Technical Memorandum Number 824

Tardiness Risk at a Steel Mill

by

Matthew J. Sobel

September 2009

Department of Operations
Weatherhead School of Management
Case Western Reserve University
330 Peter B. Lewis Building
Cleveland, Ohio 44106

TARDINESS RISK AT A STEEL MILL

Matthew J. Sobel
Department of Operations
Case Western Reserve University
Cleveland, OH 44106-7235
E-mail: matthew.sobel@case.edu

Running head: Tardiness Risk at a Steel Mill

Abstract

A customer who places a tentative order at a steel mill customarily receives a due-date quotation, namely the date by which the mill says that it will ship the order. The steel mill wants the due date to be early enough so that the customer won't balk (withdraw the order), but late enough so that the probability of tardiness is low. This note obtains expressions for the tardiness probability at a continuous caster steel mill under the assumption that orders for different steel grades arrive as independent grade-dependent renewal processes and the time spent in the melt process is the waiting time in an $M/M/1$ queue. Some expressions require less stringent assumptions. The model reflects the fact that the melt process can be initiated only after the cumulative tonnage of non-balking unprocessed orders reaches a minimum batch size. This key feature occurs in other process industries such as aluminum and copper casting.

1 INTRODUCTION

It is common for a steel mill to receive streams of tentative customer orders for assorted grades of steel. The mill responds to a tentative order with a due-date quotation which the customer may or may not accept. Although a shorter due-date quotation raises the likelihood that the customer will not balk (i.e., that the order will remain at the mill), it also increases the likelihood of subsequent tardiness. This note is *not* directly concerned with the problem of optimally balancing the balking probability with the tardiness probability. Keskinocak and Tayur [4] survey the optimization literature. Instead, the focus here is the probability of tardiness in continuous caster steel mills and other process industries with minimum batch sizes (such as aluminum and copper casting). An expression for the probability of tardiness is useful in calculating optimal due-date quotations in a steel mill (cf. Slotnick [6]).

A steel mill with a continuous caster has a single large ladle (e.g., 100 tons) that can be used to make only one grade of steel at a time. A "melt" consists of combining iron ore, alloys, coke, and other ingredients in the ladle at high temperature to make a particular grade of steel followed by casting the molten output into slabs. Further processing ("finishing") includes rolling and forming coils of product that correspond to particular customers' orders. Since a melt has a high fixed cost, a batch enters the melt process queue only when that grade's cumulative unprocessed non-balking tonnage reaches the ladle capacity. Therefore, the elapsed time until a customer's order is shipped is the sum of three times. First, there is the delay until the cumulative tonnage of unprocessed non-balking orders with the same grade as that of the current customer reaches the ladle capacity. Second, there is the waiting time in the melt process queue, and third there is the time in the caster and in subsequent rolling mills. The steel mill is said to be tardy with respect to a particular order if the total elapsed time is greater than the due-date quotation that was given to the customer.

Consider any particular grade of steel. The delay until the cumulative tonnage of non-balking orders reaches the ladle capacity can be regarded as an attribute of a batch service queueing process. The literature on the dependence of these processes on the batch size (Bailey [1] seems to have originated the literature, and see Gold and Tran-Gia [2] for references) and batch size optimization (see Xia et al. [10] and its references) is not applicable to steel mills where the batch size, i.e., the size of the ladle, is fixed.

Batch service queues exemplify *stochastic clearing systems*. Stidham ([8] and [9]) initiated the stochastic process and optimization literatures; see Kella et al. [3] and Perry et al. [5] for more recent references. There too, the batch size is a decision variable in the motivating contexts, so the results do not seem applicable to the quotation of due dates in steel mills (because the ladle size is fixed). However, the results for batch service queues and, more generally, stochastic clearing systems, may be applicable to the *design* of steel mills and factories in other process industries with minimum batch size technologies.

2 MODEL AND OVERVIEW

For each grade of steel, the combined tonnage of unprocessed non-balking customers' orders waits to enter the melt process until the aggregate is at least as large as the ladle capacity. A grade's *batch gap* at a given time is the difference between the ladle capacity and the current aggregate non-balking tonnage which is waiting to be melted.

