Competition and Diversification Effects in Supply Chains with Supplier Default Risk

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We study the effects of disruption risk in a supply chain where one retailer deals with competing risky suppliers who may default during their production lead-times. The suppliers, who compete for business with the retailer by establishing wholesale prices, are leaders in a Stackelberg game with the retailer. The retailer, facing uncertain future demand, chooses order quantities while weighing the benefits of procuring from the cheapest supplier against the advantages of order diversification. For the model with two suppliers we show that low supplier default correlations dampens competition among the suppliers, increasing the equilibrium wholesale prices. Therefore, the retailer prefers suppliers with highly correlated default events, despite the loss of diversification benefits. In contrast, the suppliers and the channel prefer defaults that are negatively correlated. However, as the number of suppliers increases our model predicts that the retailer may be able to take advantage of both competition and diversification.

1. Introduction

An increasing concern about supply disruptions due to strikes, natural disasters, production failures, or due to suppliers declaring bankruptcy, have reinforced the importance of default risk management, not only as a treasury function, but also in operational planning. This is highlighted by the increased importance of backup agreements, contingency measures, and by firms establishing multiple sources of supplies. This paper addresses the problems faced by a retailer who deals with multiple competing suppliers who may default on their obligations to deliver order quantities at the end of a given production lead time. Using a simple one-period model of a supply chain with one retailer and multiple risky suppliers, this paper studies questions of supplier selection, pricing and ordering policies among firms.

Recognition of supply disruptions and default risk among counterparties in a supply chain is heightened for several reasons. First, the credit quality of non financial firms have continued to slide over the last two decades and the performance of many suppliers can no longer be taken as guaranteed. Fears of counterparties defaulting are not unfounded. By 2005, for example, there were fewer than 10 non-financial firms with AAA ratings. The combined volume of defaults in 2001 and 2002, for example, exceeded the total volume of defaults in the US over the previous 20 years. The filing for bankruptcy in 2005 by Delphi, a very large visible supplier who makes nearly every component found in a car and employs 185,000 people worldwide, has brought enormous attention to the fact that diversifying sources of supply is important. Finally, Barton, Newell and Wilson (2003) share the following experience that highlights the importance of financial default risk:

"In 1997, one South Korean automaker saw many of its parts suppliers go under. Without backup suppliers, it couldn't increase production for export when the won was devalued. While foreign distributors begged for more cars to sell, production lines were idle back at home for lack of critical parts. The company weathered the storm but never fully recovered its market position and was eventually acquired by another domestic automaker".

A second reason for being concerned about supply chain disruptions is the high cost of disruptions. While some of this vulnerability has been due to major one-time events such as 9/11, the west-coast port strike in 2002, the 2003 Northeast blackout, the Katrina floods in 2005, and other act of nature, many supply chain disruptions could have been better managed if firms had better control over their supply chain networks. The economic consequences of disruptions can be massive. Indeed, in a comprehensive study, Hendricks and Singhal (2005) investigate over 800 cases of disruptions in supply chains and conclude that firms suffering from supply chain disruptions experience about 30% lower stock returns then their matched benchmarks. Their study makes a compelling argument that ignoring the risk of supply chain disruptions can have serious economic consequences.

To deal with possible supply disruptions the retailer might consider diversifying order quantities among competing suppliers. Indeed, the diversification is widely employed in some industries (according to some reports in business press, Japanese companies single-source only 15% of the components in their domestic market, see Lester (2002)). From the methodological point of view, if the wholesale prices provided by the suppliers are taken as *exogenous*, then the retailer is faced with a classical portfolio problem, where defaults can be optimally diversified by splitting orders among multiple suppliers. The benefits of diversification increase as the correlation between the different supplier defaults decreases. If, however, the wholesale prices charged by the suppliers are *endogenous* then the benefits from diversification for the retailer may depend on the actions of suppliers. In this case, the analogy with a classical portfolio selection problem is no longer valid and the analysis requires game theory tools rather than simple portfolio optimization tools.

When a firm uses several supply sources for the same product, new questions arise concerning the management of supplier-supplier relationships and the extent to which the firm has the power of affecting them. The firm has an incentive to intensify the competition among its suppliers, in order for the component prices to remain low. Empirical studies that have been conducted using data from automotive and other industries (see Asanuma (1985), Kamath and Liker (1994), Wu and Choi (2005)) have led to the conclusion that building long-term relationships with suppliers and enabling supplier improvement processes is usually a secondary concern compared with ensuring that suppliers are in constant competition with each other. As we will see later in the paper, it turns out that the correlation of the default processes of the suppliers that affects not only the benefits from the diversification but also the intensity of the competition.

