cases}$$

then the optimal order quantity for the centralized system is

$$z^{cental} = G^{-1}\left(1 - \frac{c}{e^{-r}(1-\pi)s}\right), \tag{8}$$

and the results of Proposition 3 hold without the IGFR assumption.

5.2 The Supplier's Problem

According to equation (6), the discounted expected profit of the supplier, given that she induces the retailer to order z is given by

$$S(K) = (K - c)z(K).$$

Because there is a one-to-one correspondence between the wholesale price K and the order quantity z, defined by equation (7), we can rewrite the supplier's discounted expected profit as a function of z:

$$S(z) = [P'(z) - c]z. \tag{9}$$

Proposition 4. *There exists a solution to the supplier's problem (9). The optimal order quantity satisfies the following equation:*

$$E\left\{\beta\overline{G}(\beta z^*)[1 - h(\beta z^*)]\right\} = \frac{c}{se^{-r}}. \tag{10}$$

Proof. See Appendix. ■

In general, equation (10) may have several solutions. To ensure that the supplier's problem is unimodal additional assumptions are needed. Assume that the recovery rate has a Bernoulli distribution with default probability π. Then the supplier's problem is to maximize

$$S(z) = \left[e^{-r}(1 - \pi)s\overline{G}(z) - c\right] z. \tag{11}$$

This problem is equivalent to the problem studied in Lariviere and Porteus (2001) with unit sales revenues given by $s(1 - \pi)e^{-r}$ and the next proposition follows directly from their Theorem 1.

Proposition 5. *Suppose that the demand distribution has finite mean, support on $[a, b)$, and function $G(\cdot)$ has an increasing generalized failure rate (IGFR). Then:*

(i) *The first order condition for supplier's problem is:*

$$\overline{G}(z)\,[1 - h(z)] = \frac{c}{s(1 - \pi)e^{-r}}. \tag{12}$$

(ii) *The supplier's profit function is unimodal on $[0, +\infty)$, linear and strictly increasing on $[0, a)$, strictly concave on $[a, \overline{z})$, strictly decreasing on $(\overline{z}, +\infty)$. Any solution z^* to equation (12) is unique and must lie in the interval $[a, \overline{z}]$. The supplier's optimal order quantity is either z^* or a.*

Thus, the IGFR property of the demand distribution guarantees the uniqueness of the solution to the supplier's problem.

Next, consider the effects of credit risk. From equations (7) and (12), the equilibrium wholesale price is

$$K^* = e^{-r}(1-\pi)s\overline{G}(z^*). \tag{13}$$

Conversely, if the supplier charges wholesale price $K^* \geq c$, the retailer orders

$$z^* = G^{-1}\left(1 - \frac{K^*}{e^{-r}(1-\pi)s}\right). \tag{14}$$

Comparing (14) with (8), because $K^* \geq c$, it follows that $z^* \leq z^{central}$, which is the familiar effect of double marginalization. It is also possible to show that total system profit is lower in the decentralized system.

Similarly to the centralized system, the performance of the decentralized system deteriorates as the default probability increases

Theorem 1. *For the Stackelberg game between the supplier and the retailer, the equilibrium order quantity, z^*, the optimal supplier's profit, S^*, and the optimal retailer's profit, R^*, are all decreasing in the default probability, π.*

Proof. See Appendix. ■

5.3 Performance Measures

Theorem 1 indicates that as the default probability decreases the profits for both the supplier and the retailer rise. However, Theorem 1 does not specify which of the two firms is made relatively better off with an improvement in the credit quality. Let $\eta(\pi) \equiv \frac{S^*}{R^*}$ represent the ratio of equilibrium profits, for a given default probability, π. If $\eta(\cdot)$ is decreasing in π, then the supplier is relatively worse off if default risk increases. If $\eta(\cdot)$ is increasing then in π, then the retailer is relatively worse off if default risk increases. Unfortunately, it is difficult to characterize function $\eta(\cdot)$ analytically. However, using expressions (12) and (13), we can derive a lower bound for η as follows:

$$\eta(\pi) = \frac{S^*}{R^*} = \frac{(K^*-c)z^*}{e^{-r}(1-\pi)sE\min(D,z^*) - K^*z^*} \geq \frac{K^*-c}{e^{-r}(1-\pi)s - K^*} = z^*\frac{g(z^*)}{G(z^*)} = \gamma[z^*(\pi)],$$

where $\gamma(z) = z\frac{g(z)}{G(z)}$. The lower bound $\gamma(\pi)$ represents the ratio of supplier's and retailer's profit per each sold unit. While it is not the same as the ratio of profits $\frac{S^*}{R^*}$, it serves as an approximation, which would be fairly precise if the probability that the retailer sells the entire order z^* is high.

This probability is related to the service level of the system, defined as $Pr(D < z^*\beta)$. This is another important measure of the supply chain performance and is equal to

$$Pr(D < z^*\beta) = (1-\pi)G(z^*) = \frac{e^{-r}(1-\pi)s - K^*}{e^{-r}s}. \tag{15}$$

The following proposition characterizes the behavior of the lower bound , γ, and the behavior of the ratio of the service levels of the supply chain relative to a centrally coordinated system.

Proposition 6. *Suppose that the lowest possible default probability is $\pi_0 \geq 0$, so that for all π, $\pi \geq \pi_0$. Let z_0^* be the optimal order quantity corresponding to π_0.*

Then, if cumulative distribution function of demand is concave (convex) on the interval $[0, z_0^*]$, then for all π:

(i) *The ratio of the service level of the decentralized system, $\frac{e^{-r}(1-\pi)s-K^*}{e^{-r}s}$, over the service level of the centralized system, $\frac{e^{-r}(1-\pi)s-c}{e^{-r}s}$, is decreasing (increasing) in π.*

(ii) *The ratio of supplier's and retailer's profits per unit of sold product, $\frac{K^*-c}{e^{-r}(1-\pi)s-K^*}$, is increasing (decreasing) in π.*

Proof. See Appendix. ∎

The ratio of service levels is equal to the conditional probability of meeting customer's demand in the decentralized system, given that the demand is met in the centralized system. According to Proposition 6, when the cumulative demand distribution function is concave, we overestimate this conditional probability if we ignore the credit risk in the system. Thus, we underestimate the severity of the drop in the service level. On the other hand, when the cumulative demand distribution function is convex, by ignoring credit risk, we are being too pessimistic about the service levels in the decentralized system.