Notation

$g =$ grade of current customer who is receiving a due-date quotation;

$\gamma =$ grade g batch gap *assuming that the current customer does not balk;*

$T =$ elapsed time to fill the grade g batch gap;

$\lambda_g =$ arrival rate of *non-balking* grade g customers;

$\mathcal{A} =$ average tonnage per non-balking grade g customer;

$\Lambda = \sum_k \lambda_k$; aggregate non-balking arrival rate;

$\beta =$ melt rate (in tons per unit time);

$W =$ time that an order spends in the melt *system* after the batch gap is filled;

$\delta =$ due-date quotation;

$K =$ batch size $=$ ladle capacity;

$\Phi, \phi =$ distribution and density functions of the (standard) normal random variable with mean 0 and variance 1.

If the grade g batch gap was γ_1 immediately before the current customer arrived, and the tonnage of the current customer's order is A, then the updated grade g batch gap is $\gamma = (\gamma_1 - A)^+$. Henceforth, this note considers one particular grade of steel, so g is suppressed in the notation.

This note employs the following steps to obtain a computationally practical expression for $P\{T+W > \delta\}$. The first step models W as the equilibrium waiting time in an M/M/1 queue with arrival rate Λ and service rate β. The second step calculates the tardiness probability by treating T and W as independent random variables, with T having a normal distribution. *The mean and variance of T are parameters in the second step.* The third step deduces the mean and variance of T from properties of renewal processes and renewal-reward processes. Section 5 sketches the first two steps and, until then, this note obtains expressions for the mean and variance of T, i.e., the third step.

2.1 OVERVIEW

Let $X_1, X_2, ...$ be the inter-arrival times of future non-balking customers, and let $A_1, A_2, ...$ be the tonnages in their orders. Assume that $X_1, X_2, ...$ and $A_1, A_2, ...$ generate independent renewal processes, i.e., they are mutually independent and each sequence consists of independent and identically distributed nonnegative random variables. Let $H(\cdot)$ and $J(\cdot)$ denote the respective distribution functions of the A's and X's.

The sequence $X_1, X_2, ...$ is the output from filtering the exogenous arrival process of orders with the balking process, which depends on the due-date quotations. This note does not consider the exogenous process or the filtration; see Slotnick [6] and Slotnick and Sobel [7] for optimal filtrations.

The next section derives two expressions for μ_T. The first, (3.11), is valid without any additional assumptions, and the second, (3.21), is valid if the arrival process is Poisson and the order sizes are exponentially distributed. The latter expression coincides with an upper bound on μ_T which is obtained in §3.1.

Section 4 begins with a formula for σ_T^2 in the general case. The assumption in §4.1 of Poisson arrivals specializes the formula, and the additional assumption in §4.2 of exponential order sizes further specializes it.

Section 5 uses the results in §3 and §4, treating T and W as independent random variables and T as approximately normally distributed, to calculate the tardiness probability. The note is summarized in §6.

3 $\mu_T = E(T)$

Let $\{N(t) : t \geq 0\}$ be the renewal process generated by $X_1, X_2, ...$:

$$N(t) = \sup\{n : \sum_{j=1}^{n} X_j \leq t\} \quad (t \geq 0)$$

That is, $N(t)$ is the cumulative number of non-balking grade g customers who have arrived t time units hence. Let Z_n be the cumulative tonnage brought by the first n non-balking customers :

$$Z_n = \sum_{j=1}^{n} A_j \qquad (n = 0, 1, 2, ...) \tag{3.1}$$

The batch gap is filled at the earliest time T at which this sum is larger than the batch gap, γ:

$$T = \inf\{t : Z_{N(t)} > \gamma\} \tag{3.2}$$

More generally, $T = T(\gamma)$ with the definition

$$T(y) = \inf\{t : Z_{N(t)} > y\} \qquad (y \geq 0) \tag{3.3}$$

Denote the distribution functions of $T(y)$ and $Z_{N(t)}$ with

$$F_T(y,t) = P\{T(y) \leq t\} \qquad F_N(y,t) = P\{Z_{N(t)} \leq y\} \tag{3.4}$$

The distribution functions are related because

$$T(y) \leq t \Longleftrightarrow Z_{N(t)} > y \tag{3.5}$$

From (3.3),

$$F_T(y,t) = 1 - F_N(y,t) \tag{3.6}$$

Let $\mu_T(y) = E[T(y)]$ and use (3.5):

$$\mu_T(y) = \int_0^{\infty} [1 - F_T(y,t)]dt = \int_0^{\infty} F_N(y,t)dt \tag{3.7}$$

In order to exploit (3.7), let $H_k(\cdot)$ denote the distribution function of the cumulative tonnage of the first k non-balking customers, namely of $\Sigma_{j=1}^{k} A_k$, so $H(r) = H_1(r)$ and $H_0(r) \equiv 1$ if $r \geq 0$.