Thus, the basic idea of the paper is that a retailer, concerned with random supply shocks, faces a tradeoff between *diversification* and *price competition* effects, both of which might depend on the correlation of supplier defaults. For example, consider two identical suppliers with correlated default processes. If their default processes were perfectly positively correlated, then, from the retailer's perspective, the two suppliers are identical and from the typical portfolio viewpoint, there is no diversification benefit. However, in this case, since the goods are completely substitutable, one might expect fierce price competition between the suppliers. Indeed, the competition from the loser holds down the price the winner can charge to the production cost and this increases the retailer's profits, allowing him to extract all the surplus from the supply chain. Now, as correlation of the supply shocks decreases, the usual portfolio viewpoint would indicate that diversification benefits should accrue. However, as correlation decreases, the goods essentially become less than perfect substitutes. This reduces the price competition, and, in equilibrium, allows room for the suppliers to profit by charging prices higher than the production cost. To see this more clearly, consider the extreme case where the supply shocks are perfectly negatively correlated in such a way that exactly one supplier will be able to deliver any order quantity, although it is uncertain which one. In this case, since the two suppliers do not coexist in the same probabilistic states of nature, they have no need to compete over prices for business from the retailer, and, acting as monopolists, they both can charge the retailer up to the full marginal value of the additional unit. Thus, the two suppliers, together, are capable of extracting all the surplus from the supply chain.

In general, then, when prices are endogenous, tradeoffs exist between diversification benefits, which improve with decreasing default correlation, and pricing effects, which improve with increasing default correlation.

Throughout this paper we will assume that the suppliers can set the wholesale prices. If the reverse allocation of bargaining power was assumed, specifically, that the retailer could always fix the supplier prices at their marginal values, then we would obtain uninteresting results, namely that the retailer could extract all the rents from the supply chain. Interestingly, we find for our setup and for the case where there are $N = 2$ suppliers who provide wholesale prices to the retailer, that the retailer would have strong preference for the supplier default processes to be highly correlated! That is, the price competition effects, induced by positive correlations, exceed the diversification benefits, that arise from lower or even negative correlations. A similar logic applies when there are more than two suppliers, although in this case the very stark tradeoff between diversification and price effects is reduced by the fact that the degree of competition among the suppliers depends on the joint codependence structure, and the retailer may be able to obtain both price and diversification benefits.

The supply chain considered in this paper is not coordinated and the greatest loss, due to the lack of coordination, is incurred by the system when supplier defaults are negatively correlated and the smallest loss is incurred when supplier defaults are positively correlated. A numerical study of the model with $N = 2$ suppliers demonstrates that despite the decreasing coordination, the system still benefits from the negative correlation between supplier defaults.

We analyze how the magnitude of disruption probabilities affects all agents in a supply chain. Not surprisingly, we find that increasing probability of disruptions hurts all firms in the chain with a single supplier, $N = 1$. Surprisingly, we found a numerical example for a model with two suppliers, $N = 2$, in which suppliers profits are non-monotone in the disruption probabilities. Thus, the suppliers could be benefiting from increasing disruptions probabilities.

In addition to determining the effects of codependence among supplier defaults on the performance of firms in a supply chain, this paper also examines the consequences of the suppliers offering different payment policies, ranging from up-front payments for the entire order quantity, to on-delivery payments where only the goods that are delivered are paid for. We prove that in equilibrium all linear payment policies are equivalent and illustrate the effect of timing of payments on the equilibrium wholesale prices using a numerical example.

The paper proceeds as follows. In section 2 we review the related literature on supply chains with disruptions. In section 3 we introduce our basic model, describe the default processes and the nature of competition. Our main results are provided in Section 4 where we focus on the effect of correlation under deterministic demand, first for the model with two suppliers, then three suppliers, and finally for the more general model with N suppliers. The insights derived from the models and their implications on the strategic behavior of the firms are carefully examined. Section 5 extends the analysis to the models where, in addition to supply uncertainty, there is demand uncertainty. Section 6 discusses the effects of the timing of payments on the equilibrium solution. Section 7 summarizes the findings.

2. Literature Review

Our work is related to the research on random yields. An excellent review of the random yield literature is offered by Yano and Lee (1995). The retailer's problem in our model with two suppliers is similar to a single-period model by Anupindi and Akella (1993) and falls into the "multiple suppliers of the same item" category of the taxonomy proposed by Yano and Lee (1995). Anupindi and Akella (1993) study one- and multi- period discrete-time problems of a retailer who can order from one or two suppliers whose failure processes are uncorrelated. The authors derive optimal ordering policies under various stochastic yield assumptions including all-or-nothing, partial recovery, and delayed delivery. Our analysis generalizes their findings in that we consider suppliers with correlated default processes and who control the wholesale prices.

The problem of a single supplier controlling wholesale price while selling to a newsvendor has been addressed by Lariviere and Porteus (2001). In our paper, we add a possibility of supplier's default to the problem in Lariviere and Porteus (2001) and focus on the effects of the supply risk on the performance of the supply-chain. We further generalize the problem in Lariviere and Porteus (2001) by considering a game with more than one supplier.