According to part (ii) of Proposition 6, and assuming the actual ratio of equilibrium profits behaves similarly to the lower bound, if the demand distribution function is concave then the optimal retailer's profit decreases at a faster rate than the optimal supplier's profit. Conversely, if the demand distribution function is convex then the supplier's profit decreases faster than the retailer's profit as credit risk increases.

Numerical results suggest that the actual ratio of optimal supplier's and retailer's profits, $\eta(\pi)$, behaves similarly to the lower bound, $\gamma(z^*)$, as illustrated by the following example.

Example 4.

Suppose that $s = 100, c = 30$ and $r = 0.1$. First, consider the case when demand is exponential with mean 150 units. According to Proposition 2 and Theorem 1, as credit risk increases, the optimal supplier's profit, the optimal retailer's profit and the coordinated channel profit are decreasing. These properties are illustrated in Figure 1.

Since the demand distribution function is concave, as predicted by Proposition 6, the ratio $\frac{S^*}{R^*}$ and $\gamma(z^*)$ should increase in π. Panel B of Figure 1 shows that this is indeed the case. Panel C of Figure 1 compares the profit of the decentralized system $(S^* + R^*)$ to the profit of the centralized system (C^*). The figure shows that the ratio $\frac{S^*+R^*}{C^*}$ is slightly increasing in π.

Figure 1 also shows the results for the case where the cost and interest rate parameters are the same but the demand distribution is normal with mean 150 units and standard deviation 60. For small values of z (corresponding to large values of π) the cumulative demand distribution function is convex. Therefore, according to Proposition 6 and as shown in Figure 1, $\gamma(z^*)$ is decreasing for large π, and $\frac{S^*}{R^*}$ has a similar behavior.

In addition, Panel C shows that the ratio of the decentralized system profit over the centralized system profit $\frac{S^*+R^*}{C^*}$ is decreasing for large π. ■

6 The Effect of Correlation

As was shown in section 5, credit risk reduces firms' profits as well as channel profit. To reduce default risk exposure, the retailer might consider placing orders with both two suppliers. Ceteris paribus, if the wholesale prices are exogenously fixed, because of the diversification, the retailer benefits from the decreasing correlation between suppliers' defaults. This section shows how the correlation affects retailer's and suppliers' profits if wholesale prices are determined endogenously.

In this section we assume that recovery rates β_i for each supplier follow Bernoulli distributions with probabilities of default $\pi_i, i = 1, 2$. Furthermore, let p_{11} be the probability that both suppliers will default; p_{01} be the probability that the supplier 1 will survive and the supplier 2 will default, etc. The joint default distribution and the marginal probabilities, π_k, satisfy the following:

$$p_{00}, p_{01}, p_{10}, p_{11} \geq 0; \quad p_{00} + p_{01} + p_{10} + p_{11} = 1;$$

$$p_{00} + p_{01} = 1 - \pi_1; \quad p_{00} + p_{10} = 1 - \pi_2; \quad p_{11} + p_{01} = \pi_2; \quad p_{11} + p_{10} = \pi_1.$$

We model the correlation between suppliers' defaults through values of p_{ij}. For example, if the defaults are perfectly positively correlated, then $p_{01} = p_{10} = 0$ and $p_{00} = 1 - \pi_1 = 1 - \pi_2$ (hence $\pi_1 = \pi_2$). As the correlation decreases, p_{01} and p_{10} increase and p_{00} decreases. When defaults are perfectly negatively correlated, $p_{11} = p_{00} = 0$ and $p_{01} = 1 - \pi_1, p_{10} = 1 - \pi_2$.

6.1 Deterministic Demand

As we have seen in the one-supplier model, the supplier's problem could be difficult to solve. For the two-supplier model the difficulty is compounded by the game between suppliers. To develop intuition on the role of the correlation, we initially ignore one of the sources of uncertainty and assume that demand is deterministic. The insights will still be valid for the more general case of stochastic demand, where analytical solutions are not generally attainable.

6.1.1 The Retailer's Problem

Given information about the default distribution and the supplier's wholesale prices $K_i, i = 1, 2$, the retailer determines how much to order from each of the suppliers so as to maximize

$$R(z_1, z_2) = P(z_1, z_2) - K_1 z_1 - K_2 z_2, \tag{16}$$

where

$$P(z_1, z_2) = e^{-r} s \left[p_{01} \min(D, z_1) + p_{10} \min(D, z_2) + p_{00} \min(D, z_1 + z_2) \right]. \tag{17}$$

The solution to the retailer's problem is described in the following proposition.

Proposition 7. *Assume that $e^{-r} s(1 - \pi_i) \geq K_i, i = 1, 2$. Then*

$$(z_1^*, z_2^*) = \begin{cases} (D, D) & \text{if } K_1 \leq e^{-r} s p_{01} \text{ and } K_2 \leq e^{-r} s p_{10} \\ (0, D) & \text{if } e^{-r} s p_{01} < K_1 < e^{-r} s(1 - \pi_1) \text{ and } K_2 < K_1 + e^{-r} s(\pi_1 - \pi_2) \\ (D, 0) & \text{if } e^{-r} s p_{10} < K_2 < e^{-r} s(1 - \pi_2) \text{ and } K_2 > K_1 + e^{-r} s(\pi_1 - \pi_2) \\ z_1^* + z_2^* = D; z_i^* \geq 0 & \text{if } K_1 > e^{-r} s p_{01},\ K_2 > e^{-r} s p_{10}, \text{ and } K_2 = K_1 + e^{-r} s(\pi_1 - \pi_2) \end{cases}$$

Proof. See Appendix ■

Figure 2 provides a graphical representation of the retailer's response described in Proposition 7.

6.1.2 Equilibrium Solution of the Game between Suppliers

The suppliers compete by selecting wholesale prices K_i that maximize their discounted expected profits as given in equation (6). Based on the retailer's response function the solution to the game between the suppliers is given in the following proposition.