Use definition (3.4) and condition on $N(t)$:

$$F_N(y,t) = \sum_{k=0}^{\infty} P\{Z_{N(t)} \leq y \,|N(t) = k\}P\{N(t) = k\} \tag{3.8}$$

Let $p_k(t) = P\{N(t) = k\}$ and use $H_k(y) = P\{Z_{N(t)} \leq y \,|N(t) = k\}$:

$$F_N(y,t) = \sum_{k=0}^{\infty} H_k(y)p_k(t) \tag{3.9}$$

Substitute in (3.7):

$$\mu_T(y) = \int_0^{\infty} F_N(y,t)dt = \int_0^{\infty} \sum_{k=0}^{\infty} H_k(y)p_k(t)dt \tag{3.10}$$

Interchange the order of integration and summation to obtain a general expression for the mean time to close a bucket gap:

$$\mu_T(y) = \sum_{k=0}^{\infty} c_k H_k(y) \qquad \textit{where} \;\; c_k = \int_0^{\infty} p_k(t)dt \tag{3.11}$$

3.1 μ_T BOUNDS WITH POISSON ARRIVALS

Since $Z_{N(t)}$ is the cumulative tonnage that has arrived by time t,

$$\gamma \leq Z_{N(T)} \leq \gamma + A_{N(T)+1} \tag{3.12}$$

Let $M_X(\cdot)$ denote the renewal function for $\{N(t), t \geq 0\}$ which has interarrival times $X_1, X_2, ...$ That is, $M_X(t) = E[N(t)]$. Therefore,

$$E[Z_{N(T)}] = \mathcal{A}E[M_X(T)] \tag{3.13}$$

Assuming that $\{N(t), t \geq 0\}$ is a Poisson process with rate λ, $M_X(t) = \lambda t$ and

$$E[Z_{N(T)}] = \mathcal{A}\lambda E(T) = \mathcal{A}\lambda\mu_T \tag{3.14}$$

It follows from (3.12) that $\gamma \leq E[Z_{N(T)}] < \gamma + \mathcal{A}$. These inequalities and (3.14) yield

$$\gamma/(\lambda\mathcal{A}) \leq \mu_T \leq (1 + \gamma/\mathcal{A})/\lambda \tag{3.15}$$

3.2 μ_T: POISSON ARRIVALS AND EXPONENTIAL ORDER SIZES

In this subsection, the arrival process is a Poisson process with rate λ and the order sizes are exponentially distributed with rate $\theta = 1/\mathcal{A}$. So $p_k(t) = e^{-\lambda t}(\lambda t)^k/k!$ $(k = 0, 1, ...)$, $H(y) = 1 - e^{-\theta y}$,

$$H_k(y) = \int_0^y \theta e^{-\theta r}(\theta r)^{k-1} dy/(k-1)! \quad (k = 1, 2, ...), \; \textit{and} \tag{3.16}$$

$$1/\lambda = \int_0^\infty p_k(t)dt \qquad (k = 0, 1, ...) \tag{3.17}$$

Let $M(\cdot)$ be the renewal function that corresponds to $H(\cdot)$. So

$$M(y) = \sum_{k=1}^{\infty} H_k(y) \tag{3.18}$$

Also, $M(y) = E[N_A(y)]$ with the definition $N_A(y) = \sup\{n : \sum_{j=1}^n A_j \leq y\}$, but there is no further mention of $N_A(\cdot)$ in this note.

Combining the definition of $H_k(\cdot)$, (3.10), (3.11), (3.17), (3.18), and $H_0(\cdot) \equiv 1$,

$$\mu_T(y) = (1/\lambda) \sum_{k=0}^{\infty} H_k(y) = (1/\lambda)[1 + M(y)] \tag{3.19}$$

Since exponentiality implies $M(t) = \theta t$,

$$\mu_T(y) = (1/\lambda) \sum_{k=0}^{\infty} H_k(y) = (1/\lambda)(1 + \theta y) \tag{3.20}$$

Substitute $y = \gamma$ and $\theta = 1/\gamma$ to obtain

$$\mu_T = E(T) = (1 + \gamma\theta)/\lambda \tag{3.21}$$

This expression is the upper bound in (3.15) because $\theta = 1/\mathcal{A}$. Notice that the right side of (3.21) is the sum of the means of $1+\gamma\theta$ exponential random variables with parameter λ. An obvious renewal-theoretic argument permits the interpretation of $1 + \gamma\theta$ as the expected number of grade g non-balking customers which are needed to close the gap.