A relatively recent area of research in the supply chain field is the design of reliable logistics distribution systems (usually global systems). See, for example, Vidal and Goetschalckx (1997), Vidal and Goetschalckx (2000), Snyder and Daskin (2003), Bundschuh, Klabjan and Thurston (2003) and references therein. The terrorist attack on September 11, 2001 prompted many companies to review their supply chains for potential vulnerabilities. Both academics and practitioners have published a number of articles addressing questions of managing supply chains under the threat of disruptions (see Sheffi (2001), Rice and Caniato (2003a), Rice and Caniato (2003b), Chopra and Sodhi (2004), and Tang (2005)). Besides modeling methodology, the biggest difference between those papers and our work is our focus on the *strategic* interaction and games between suppliers and retailers and among suppliers.

One could also interpret the problem considered in this paper as a multi-supplier sourcing problem. Recent survey articles by Elmaghraby (2000) and Minner (2003) describe a variety of models proposed in a multi-supplier supply chain management literature. In the description of future research, Minner (2003) suggests that models with competing suppliers and inventory considerations due to the demand or lead time uncertainty have not been explored sufficiently yet. Our paper attempts to rectify this shortfall.

Typically, historical data on supply disruptions is sparse (particularly for disruptions caused by natural disasters). However, if we focus on the financial default events, then rather than estimating true default probabilities, it may feasible to estimate risk-neutralized probabilities. Indeed, pricing models for defaultable claims (e.g. Merton (1974), Jarrow and Turnbull (1995), Duffie and Singleton (1999), Lando (1998) and others) all require risk-neutralized processes rather than the true data-generating processes. If the suppliers are large firms that have traded equity, debt and perhaps other claims on the assets of their respective firms, then these prices contain information on the parameters of the default processes. For example, if the price of a supplier's debt falls, then this is a signal that default is more likely. The idea, then, is to use traded prices to infer parameter estimates for processes that control the well being of the firm.

Finally, similar to this work, Cachon and Zhang (2005) observe that if one considers strategic behavior of the suppliers (e.g. competition among suppliers) the traditional, optimization-based, intuition for buyer's preferences may be reversed.

3. Model Assumptions

Consider a model of a simple supply chain with one retailer and several suppliers, who produce perfectly substitutable products using technologies with identical production lead-times. Without loss of generality, assume that the common lead time is one period and that production begins at date 0 and ends at date 1. At date 0, the suppliers determine their pricing policies. The retailer responds by choosing order quantities. Thus, the suppliers compete with each other for the retailer's business, and collectively, they are Stackelberg leaders in a game with the retailer. As soon as the suppliers receive orders, they commence production. The per unit production cost for supplier i is c_i and the bulk of production costs is incurred up-front (at date 0).

At date 0 the retailer is faced with ordering decisions to satisfy the uncertain demand, D, that is realized at date 1. The cumulative distribution function of demand, $G(\cdot)$, is continuous with probability density function $g(\cdot)$. We will also consider models with deterministic demand.

We assume that suppliers are subject to random defaults. If a supplier defaults during the production cycle the entire order placed to her by the retailer is lost. The time of disruption for supplier i is a random stopping time independent of the pricing and payment policies and, in particular, of the order quantities. This assumption is justified if the default risk is attributed to exogenous events, such as labor strikes or natural disasters, or in the case of financial defaults, if the business that the retailer brings to the supplier is a small part of the supplier's full line of business activities. Further, to focus on the tradeoffs of diversification and competition we assume away all agency costs and consider a full information model. In particular, the joint default distribution as defined below is known by all agents.

Let δ_i be a binary random variable denoting the number of defaults of supplier i during the production period. The joint distribution of $\delta_1, \ldots, \delta_N$ is determined by the probabilities $p_{d_1 d_2 \ldots d_N} =$

$P[\delta_1 = d_1, \ldots, \delta_N = d_N], d_i \in \{0,1\}, i = 1, \ldots, N$. We will indicate marginal probabilities by replacing appropriate indices of $p_{d_1 d_2 \ldots d_N}$ by $*$.

Since our study focuses heavily on the role of default correlation structures in a supply chain, it is important to highlight the modeling of codependence between two, three, or more risky suppliers. There are a number of approaches to modeling the joint default distribution so as to reflect the effects of increasing codependence between supplier defaults. While Pearson's linear correlation coefficient works well as a codependence measure for models with elliptical distributions, Embrechts, McNeil and Straumann (2002) show that it might be inadequate for non-elliptic problems. Copula functions have been proposed as a modeling alternative to linear correlation by a number of authors (see, Nelsen (1999) and Embrechts, Lindskog and McNeil (2003)). However, the copula methodology has been developed for continuous distributions and the choice of the appropriate copula class is a non-trivial task. For the purposes of this paper, we will model changes in the joint default distributions, $p_{d_1 \ldots d_N}$, directly and will use the term *correlation* to denote *codependence* in general.