Proposition 8. *Assume that either $e^{-r}sp_{01} > c_1$ or $e^{-r}sp_{10} > c_2$. Then the equilibrium solution to the game between suppliers is unique and*

(i) *If $e^{-r}sp_{01} > c_1$ and $e^{-r}sp_{10} > c_2$, then $(K_1^*, K_2^*) = (e^{-r}sp_{01}, e^{-r}sp_{10})$. The retailer's order quantities are (D, D).*

(ii) *If $e^{-r}sp_{01} > c_1$ and $e^{-r}sp_{10} \leq c_2$, then $(K_1^*, K_2^*) = [c_2 - \varepsilon - e^{-r}s(\pi_1 - \pi_2), c_2 - \varepsilon]$ for a small ε. The retailer's order quantities are $(D, 0)$.*

(iii) *If $e^{-r}sp_{01} \leq c_1$ and $e^{-r}sp_{10} > c_2$, then $(K_1^*, K_2^*) = [c_1 - \varepsilon, c_1 - \varepsilon + e^{-r}s(\pi_1 - \pi_2)]$ for a small ε. The retailer's order quantities are $(0, D)$.*

Figure 2 shows the unique equilibrium solution described in Proposition 8.

6.1.3 Defaults Correlation and Supply Chain Profits

Part (i) of Proposition 8 is the most relevant to the study of the correlation effects, because in this case both suppliers participate in the game. Using expressions for the equilibrium prices and order quantities, under hypothesis of part (i) in Proposition 8, the equilibrium retailer's and suppliers' profits are

$$R^* = e^{-r}sD\left(p_{01} + p_{10} + p_{00}\right) - e^{-r}sp_{01}D - e^{-r}sp_{10}D = e^{-r}sp_{00}D \tag{18a}$$

$$S_1^* = \left(e^{-r}sp_{01} - c_1\right)D \tag{18b}$$

$$S_2^* = \left(e^{-r}sp_{10} - c_2\right)D. \tag{18c}$$

Therefore, the total supply chain profit is

$$U^* = \left(e^{-r}sp_{01} - c_1\right)D + \left(e^{-r}sp_{10} - c_2\right)D + e^{-r}sp_{00}D = e^{-r}s(1 - p_{11})D - c_1D - c_2D. \tag{19}$$

The coordinated channel profit is

$$C^* = e^{-r}sD\left(p_{01} + p_{10} + p_{00}\right) - c_1D - c_2D = e^{-r}s(1 - p_{11})D - c_1D - c_2D. \tag{20}$$

Using these explicit expressions for profits and noting that as the correlation between defaults changes from the perfect negative to the perfect positive the default distribution parameters change from ($p_{01} = 1 - \pi_1$, $p_{10} = 1 - \pi_2$ and $p_{00} = p_{11} = 0$) to ($p_{00} = 1 - \pi$, $p_{01} = p_{10} = 0$, $p_{11} = \pi$), we immediately obtain the following result:

Theorem 2. *Assume that $e^{-r} s p_{01} > c_1$ and $e^{-r} s p_{10} > c_2$. Then the channel profit is equal to the coordinated channel profit ($U^* = C^*$) and, therefore, equilibrium solution (K_1^*, K_2^*) are channel coordinating. In addition, as the correlation between defaults increases:*

(i) *The supply chain profit, $U^* = C^*$, decreases*

(ii) *The retailer's profit, R^*, increases*

(iii) *The suppliers' profits, S_1^* and S_2^*, decrease.*

It follows from Theorem 2 that the retailer would prefer the suppliers' defaults to be positively correlated. The positive correlation between defaults induces high competition between suppliers, which leads to lower wholesale prices, compensating the retailer for the loss of diversification benefits. Conversely, suppliers prefer to have the defaults that are negatively correlated. When the defaults are perfectly negatively correlated there is no competition between the suppliers (in the probabilistic states of nature where one of the suppliers survived the other one defaulted), therefore, each supplier behaves as a monopolist and extracts all of the system profits.

If it were feasible, the suppliers would benefit by decreasing their defaults' correlation. The correlation between defaults can be reduced by using different production technologies, different raw materials sources, by placing production facilities in different parts of the country (or different countries). This might provide firms with incentives to expand their global operations.

Finally, note that the supply chain profit increases as the correlation of defaults decreases. Therefore, what is good for the supplier is also good for the channel, but detrimental for the retailer.

6.2 Stochastic Demand

6.2.1 The Retailer's Problem

Given wholesale prices K_1 and K_2, the retailer maximizes her discounted expected profit,

$$R(z_1, z_2) = P(z_1, z_2) - K_1 z_1 - K_2 z_2, \tag{21}$$

where

$$
\begin{aligned}
P(z_1, z_2) =& e^{-r} sE\left[\min(D, z_1\beta_1 + z_2\beta_2)\right] = \\
&= e^{-r} s\left\{p_{01}E\left[\min(D, z_1)\right] + p_{10}E\left[\min(D, z_2)\right] + p_{00}E\left[\min(D, z_1 + z_2)\right]\right\}.
\end{aligned} \tag{22}
$$

The following proposition summarizes the solution of the retailer's problem with stochastic demand.

Proposition 9. *The optimal order quantities, (z_1, z_2), for the problem in (21), (22) satisfy the following systems of equations:*

$$
\textit{If} \begin{cases} K_2 \geq \frac{p_{00}}{1-\pi_1}K_1 + e^{-r}sp_{01}, \\ K_1 \leq e^{-r}s(1-\pi_1) \end{cases} \qquad \textit{Then} \begin{cases} e^{-r}s(1-\pi_1)\overline{G}(z_1) = K_1 \\ z_2 = 0. \end{cases} \tag{23}
$$

$$
\textit{If} \begin{cases} K_1 \geq \frac{p_{00}}{1-\pi_2}K_2 + e^{-r}sp_{10}, \\ K_2 \leq e^{-r}s(1-\pi_2) \end{cases} \qquad \textit{Then} \begin{cases} z_1 = 0 \\ e^{-r}s(1-\pi_2)\overline{G}(z_2) = K_2. \end{cases} \tag{24}
$$

$$
\textit{If} \begin{cases} K_1 < \frac{p_{00}}{1-\pi_2}K_2 + e^{-r}sp_{10}, \\ K_2 < \frac{p_{00}}{1-\pi_1}K_1 + e^{-r}sp_{01} \end{cases} \qquad \textit{Then} \begin{cases} e^{-r}s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] = K_1 \\ e^{-r}s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K_2. \end{cases} \tag{25}
$$

$$
\textit{Otherwise} \qquad \begin{cases} z_1 = 0 \\ z_2 = 0. \end{cases} \tag{26}
$$

Proof. See Appendix ∎

Figure 3 provides a graphical representation of the retailer's response function described in Proposition 9.