4 $\sigma_T^2 = Var(T)$

This section obtains expressions for $E[T(y)^2]$ because there are expressions for $\mu_T(y)$ ((3.11), (3.19), and (3.20)).

From (3.6) and (3.10),

$$\partial F_T(t, y)/\partial t = \partial[1 - F_N(y, t)]/\partial t = \partial[1 - \sum_{k=0}^{\infty} H_k(y)p_k(t)]/\partial t$$

So

$$\partial F_T(t, y)/\partial t = -\sum_{k=0}^{\infty} H_k(y)[dp_k(t)/dt]$$

Interchanging the order of integration and summation yields

$$E[T(y)^2] = \int_0^{\infty} t^2[\partial F_T(t, y)/\partial t]dt = \sum_{k=0}^{\infty} b_k H_k(y) \tag{4.1}$$

Define

$$b_k = -\int_0^{\infty} t^2[dp_k(t)/dt]dt \tag{4.2}$$

Therefore, (3.16) and (4.1) imply

$$\sigma_T^2(y) = \sum_{k=0}^{\infty} b_k H_k(y) - [\sum_{k=0}^{\infty} c_k H_k(y)]^2 \tag{4.3}$$

where c_k is defined in (3.11).

4.1 σ_T^2: POISSON ARRIVAL PROCESS

This subsection specializes (4.3) under the assumption that $\{N(t), t \geq 0\}$ is a Poisson process with rate λ; the order tonnages (the A's) continue to have an arbitrary distribution function $H(\cdot)$.

Use $p_k(t) = e^{-\lambda t}(\lambda t)^k/k!$ in (3.11) to obtain $c_k = 1/\lambda$. Therefore, in (4.3),

$$\sum_{k=0}^{\infty} c_k H_k(y) = [1 + M(y)]/\lambda \tag{4.4}$$

Use

$$dp_k(t)/dt = -\lambda e^{-\lambda t}[(\lambda t)^k/k! - (\lambda t)^{k-1}/(k-1)!]$$

in (3.17) to obtain

$$b_k = -\int_0^\infty t^2[dp_k(t)/dt]dt = \int_0^\infty t^2\lambda e^{-\lambda t}[(\lambda t)^k/k! - (\lambda t)^{k-1}/(k-1)!]dt$$

So

$$b_k = \int_0^\infty t^2\lambda e^{-\lambda t}(\lambda t)^k dt//k! - \int_0^\infty t^2\lambda e^{-\lambda t}(\lambda t)^{k-1}dt/(k-1)!$$

The integrals are the mean squares of gamma random variables with respective parameters $(k+1,\lambda)$ and (k,λ). Since the mean square is the sum of the variance (which is $(k+1)/\lambda^2$ for the first integral and k/λ^2 for the second) and the square of the mean (the mean is $(k+1)/\lambda$ in the first integral and k/λ in the second),

$$b_k = [(k+1) + (k+1)^2 - k - k^2]/\lambda^2 = 2(k+1)/\lambda^2$$

Therefore, the first sum in (4.3) is

$$\sum_{k=0}^\infty b_k H_k(y) = 2[1 + \sum_{k=1}^\infty (k+1)H_k(y)]/\lambda^2$$

Define

$$Q(y) = \sum_{ke1}^\infty kH_k(y) \tag{4.5}$$

From definition (3.18) of $M(y)$ and (4.5),

$$\sum_{k=0}^\infty b_k H_k(y) = 2[1 + Q(y) + M(y)]/\lambda^2 \tag{4.6}$$

Combining (4.3), (4.4), and (4.6),

$$\begin{aligned}\sigma_T^2(y) &= 2[1 + Q(y) + M(y)]/\lambda^2 - \{[1 + M(y)]/\lambda\}^2 \\ &= [2Q(y) - M(y) - M(y)^2 + 1]/\lambda^2 \qquad (4.7)\end{aligned}$$

Notice that the standard deviation of $T(y)$ is proportional to the mean non-balking customer interarrival time $(1/\lambda)$.