When there are $N = 2$ suppliers, the joint default probabilities, $p_{00}, p_{01}, p_{10}, p_{11}$, satisfy the following relationships:

$$p_{00} + p_{01} + p_{10} + p_{11} = 1 \tag{1a}$$

$$p_{11} + p_{10} = p_{1*} \tag{1b}$$

$$p_{11} + p_{01} = p_{*1} \tag{1c}$$

Using (1), the joint distribution can be completely characterized by the following three parameters, $\pi_1 = p_{1*}$, $\pi_2 = p_{*1}$, and p_{11}, i.e., the marginal default probabilities and the probability of exactly two defaults.

This parametrization allows to model the supplier correlation in a convenient way. Indeed, letting $p_{11} = \pi_1 = \pi_2$, it follows that $p_{01} = p_{10} = 0$, therefore, the defaults are perfectly positively correlated. On the other hand, when $p_{11} = 0$ and $\pi_1 + \pi_2 = 1$, then it follows that $p_{00} = 0$, $p_{01} = p_{*1}$, and $p_{10} = p_{1*}$, thus the defaults are perfectly negatively correlated.

Furthermore, by keeping π_1 and π_2 fixed and varying p_{11} we can model the effects of different correlations on the model. Specifically, as p_{11} increases (and thus p_{01} and p_{10} decrease) the default correlation increases. In order to capture the entire range from perfect negative to perfect positive correlation in this manner, it is necessary to set $\pi_1 = \pi_2 = \frac{1}{2}$ and allow p_{11} to vary between 0 and $\frac{1}{2}$.

For the case of $N = 3$ suppliers, the joint default distribution is determined by 8 probabilities: $p_{000}, p_{001}, p_{010}, p_{100}, p_{110}, p_{101}, p_{011}$, and p_{111}. These probabilities satisfy the following relationships

$$p_{000} + p_{001} + p_{010} + p_{100} + p_{110} + p_{101} + p_{011} + p_{111} = 1 \tag{2a}$$

$$p_{111} + p_{100} + p_{110} + p_{110} = p_{1**} \tag{2b}$$

$$p_{111} + p_{010} + p_{110} + p_{011} = p_{*1*} \tag{2c}$$

$$p_{111} + p_{001} + p_{101} + p_{011} = p_{**1} \tag{2d}$$

$$p_{111} + p_{011} = p_{*11} \tag{2e}$$

$$p_{111} + p_{101} = p_{1*1} \tag{2f}$$

$$p_{111} + p_{110} = p_{11*} \tag{2g}$$

We take as the data-based inputs the marginal probabilities of default of each supplier, $\pi_1 = p_{1**}$, $\pi_2 = p_{*1*}$, and $\pi_3 = p_{**1}$, together with the pairwise probabilities of default namely, p_{*11}, p_{1*1}, and p_{11*}. The remaining input is the probability of all three suppliers defaulting, p_{111}.

This parametrization of the default distribution allows us to represent a growing codependence between defaults of a specific supplier pair, for instance, supplier 2 and 3 as follows. As codependence between defaults of suppliers 2 and 3 increases, p_{*11} and p_{111} increase. The increased codependence can be accomplished without altering the marginal probabilities of default, namely, p_{1**}, p_{*1*}, p_{**1}, nor the pairwise default probabilities involving supplier 1, that is, p_{1*1} and p_{11*}. The rate of change of p_{111} depends on the codependence of defaults among all three suppliers and is typically slower than the rate of increase of p_{*11}.[1] Therefore, from system (2), we conclude that as codependence between suppliers 2 and 3 increases, p_{011} increases, and p_{101}, p_{110} decrease. We will use this fact in Section 4.2, in the analysis of the equilibrium solution.

We assume that all agents are risk-neutral, and that the riskless interest rate is r. An alternative assumption is that the supply risk arises solely from the firms' financial defaults, suppliers have debt that is publicly traded, and market is complete and arbitrage-free (hence, the bond prices reflect credit risk). In such a market, standard finance arguments guarantee the existence and the uniqueness of a pricing measure, also called risk-neutral measure, Harrison and Kreps (1979), Harrison and Pliska (1981), under which asset prices, normalized by the money fund that grows at the riskless rate r, are martingales. In such an economy, regardless of preferences, each firm in a supply chain should maximize its expected discounted profit, where expectation is taken with respect to the risk-neutral pricing measure.