The following result will be needed to prove the existence of an equilibrium in the subsequent analysis.

Corollary 2. *For any supplier i, the optimal order quantity $z_i(K_i, K_{-i})$ is a continuous function of K_i for a fixed wholesale price of the other supplier K_{-i}.*

Proof. See Appendix ∎

6.2.2 Equilibrium Solution of the Suppliers' Game

The suppliers maximize their discounted expected profits given by equation (6). Observe that $K_i > e^{-r}(1-\pi_i)s, i = 1, 2$ is a dominated strategy for each of the suppliers. Therefore, it is sufficient to consider suppliers pricing policies restricted in the rectangle $[0, e^{-r}(1-\pi_1)s] \times [0, e^{-r}(1-\pi_2)s]$. By Corollary 2, $z(\cdot,\cdot)$ is a continuous function. Therefore, from Glicksberg (1952) Theorem we derive the following

Proposition 10. *There exists a mixed-strategy equilibrium solution to the suppliers' game.*

It is difficult to show, however, that there exists a pure-strategy equilibrium for this game. The game is not supermodular, therefore, the results in Topkis (1998) cannot be applied. It is also difficult to produce parsimonious conditions that would ensure quasi-concavity of the suppliers' profit functions, even though we can verify this property for particular distributions (normal, exponential). Therefore, we cannot invoke results from Debreu (1952). For simplicity, assume that the problem is symmetric, that is $c_1 = c_2 = c$ and $\pi_1 = \pi_2 = \pi$ (consequently, $p_{01} = p_{10}$). Then, if there exists a symmetric pure-strategy equilibrium, it can be characterized in the next proposition.

Proposition 11. *If there exists a symmetric pure-strategy equilibrium, then the equilibrium order quantities, $z_1^* = z_2^* = z^*$, satisfy*

$$p_{01}\overline{G}(z)[1-h(z)] + p_{00}\overline{G}(2z)\left[1-\frac{1}{2}h(2z)\right] + \frac{p_{00}^2 g^2(2z)z}{p_{10}g(z)+p_{00}g(2z)} = \frac{c}{e^{-r}s}. \tag{27}$$

The equilibrium wholesale prices are

$$K_1^* = K_2^* = e^{-r}s\left[p_{01}\overline{G}(z^*) + p_{00}\overline{G}(2z^*)\right]. \tag{28}$$

Proof. See Appendix ■

Based on Proposition 11 we suggest the following two-step procedure for computing a symmetric pure-strategy equilibrium (if it exists):

1. Solve equation (27) to find an optimal order quantity z^*.

2. Compute the corresponding equilibrium wholesale price, K^*, using equation (28).

Note that the supplier's profit function is piecewise defined

$$S_i(K_i, K_{-i}) = \begin{cases} S_i^{all}(K_i, K_{-i}) & \text{if the retailer orders only from supplier } i \\ S_i^{share}(K_i, K_{-i}) & \text{if the retailer orders from both suppliers} \\ S_i^{none}(K_i, K_{-i}) & \text{if the retailer does not order from supplier } i \end{cases}$$

The two-step procedure above finds the equilibrium point using only the S_i^{share} part of the supplier's profit function. Because $S_i^{none}(K_i, K_{-i}) \equiv 0$, $\max_{K_i}\{S_i^{share}(K_i, K_{-i})\} \geq \max_{K_i}\{S_i^{none}(K_i, K_{-i})\}$. The following lemma provides conditions under which the maximum of S_i^{share} also dominates the maximum of S_i^{all}. Let K^{mon} correspond to the equilibrium wholesale price of a one-supplier model. A subindex, i, identifying supplier, is implied in the following statement.

Lemma 1.
Assume that the demand distribution function is IGFR. Then for all $\widehat{K} < K^{mon}$

$$\max_K S^{share}(K, \widehat{K}) \geq \max_K S^{all}(K, \widehat{K})$$

Proof. See Appendix ■

Therefore, under the conditions of Lemma 1, the maximum of S^{share} is also the global maximum of the supplier's profit function S.

6.2.3 Default Correlation and Supply Chain Profits

While it is difficult to characterize the equilibrium solution in general, special cases of perfect positive and perfect negative correlation between defaults lend themselves to analysis rather easily. To study the supply chain performance at intermediate values of the defaults' correlation, we resort to numerical analysis.

Example 5. (Arbitrary Correlation. Exponential Demand Distribution)
Suppose that the demand distribution is exponential with mean 150 units and that the values of the other parameters are $s = 100, c_1 = c_2 = c = 10, r = 0.1, \pi_1 = \pi_2 = \frac{1}{2}$. Note that a value $\pi = \frac{1}{2}$ is extremely high from a practical perspective, however, this is the only value that allows us to consider the full range of correlations (from perfect negative to perfect positive) in a symmetric game. Using the two-step procedure described in the previous subsection we establish the symmetric equilibrium order quantity, z^*, and wholesale price, K^*, for different values of p_{00}. Figure 4 shows the results.

Figure 4 illustrates that as the correlation between suppliers' defaults increases, the system profit and suppliers' profits decrease, while the retailer's profit increases. ■

Similar results are obtained for other demand distributions. Just as in the case of deterministic demand, we observe that a positive correlation between the defaults induces more intense competition between the suppliers, benefiting the retailer. While the supply chain as a whole benefits from the diversification, the retailer makes the least profits when the defaults are perfectly negatively correlated. Because of this conflict of interests, the responsibilities of the central planner in a supply chain cannot be delegated to the retailer.