4.2 σ_T^2: POISSON ARRIVAL PROCESS AND EXPONENTIAL ORDER SIZES

This section obtains the variance and specializes (4.3) under the assumptions that $\{N(t), t \geq 0\}$ is a Poisson process with rate λ and the order tonnages are exponentially distributed with rate $\theta = 1/\mathcal{A}$. The starting point is (4.7) with $M(y) = \theta y$. Since $Q(y)$ is needed for (4.7), here

$$Q(y) = \sum_{k=1}^{\infty} kH_k(y) = \sum_{k=1}^{\infty} k \int_0^y \lambda e^{-\lambda t}(\lambda t)^{k-1} dt/(k-1)!$$

Interchanging the order of summation and integration,

$$Q(y) = \int_0^y \theta e^{-\theta t}[\sum_{k=1}^{\infty} k(\theta t)^{k-1}/(k-1)!]dt \tag{4.8}$$

The sum is

$$\sum_{k=1}^{\infty} k(\theta t)^{k-1}/(k-1)! = \sum_{k=0}^{\infty}(k+1)(\theta t)^k/k! \tag{4.9}$$

which can be split into two terms. The first term is

$$\sum_{k=0}^{\infty} k(\theta t)^k/k! = \sum_{k=1}^{\infty} k(\theta t)^k/k! = \sum_{k=1}^{\infty} \theta t(\theta t)^{k-1}/(k-1)! = \theta t \sum_{k=0}^{\infty}(\theta t)^k/k!$$

Since the sum in the right-most term is $e^{\theta t}$,

$$\sum_{k=1}^{\infty} k(\theta t)^{k-1}/(k-1)! = \theta t e^{\theta t} \tag{4.10}$$

The other term on the right side of (4.9) is

$$\sum_{k=0}^{\infty}(\theta t)^k/k! = e^{\theta t} \tag{4.11}$$

Substituting (4.10) and (4.11) in (4.9), and inserting the result in (4.8),

$$Q(y) = \int_0^y \theta e^{-\theta t}(1+\theta t)e^{\theta t}dt = \theta y + \theta^2 y^2/2 \tag{4.12}$$

Finally, substituting (4.12) and $M(y) = \theta y$ in (4.7) yields the following expression for $\sigma_T^2(y)$ when non-balking customers arrive according to a Poisson process and those customers bring exponentially distributed order tonnages:

$$\sigma_T^2(y) = (1+\theta y)/\lambda^2 \tag{4.13}$$

Notice that the variance of $T(y)$ is a linear function of the size of the gap (y) and of the reciprocal of the mean order size $(\theta = 1/\mathcal{A})$. Notice too that it is the sum of the variances of $1+\gamma\theta$ independent and identically distributed exponential random variables with parameter λ. An obvious renewal-theoretic argument permits the interpretation of $1+\gamma\theta$ as the expected number of grade g non-balking customers which are needed to close the gap.

5 TARDINESS PROBABILITY CALCULATION

Under the assumption that the duration of the melt process is the equilibrium waiting time in an M/M/1 queue, W is an exponentially distributed random variable with rate $\beta - \Lambda$. So its mean and variance are

$$\mu_W = E(W) = 1/(\beta - \Lambda) \qquad \sigma_W^2 = Var(W) = 1/(\beta - \Lambda)^2 \tag{5.1}$$

One way to calculate the tardiness probability would be to treat W and T as independent normal random variables and to calculate

$$P\{T + W > \delta\} \approx 1 - \Phi\left[(\delta - \mu_W - \mu_T)/\sqrt{\sigma_W^2 + \sigma_T^2}\right] \tag{5.2}$$

using (3.21) for μ_T, (4.13) for σ_T^2, and (5.1) for μ_W and σ_W^2.

An alternative calculation of the tardiness probability exploits the exponential assumption regarding W in its entirety, treats T and W as independent random variables, and represents T as a normal random variable with parameters given by (3.21) and (4.13). Proceeding with this alternative calculation and letting $c = \beta - \Lambda$,

$$\begin{aligned} P\{T + W > \delta\} &\approx \int_{-\infty}^{\infty} P\{W > \delta - b\}\phi\left(\frac{b - \mu_T}{\sigma_T}\right) db \\ &= e^{-c\delta} \int_{-\infty}^{\delta} e^{cb}\phi\left(\frac{b - \mu_T}{\sigma_T}\right) db + 1 - \Phi\left(\frac{\delta - \mu_T}{\sigma_T}\right) \end{aligned} \tag{5.3}$$

Standard manipulations of the integral in (5.3) yield

$$\int_{-\infty}^{\delta} e^{cb}\phi\left(\frac{b - \mu_T}{\sigma_T}\right) db = \sigma_T e^{c(\mu + c\sigma^2/2)}\Phi\left(\frac{\delta - \mu}{\sigma_T} - c\sigma_T\right)$$