The default and demand random variables are independent and the per unit retail sales price, s, is predetermined. One can think of s as the expected present value of the future random price, $S(T)$, where $S(T)$ is independent from other random variables in the model, $s = e^{-r}E[S(T)]$. We assume, for simplicity, that any unsatisfied demand is lost and any unsold goods are costlessly

[1] For example, if supplier 1 is independent from the rest, then both $p_{111} = p_{1**}p_{*11}$ and $p_{011} = p_{0**}p_{*11}$ have to increase as p_{*11} increases. If supplier 1 is perfectly positively correlated with supplier 2, then $p_{011} = 0$ and $p_{111} = p_{*11}$. If supplier 1 is perfectly negatively correlated with supplier 2, then $p_{111} = 0$ and $p_{011} = p_{*11}$. So, in the extreme correlation cases, p_{111} and p_{011} either do not change or change as much as p_{*11}.

discarded. Holding and shortage costs could be easily added to our model. However, because they do not alter the nature of our findings we omit them, again, for ease of exposition.

The payments from the retailer to the suppliers are made at date 0. Let K_i be the wholesale price charged by supplier $i = 1, \ldots, N$. One could think that at date 0 the suppliers bid for the retailer's business by quoting their unit wholesale prices. Based on those quotes, the retailer decides how to divide the business among suppliers.

The problem of the retailer, who can place orders with N suppliers, is

$$\sup_{z_1 \geq 0, z_2 \geq 0, \ldots, z_N \geq 0} \left(s\, E \left\{ \min \left[D, \sum_{i=1}^{N} (1 - \delta_i) z_i \right] \right\} - \sum_{i=1}^{N} K_i z_i \right). \tag{3}$$

The objective function in (3) is bounded, continuous, decreasing for large z_i, and concave. Therefore, problem (3) has a unique solution. Denote by $z_i(K_1, ..., K_N)$ the retailer's order quantity to supplier i. The suppliers compete with each other for the retailer's business and solve the following optimization problems

$$\sup_{K_i \geq 0} \, (K_i - c_i)\, z_i(K_1, ..., K_N), \quad i = 1, 2, ..., N. \tag{4}$$

4. The Effect of Correlation under Deterministic Demand

If the demand is known and the default risk is not present, then, if the wholesale prices quoted by the suppliers are exogenously provided, the optimal response by the retailer is to order the *full demand quantity*, D, from the *cheapest* supplier. When default risk is present, the retailer may deviate from this strategy and split the order among many suppliers. Purchasing units from multiple sources and at higher costs is viewed as a cost for providing a partial hedge against default risks. Further, in this case, even though demand is certain, since the amount received is uncertain, the total quantity ordered across all suppliers could exceed the deterministic demand, D.

For deterministic demand, the retailer's problem in (3) reduces to:

$$\max_{z_1 \geq 0, z_2 \geq 0, \ldots, z_N \geq 0} \left\{ s \sum_{d_1=0}^{1} \sum_{d_2=0}^{1} \ldots \sum_{d_N=0}^{1} \min \left(D, \sum_{i=1}^{N} (1 - d_i) z_i \right) p_{d_1 d_2 \ldots d_N} - \sum_{i=1}^{N} K_i z_i \right\}. \tag{5}$$

Problem (5) is bounded, continuous, concave and piece-wise linear in $\{z_i\}_{i=1}^{N}$. Therefore, it has a corner point solution, i.e., $z_i = 0$ or D, for $i = 1, \ldots, N$ (which need not be unique). Finding this solution requires a simple numerical calculation, and for small values of N, as will be presented below, one can provide analytical expressions for it.

If the wholesale price, K_i, quoted by any supplier is greater than the expected benefit from ordering from that supplier alone, $s(1-\pi_i)$, the retailer will not order from this supplier. Therefore, in the analysis we should consider only the suppliers that satisfy

$$s(1-\pi_i) \geq K_i. \tag{6}$$

In particular we will assume that, for any i, $s(1-\pi_i) \geq c_i$. This is reasonable because in most applications π_i is small.

4.1 Two suppliers

The solution to the retailer's problem is described in the following proposition.

Proposition 1. *The retailer's optimal purchase quantities (z_1^*, z_2^*) from the two risky suppliers, given that their per unit charges are K_1 and K_2 respectively, is given by:*

$$(z_1^*, z_2^*) = \begin{cases} (D, D) & \text{if } K_1 \leq sp_{01} \text{ and } K_2 \leq sp_{10} \\ (0, D) & \text{if } sp_{01} < K_1 \text{ and } sp_{01} - K_1 < sp_{10} - K_2 \\ (D, 0) & \text{if } sp_{10} < K_2 \text{ and } sp_{01} - K_1 > sp_{10} - K_2 \\ (\alpha D, (1-\alpha)D) \quad 0 \leq \alpha \leq 1 & \text{if } K_1 > sp_{01},\ K_2 > sp_{10},\text{ and } sp_{01} - K_1 = sp_{10} - K_2 \end{cases}$$

Proof. See Appendix ■

Figure 1 provides a graphical representation of the retailer's response described in Proposition 1.