7 Conclusion

The recent experience with high level of corporate defaults have reinforced the importance of credit risk management, not only as a treasury function but also in the context of operational planning. While related operational random yield and financial defaults literatures are quite extensive, we believe that this paper is one of the first to address supply-chain management questions in the presence of financial credit risk. Specifically, using a simple one-period model of a supply chain with one retailer and two risky suppliers, this paper studies questions of supplier selection, pricing and ordering policies among firms. In our model, the suppliers compete for business from the retailer, and are, collectively, Stackelberg leaders in a game with the retailer.

Although, in general, the timing of the payments from the retailer to the suppliers is important, we prove that a family of general linear pricing policies can be divided into equivalence classes such that, in equilibrium, the suppliers, the retailer, and the channel are not concerned with the timing of payments.

A one-supplier model confirms that default risk has detrimental effect on firms in a supply chain. We identify sufficient conditions for the existence and the uniqueness of the equilibrium in the game between the supplier and the retailer and provide an equation that the equilibrium order quantity must satisfy. Analysis of the one-supplier model shows that the supplier, the retailer, the channel, and the coordinated channel profits are decreasing in the default probability. Furthermore, the rate of profits decline for firms in different echelons of the supply chain depends on the concavity or convexity of the demand's cumulative distribution function.

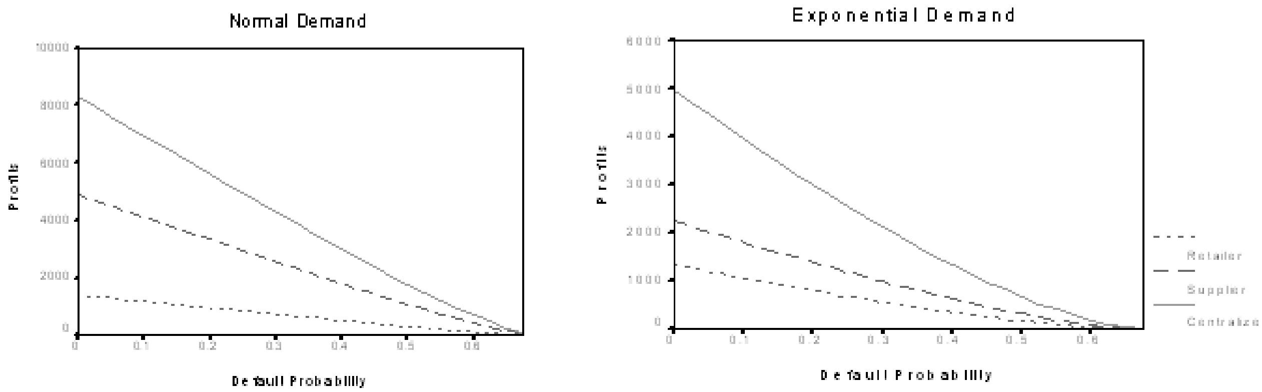
With more than one supplier, the retailer may decide to hedge default risk by splitting orders. If the wholesale prices were *exogenously* fixed, then, as one would expect, the negative correlation between default events yields higher diversification benefits to the retailer. However, in our competitive environment, the wholesale prices are determined *endogenously* by the suppliers. We are able to find equilibrium solutions analytically when demand is deterministic or when demand is stochastic and default correlation is either one or minus one. For the model with stochastic demand and arbitrary correlation we compute the equilibrium solution numerically. The analysis of the equilibrium solution shows that the positive correlation between default events stimulates competition between suppliers leading to lower wholesale prices. The benefits to the retailer, due to the lower wholesale prices, far outweigh the losses due to the weaker diversification. Therefore, contrary to initial intuition about the advantages of the diversification, positive default correlation benefits the retailer. We also show that a negative default correlation benefits the suppliers and the channel as a whole. Thus, incentives of the retailer and the channel are misaligned. The retailer should not be delegated to coordinate the channel. Further, the suppliers can benefit by making their defaults processes as negatively correlated as possible. For example, they may attempt to sell to different customers, use different production technologies, procure from different raw materials sources, and reduce exposures to common country specific risks or common catastrophic events.

In our analysis we have made several simplifying assumptions. For example, we assumed that the retailer cannot default, the default distribution of suppliers does not depend on the order quantities, the production lead times for both suppliers are equal. However, even in a simple model, including credit risk considerations into operational planning significantly affects ordering and pricing decisions in a supply chain and alters the nature of competition among firms. It remains for future research to study the effects of weakening these assumptions.

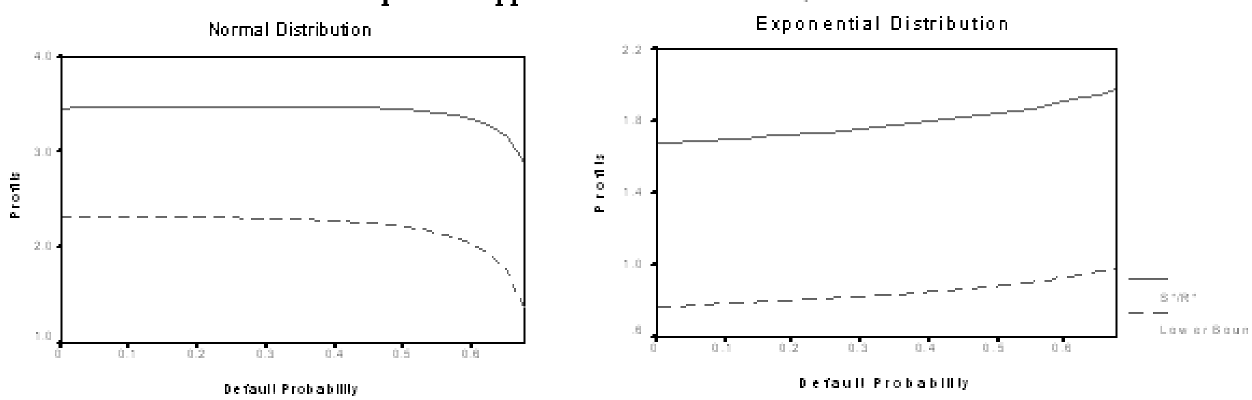
Figure 1: Comparisons of profits for normal and exponential demand distributions

The figure shows the optimal profits, ratio of supplier and retailer profits with lower bounds, and ratio of decentralized to centralized system profits for the normal and exponential demands. The case parameters are s=100; c=30; r=0.10. For the normal demand, the mean is 150 and std. deviation is 60. For the exponential demand, the mean is 150 units.