Therefore,

$$\begin{aligned} &P\{T + W > \delta\} \approx \sigma_T\ exp\{-(\beta - \Lambda)[\delta - \mu_T - (\beta - \Lambda)\sigma_T^2/2\} \\ \times\ &\Phi\left[\frac{\delta - \mu_T}{\sigma_T} - (\beta - \Lambda)\sigma_T\right] + 1 - \Phi\left(\frac{\delta - \mu_T}{\sigma_T}\right) \end{aligned} \tag{5.4}$$

The computer calculation of (5.5) calls the standard normal distribution function twice, whereas (5.2) calls it only once and doesn't evaluate an exponential as (5.5) requires. More importantly, if data on W are decidedly not exponential, (5.2) may provide a superior estimate of the tardiness probability.

6 SUMMARY

A steel mill exemplifies process industry factories which receive streams of tentative orders for assorted grades of their product. In response to an order, the mill quotes a delivery lead time which may be acceptable to the customer, or the customer may balk and withdraw the order. In these contexts it is important to quantify the risk of tardiness which is associated with a particular lead time quotation.

Only one grade of product can be processed at a time in the first stage of manufacture (the melt process in a steel mill) and the cost structure results in batch processing. The batch size, denoted K in this note, is determined by technology (the size of the ladle in a steel mill). So each grade's stream of non-balking orders cannot enter the first stage of manufacture (the melt process in a steel mill) until the cumulative tonnage of those orders reaches K. The elapsed time for that cumulative tonnage to reach K is denoted T. When a grade's cumulative tonnage reaches K, the grade's batch is moved into the first stage of manufacture, the running total of that grade's cumulative tonnage prior to the first stage is reset to zero, and the process repeats itself.

The first stage is (in continuous caster steel mills) a single-server process with a conceptual queue of batches which are waiting to be processed (melted in a steel mill). After a batch completes its first stage, it moves on to a second and possibly third stage (in a steel mill, these are the hot and cold rolling mills). So a tandem queueing model is an appropriate description. This note uses W for the waiting time in that system.

The elapsed time until the order is shipped is $T + W$. Under the assumption that different grades' streams of non-balking orders arrive according to independent grade-dependent renewal processes, this note obtains expressions for $E(T)$ and $Var(T)$. The expressions are sharpened under the further assumptions that the arrival processes are Poisson processes and the order sizes have exponential distributions. It uses the sharpened expressions together with the assumption that W is the equilibrium waiting time in an $M/M/1$ queue. This yields an easily computed approximate formula for the probability that an order will be tardy, i.e., that $T + W$ will be greater than the quoted due date.

References

1. N.T.J. Bailey, N. T. J. (1954) On queueing process with bulk service. *Journal of the Royal Statistical Society, Series B* 16: 80-87.

2. Gold, H. & Tran-Gia, P. (1993) Performance analysis of a batch service queue arising out of manufacturing system modelling. *Queueing Systems* 14:413–426.

3. Kella, O., Perry, D., & Stadke, W. (2003) A stochastic clearing model with a Brownian and a compound Poisson component. *Probability in the Engineering and Informational Sciences* 17: 1–22.

4. Keskinocak, P. & Tayur, S. (2004) Due-date management policies. In D. Simchi-Levi, S. D. Wu, & Z. M. Shen (eds.), *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*. Norwell, MA: Kluwer Academic Publishers, pp. 485–553.

5. Perry, D., Stadje, W., & Zacks, S. (2005) Sporadic and continuous clearing policies for a production/inventory system under an M/G demand process. *Mathematics of Operations Research* 30: 354–368.

6. Slotnick, S. A. (2009). Lead-time Estimation for an Integrated Steel Mill. unpublished manuscript.

7. Slotnick, S. A. & Sobel, M. J. (2005). Manufacturing Lead-Time Rules: Customer Retention vs. Tardiness Costs. *European Journal of Operations Research* 163: 825–856.

8. Stidham, S., Jr. (1974) Stochastic clearing systems. *Stochastic Processes and their Applications* 2: 85–113.

9. Stidham, S., Jr., (1977) Cost models for stochastic clearing systems. *Operations Research* 25: 100–127.

10. Xia, C. H., Michailidis, G., Bambos, N., & Glynn, P. W. (2002) Optimal control of parallel queues with batch service. *Probability in the Engineering and Informational Sciences* 16: 289–307.