As the correlation between supplier defaults increases, the probabilities that only one supplier delivers the order, p_{01} and p_{10}, decrease, and the probability that both suppliers deliver orders, p_{00}, increases. Therefore, the region where the retailer orders from both suppliers shrinks. Consequently, the optimal order quantities to each supplier are nonincreasing in the default correlation.

If prices, $\{K_i\}_{i=1}^2$, are exogenously fixed, then the intuition regarding the role of diversification applies, with the retailer benefiting from the negative correlation of supplier defaults.

Specifically, as the correlation decreases, the potential diversification benefits increase and the retailer decides to order from two suppliers. The additional cost paid by the retailer for the higher total quantity is offset by the benefits of the hedge against default risk. As the correlation continues to decrease, the total order quantity and the cost of the order remain unchanged but the diversification benefits for the retailer continue to accrue.

This discussion, however, ignores the competition effects and changing wholesale prices. We now examine the effects of correlation in the presence of competition.

The suppliers compete by selecting wholesale prices, K_i, that maximize their discounted expected profits as given in equation (3). Based on the retailer's response function the solution to the game between the suppliers is given in the following proposition.

Proposition 2. *Suppose that $sp_{01} - c_1 \geq sp_{10} - c_2$.*

(i) *The equilibrium solution to the game between suppliers is unique and:*

$$(K_1^*, K_2^*) = \begin{cases} (sp_{01}, sp_{10}) & \text{if } sp_{10} - c_2 \geq 0 \\ (sp_{01} + c_2 - sp_{10}, c_2) & \text{if } sp_{10} - c_2 < 0 \end{cases} \tag{7}$$

(ii) *The retailer's order quantities are:*

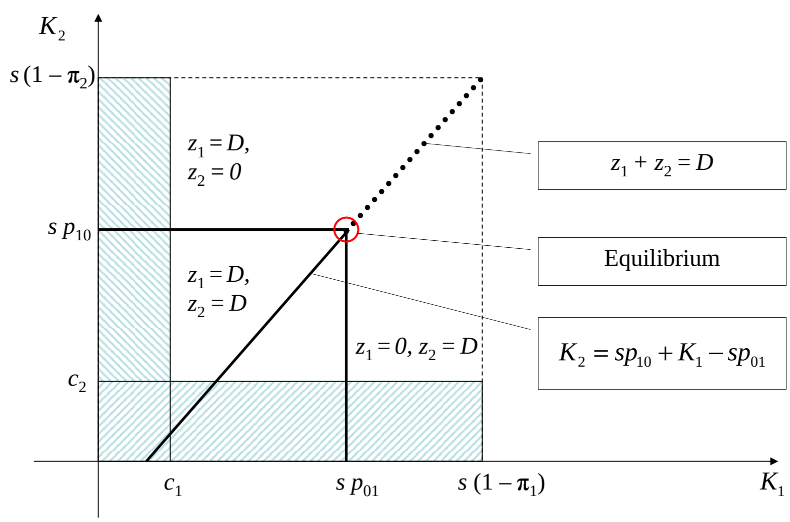
$$(z_1^*, z_2^*) = \begin{cases} (D, D) & \text{if } sp_{10} - c_2 \geq 0 \\ (D, 0) & \text{if } sp_{10} - c_2 < 0 \end{cases} \tag{8}$$

Proof. See Appendix ■

The quantities $sp_{01} - c_1$ and $sp_{10} - c_2$ represent the profit margins of the suppliers if both are working with the retailer. Since suppliers can be reordered in increasing value of this index, the condition $sp_{01} - c_1 \geq sp_{10} - c_2$ can be assumed without loss of generality.

Figure 1 shows the unique equilibrium solution described in Proposition 2.

Figure 1: Retailer's response function and equilibrium solution to the game between suppliers when demand is deterministic.



As the correlation between supplier defaults increases, the equilibrium wholesale prices decrease. To understand this result one can think of the default as being one of the attributes of the product offered by the suppliers. When supplier defaults are perfectly correlated, the goods the suppliers sell are *perfect* substitutes and the force of competition between suppliers drives the wholesale prices down to the production costs (this is an example of Bertrand competition). As the defaults become less correlated, the goods offered by suppliers become *less* substitutable, and the competition is less effective in holding the prices down. Ultimately, when supplier defaults are perfectly negatively correlated, the suppliers deliver goods in different probabilistic states of nature and do not compete at all. As a result, the lower the correlation, the weaker the price competition, and the higher the wholesale prices. This effect has to be traded off against the diversification benefits afforded by lower correlations.