Panel A: Optimal Supplier, Retailer and Centralized Profits



Panel B: Ratio of Optimal Supplier and Retailer Profits, and its Lower Bound



Panel C: Ratio of Decentralized to Centralized System Profits

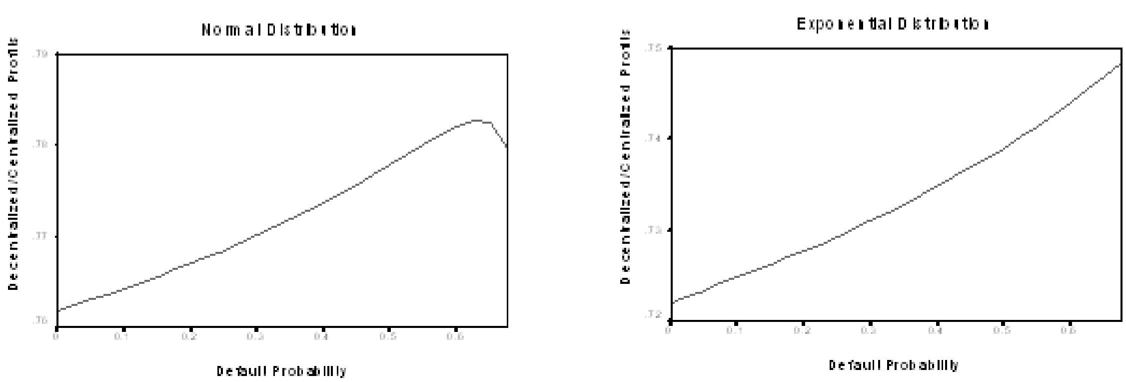


Figure 2: Retailer's response function and equilibrium solution to the game between suppliers when demand is deterministic.

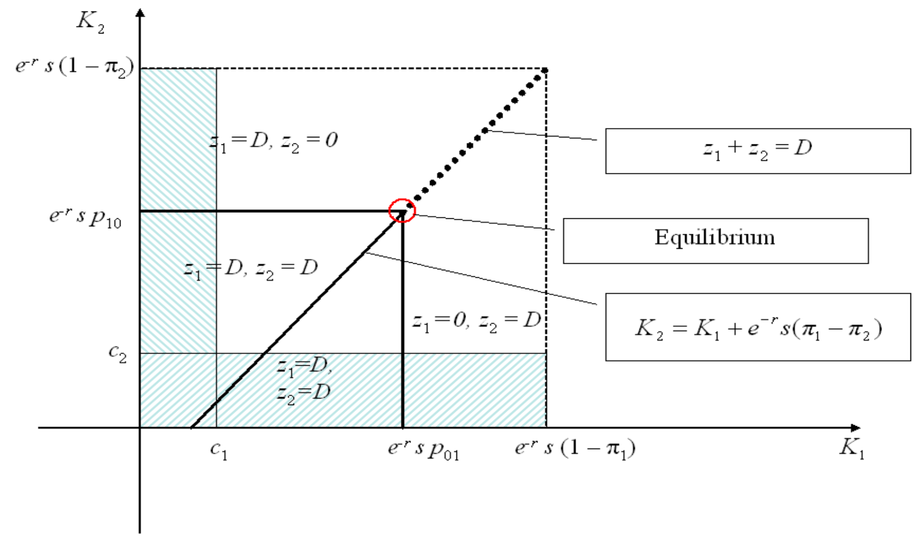


Figure 3: Retailer's response function to wholesale prices $K_i, i = 1, 2$ when demand is stochastic.

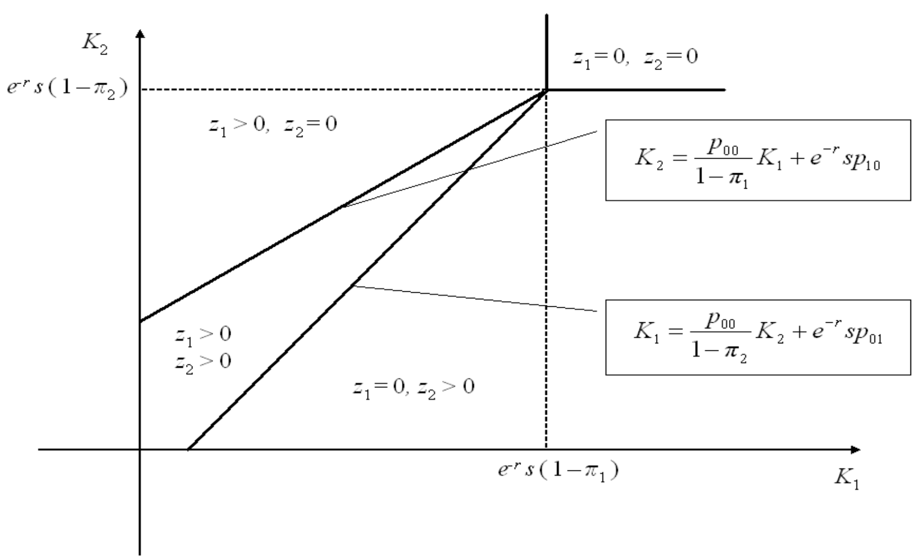
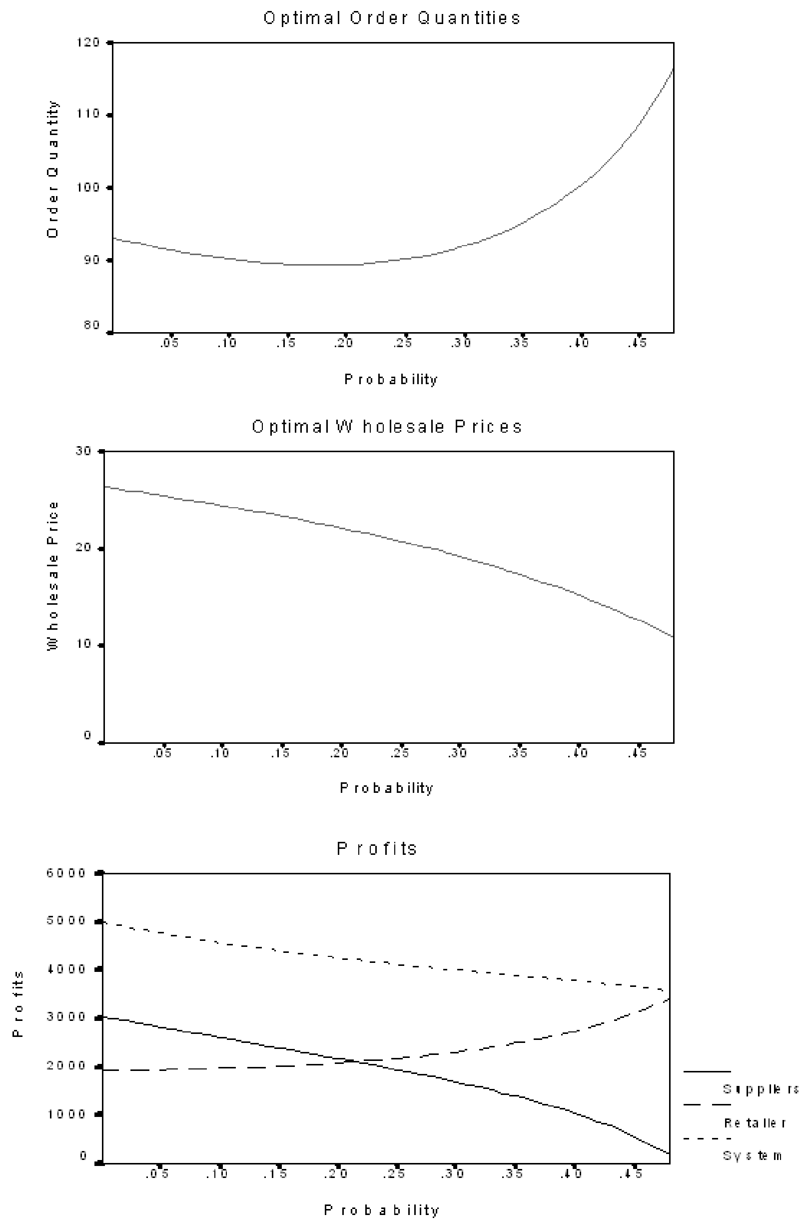