It will be convenient to use the following compact notation for the equilibrium solution

$$(K_1^*, K_2^*) = (sp_{01} + (c_2 - sp_{10})^+, sp_{10} + (c_2 - sp_{10})^+) \tag{9}$$

$$(z_1^*, z_2^*) = (D, D \cdot 1_{\{sp_{10} > c_2\}}) \tag{10}$$

Using expressions for the equilibrium prices (9) and order quantities (10), the equilibrium retailer's (R^*), suppliers' (S_1^* and S_2^*), uncoordinated system's (U^*), and coordinated system's (C^*) profits are

$$R^* = sp_{00}D - (c_2 - sp_{10})^+ D, \tag{11a}$$

$$S_1^* = (sp_{01} - c_1)\, D + (c_2 - sp_{10})^+ D, \tag{11b}$$

$$S_2^* = (sp_{10} - c_2)\, D + (c_2 - sp_{10})^+ D, \tag{11c}$$

$$U^* = C^* = R^* + S_1^* + S_2^* = s(1 - p_{11})D - c_1 D - c_2 D + (c_2 - sp_{10})^+ D. \tag{11d}$$

By uncoordinated system we mean the system where all firms act independently. By coordinated system we mean the system with a single decision maker — retailer. Because order quantities for the coordinated and uncoordinated system are the same when demand is deterministic, the equilibrium profits for both systems are equal. Contracts between the suppliers and the retailer serve only to divide the total system profits.

We can now state the following result, the proof of which is immediate using the profit expressions in (11)

Theorem 1. *As the default correlation between the two suppliers increases, the supply chain profit and both supplier's profits are non-increasing, while the profit of the retailer is non-decreasing.*

Perhaps the most surprising statement in Theorem 1 is that, all things being equal, the retailer would prefer that the suppliers have highly positively correlated default processes. This is contradictory to the intuition built only on diversification considerations. With competition, positive correlation between defaults leads to lower wholesale prices, compensating the retailer for the loss of diversification benefits. Overall, the benefits of competition outweigh the benefits of diversification, and the retailer has a preference for suppliers with high default correlations.

Conversely, all things being equal, each supplier would prefer that their competitor have a default process that is highly negatively correlated with their own default processes. When defaults are perfectly negatively correlated there is no competition between the suppliers. Indeed, in this extreme case, whenever one supplier is able to deliver, then the other supplier will have defaulted. Hence, each supplier behaves as a monopolist, extracting all of the system profits.

If it were feasible, the two suppliers would benefit by decreasing their default correlation. Depending on the causes of defaults, the correlation between them can be reduced by using different production technologies, different raw materials sources, by placing production facilities in different parts of the country or in different countries.

Finally, note that the supply chain profit increases as the correlation of defaults decreases. Therefore, what is good for the supplier is also good for the channel, but detrimental for the retailer.

4.2 Three Suppliers

How do the results change when there are more than two potential suppliers? In this case, possibilities exist for the retailer to profit from the fierce price competition between two suppliers that might have highly correlated default processes, and, at the same time, benefit from diversification, if the default process of the third supplier is negatively correlated with the other two. That is, rather than diversification and price competition working in different directions, with three suppliers, the two effects may be more complementary. We explore this possibility.

The following proposition provides sufficient conditions for the equilibrium solutions to involve all three, two, or just one supplier.

Proposition 3.

(i) *If* $sp_{011} - c_1 > sp_{101} - c_2 > sp_{110} - c_3 \geq 0$*, then there exists a three-supplier equilibrium with prices* $(K_1^*, K_2^*, K_3^*) = (sp_{011}, sp_{101}, sp_{110})$ *and corresponding order quantities* $(z_1^*, z_2^*, z_3^*) = (D, D, D)$*.*

(ii) *If* $c_1 \leq sp_{01*}$*,* $c_2 \leq sp_{10*}$*, and* $c_3 > \max(sp_{*10}, sp_{1*0}, sp_{*10} + sp_{100} - sp_{001})$*, then there exists a two-supplier equilibrium with prices* $(K_1^*, K_2^*, K_3^*) = (sp_{01*}, sp_{10*}, c_3)$ *and corresponding order quantities* $(z_1^*, z_2^*, z_3^*) = (D, D, 0)$*.*

(iii) *Let*

$$\widehat{K}_1 = \min\left(sp_{01*} + c_2 - sp_{10*}, sp_{0*1} + c_3 - sp_{1*0}, c_2 - sp_{101} + c_3 - sp_{110} + sp_{011} - sp_{100}\right)$$

If $c_2 > sp_{10*}$*,* $c_3 > sp_{1*0}$*,* $c_1 \leq \widehat{K}_1$*, then there exists a one-supplier equilibrium with prices* $(K_1^*, K_2^*, K_3^*) = (\widehat{K}_1, c_2, c_3)$ *and corresponding order quantities* $(z_1^*, z_2^*, z_3^*) = (D, 0, 0)$*.*

Proof. See Appendix. ■

Using expressions for the equilibrium prices from Proposition 3, we can examine the effects of default correlation on wholesale prices in the supply chain.

For case (i) as the correlation between defaults of a pair of supplier, say, suppliers 2 and 3, increases, equilibrium prices K_2^* and K_3^* decrease and K_1^* increases. Intuitively, as the benefits of diversification due to the increasing correlation between suppliers 2 and 3 defaults are decreasing, the retailer is willing to pay more for the benefits of diversification provided by supplier 1.