Figure 4: Symmetric Equilibrium Results for Exponential Demand

The figures show the symmetric equilibrium retailer order quantities, wholesale prices and profits as a function of the probability parameter, p_{00}, that controls default correlation. The case parameters are s=100; c=30; r=0.10. Exponential demand with mean 150 units.



Appendix

Proof of Proposition 3.

Recall that $A(z,\beta) = \beta\overline{G}(z\beta)$ and $A_\beta(z,\beta) = \overline{G}(\beta z)\,[1 - h(\beta z)]$. Because demand distribution is IGFR and $\beta \leq 1$, $h(z\beta) \leq h(z) < 1$ for all $z < \overline{z}$ and for all β. Therefore, for all $z \leq \overline{z}$, $A_\beta(z,\beta) > 0$ and, hence $A(z,\cdot)$ in an increasing function. Thus, for all $z \leq \overline{z}$ as $\beta \downarrow_{st}$, $E[A(z,\beta)]$ decreases. In addition, observe that $E[A(z,\beta)]$ is decreasing in z for any given random variable β. Therefore, by the proposition hypothesis, the optimal order quantity corresponding to β_{max}: $z^{central}(\beta_{max}) < \overline{z}$. It follows that for all $\beta_1 <_{st} \beta_2 <_{st} \beta_{max}$, $z(\beta_1) < z(\beta_2) < z(\beta_{max})$. This proves the first part of Proposition 3.

Next, observe that

$$Pr(D < z^{central}\beta) = E[G(z^{central}\beta)].$$

Because $G(z\beta)$ is an increasing function of β for all z and because $z^{central}$ decreases as credit risk increases, it follows that, as $\beta \downarrow_{st}$, $Pr(D < z^{central}\beta)$ decreases. $\blacksquare$

Proof of Proposition 4.

As $z \to +\infty$, by the Monotone Convergence Theorem, $S(z) \to -\infty$. Therefore, there exists $\widehat{z}$ such that for all $z > \widehat{z}$, $S(z) < 0$. Hence, we can restrict the search for an optimal z to the interval $[0,\widehat{z}]$. Function $S(\cdot)$ is bounded from above on this interval and hence, achieves the maximum.

The maximum satisfies the first order conditions (10). $\blacksquare$

Proof of Theorem 1.

Consider the first order condition (12) that determines optimal order quantity z^*. As π increases, the right hand side of the expression (12) increases. Because left hand side of the expression (12) is nondecreasing in z it follows that z^* is decreasing in π.

From expression (11) for supplier's profit, we see that for all z, $S(z)$ is decreasing in π. It follows that the optimal supplier's profit $S(z^*)$ is decreasing in π.

Finally, if the supplier charges wholesale price $K^* = e^{-r}(1-\pi)s\overline{G}(z^*_\pi)$ then the retailer's profit

$$R_\pi(z) = e^{-r}(1-\pi)s\left[E\min(D,z) - \overline{G}(z^*_\pi)z\right]$$

is decreasing in π for all z. Hence, $R^* = R(z^*)$ is decreasing in π. $\blacksquare$

Proof of Proposition 6.

If $G(\cdot)$ is concave (convex) then $\gamma(\cdot)$ is decreasing (increasing). Observe that

$$\frac{K^* - c}{e^{-r}(1-\pi)s - K^*} = \gamma(z^*)$$

and

$$\frac{\frac{e^{-r}(1-\pi)s-K^*}{e^{-r}(1-\pi)s}}{\frac{e^{-r}(1-\pi)s-c}{e^{-r}(1-\pi)s}} = \frac{1}{1+\gamma(z^*)}$$

As π increases, z^* decreases. Hence $\gamma(z^*)$ increases (decreases) and the conclusion of Proposition 6 follows. ■

Proof of Proposition 7.

Observe that, by the proposition hypothesis, it is not optimal for the retailer to order amounts from the suppliers that add up to a quantity lower than D. Therefore, we restrict the search for the optimal order quantities to $z_1^* + z_2^* \geq D$ and $z_i^* \leq D, i = 1, 2$. Using equation (17) we derive the following expression for the retailer's profit:

$$R(z_1, z_2) = (e^{-r}sp_{01} - K_1)z_1 + (e^{-r}sp_{10} - K_2)z_2 + p_{00}D.$$

The three cases now follow easily:

If $K_1 \leq e^{-r}sp_{01}$ and $K_2 \leq e^{-r}sp_{10}$, then order quantities (D, D), maximize retailer's profits.

If $K_1 \leq e^{-r}sp_{01}$ and $K_2 > e^{-r}sp_{10}$, then the optimal order quantities are $(D, 0)$.

If $K_1 > e^{-r}sp_{01}$ and $K_2 \leq e^{-r}sp_{10}$, then the optimal order quantities are $(0, D)$.

Suppose $K_1 > e^{-r}sp_{01}$ and $K_2 > e^{-r}sp_{10}$. Then the retailer would like to order as little as possible from the suppliers subject to the constraint $z_1 + z_2 \geq D$. Therefore, the retailer will order D from one of the suppliers and 0 from the other unless she is indifferent between the two [which occurs when $K_2 = K_1 + e^{-r}s(\pi_1 - \pi_2)$]. ■

Proof of Proposition 9.

From the first order conditions, $(z_1, 0)$ is the optimal retailer's response if

$$\left.\frac{\partial R}{\partial z_1}\right|_{z_2=0} = 0 \quad \text{and} \quad \left.\frac{\partial R}{\partial z_2}\right|_{z_2=0} \leq 0,$$

Or equivalently,

$$\begin{cases} e^{-r}s(1-\pi_1)\overline{G}(z_1) = K_1 \\ e^{-r}s\left[p_{10} + p_{00}\overline{G}(z_1)\right] \leq K_2. \end{cases}$$

Equivalently, $e^{-r}s(1-\pi_1)\overline{G}(z_1) = K_1$ and $K_2 \geq \frac{p_{00}}{1-\pi_1}K_1 + e^{-r}sp_{10}$. The proof for the remaining cases is similar. ∎

Proof of Corollary 2.

Without loss of generality, assume that $i = 1$. Solution of the system (25) is unique. The conclusion follows from an observation that system of equations (25) is equivalent to the system (23) when $K_1 = \frac{(1-\pi_1)}{p_{00}}\left[K_2 - e^{-r}sp_{01}\right]$ and is equivalent to the system (24) when $K_1 = \frac{p_{00}}{1-\pi_2}K_2 + e^{-r}sp_{10}$. ∎

Proof of Proposition 11.

Because the equilibrium is symmetric, the equilibrium order quantity $z_1 = z_2 = z > 0$. Thus, we consider the supplier's profit function over the region where both order quantities are positive.

For supplier 1:

$$\max_{L(K^*)\leq K_1 \leq R(K^*)} (K_1 - c)z_1(K_1, K^*),$$

where $z_1(K_1, K^*)$ satisfies the system of equations (25), $R(K^*) = \frac{p_{00}}{1-\pi_2}K^* + e^{-r}sp_{10}$, and $L(K^*) = \frac{1-\pi_1}{p_{00}}\left(K^* - e^{-r}sp_{01}\right)$. For this optimization problem we can change the variable from K_1 to z_1, z_2, as long as (25) is satisfied. Then the optimization problem becomes:

$$\max_{z_1,z_2:L(K^*)\leq K_1(z_1,z_2)\leq R(K^*)} \left\{e^{-r}s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] - c\right\} z_1,$$

subject to

$$e^{-r}s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K^*.$$

Taking the Lagrangian:

$$\begin{aligned}\max_{z_1,z_2:L(K^*)\leq K_1(z_1,z_2)\leq R(K^*)} &\left\{e^{-r}s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] - c\right\} z_1 - \\ &- \lambda\left\{e^{-r}s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] - K^*\right\},\end{aligned}$$

the first order necessary conditions for an interior maximum point are

$$\begin{aligned}&e^{-r}s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] - c - e^{-r}s\left[p_{01}g(z_1) + p_{00}g(z_1+z_2)\right]z_1 + \\ &+ \lambda e^{-r}sp_{00}g(z_1+z_2) = 0, \\ &- e^{-r}sp_{00}g(z_1+z_2)z_1 + \lambda e^{-r}s\left[p_{10}g(z_2) + p_{00}g(z_1+z_2)\right] = 0, \\ &e^{-r}s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K^*.\end{aligned}$$

After eliminating λ from the first two equations we obtain:

$$\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] - \left[p_{01}g(z_1) + p_{00}g(z_1+z_2)\right] z_1 +$$
$$+ \frac{p_{00}^2 g^2(z_1+z_2)z_1}{p_{10}g(z_2) + p_{00}g(z_1+z_2)} = \frac{c}{e^{-r}s},$$

and

$$e^{-r}s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K^*.$$

For a symmetric equilibrium, $z_1 = z_2 = z$. Hence, the equilibrium order quantity must satisfy

$$p_{01}\overline{G}(z)[1-h(z)] + p_{00}\overline{G}(2z)\left[1 - \frac{1}{2}h(2z)\right] + \frac{p_{00}^2 g^2(2z)z}{p_{10}g(z) + p_{00}g(2z)} = \frac{c}{e^{-r}s},$$

where $h(z) = z\frac{g(z)}{\overline{G}(z)}$ is the generalized failure rate function. The symmetric equilibrium order quantity is related to the symmetric equilibrium wholesale prices by

$$e^{-r}s\left[p_{10}\overline{G}(z) + p_{00}\overline{G}(2z)\right] = K^*.$$

■

Proof of Lemma 1.

It follows from the solution of the one-supplier model (Proposition 5) and from $\widehat{K} < K^{mon}$, that function S^{all} is increasing over its domain achieving maximum at the right boundary. From Corollary 2 it follows that the supplier's profit function is continuous. Therefore the maximum of S^{share} is at least as large as the value of S^{all} at the right boundary of its domain. ■

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