For case (ii) the equilibrium wholesale prices do not depend on the default correlation between suppliers 2 and 3 or 1 and 3. In this case the cost structure of supplier 3 is too high for the retailer to obtain any diversification benefit by using her. In fact, supplier 3 is so expensive, that her existence does not threaten the other two suppliers.

Finally, for case (iii), the cost structure for supplier 2 and 3 are both too high, relative to supplier 1, for the retailer to use either of them. Interestingly, the price of supplier 1 may depend on the magnitude of the default correlation between the other two suppliers who do not participate.

Using expressions for the equilibrium prices and order quantities from case (i) of Proposition 3, one can derive the equilibrium profit for the retailer (R^*), the suppliers (S_1^*, S_2^*, S_3^*), the uncoordinated system (U^*), and the coordinated system (C^*) as follows:

$$R^* = sD(1 - p_{111} - p_{011} - p_{101} - p_{110}), \tag{12a}$$

$$S_1^* = (sp_{011} - c_1)\, D, \tag{12b}$$

$$S_2^* = (sp_{101} - c_2)\, D, \tag{12c}$$

$$S_3^* = (sp_{110} - c_3)\, D, \tag{12d}$$

$$U^* = C^* = sD(1 - p_{111}) - c_1 D - c_2 D - c_3 D. \tag{12e}$$

Changes in suppliers' profits (S_1^*, S_2^*, and S_3^*) are driven by changes in wholesale prices ($K_1^* = sp_{011}$, $K_2^* = sp_{101}$, $K_3^* = sp_{110}$) and, therefore, as the correlation between defaults of any pair of suppliers increases, the profits of this pair decrease (due to falling wholesale prices), while the profit of the other supplier increases (due to increasing wholesale prices).

The system profits (both for uncoordinated and coordinated systems, $U^* = C^*$) decrease as the correlation of supplier defaults increases.

Recall that, for the two supplier model, the retailer benefited from rising default correlation because equilibrium wholesale prices fell. For the three supplier model, rising correlations between one pair of suppliers may not result in lowering of all wholesale prices. Indeed, it may be the case that the wholesale price charged by the third supplier will increase. Further, this increase in price will arise when the supplier becomes less correlated with the other two. This supplier is able to increase prices, because she recognizes that she offers the retailer a better hedge against defaults. Using the analogy with multi-attribute product competition, the product of the third supplier becomes less substitutable with the products of the other two, therefore the supplier can begin to act more like a monopolist.

To obtain further insights, assume that the default of supplier 1 is independent of the defaults of suppliers 2 and 3. Then, the retailer's profit reduces to:

$$R^* = sD(1 - p_{111} - p_{011} - p_{101} - p_{110}) = sD\left[1 - p_{*11}(1 - 2\pi_1) - \pi_1(\pi_2 + \pi_3)\right]. \tag{13}$$

As the correlation between defaults of supplier 2 and 3 increases, p_{*11} increases and the retailer's profit increases if $\pi_1 > \frac{1}{2}$ and decreases if $\pi_1 < \frac{1}{2}$. If supplier 1 is fairly reliable, then the retailer benefits from the lowered wholesale prices of suppliers 2 and 3 more than it loses from increasing price of supplier 1 and decreased benefits of diversification. If supplier 1 is unreliable, then the retailer suffers from the increasing default correlation between suppliers 2 and 3. The following Theorem summarizes the above discussion.

Theorem 2. *Assume that the conditions of case (i) of Proposition 3 hold where all three suppliers participate. Then, as the correlation between defaults of suppliers 2 and 3 increases:*

(i) *The supply chain profit, $U^* = C^*$, decreases.*

(ii) *The suppliers' profits, S_2^* and S_3^* decrease, while S_1^* increases.*

(iii) *If the default of supplier 1 is independent of the defaults of suppliers 2 and 3, then the retailer's profit increases if $\pi_1 > \frac{1}{2}$ and decreases if $\pi_1 < \frac{1}{2}$.*

For case (ii) of Proposition 3, where two suppliers participate, the effects of correlation between defaults are the same as those described in Theorem 1. The final case, case (iii) of Proposition 3, where only one supplier participates is interesting in that the equilibrium prices may depend on the default correlations. As a result, the profits of the retailer and participating supplier also depend on default correlations as well. The system profit, however,

$$U^* = C^* = sD(1 - \pi_1) - c_1 D, \tag{14}$$

does not depend on default correlations.

4.3 $N > 3$ suppliers

The effects of competition and the benefits of diversification obtained by the retailer when considering three suppliers could be quite different from the case where there are only two suppliers competing for business. When moving from 3 to $N > 3$ suppliers, the analysis becomes even more complex. However, the overall direction of the results does not change. That is, competition among highly correlated suppliers tends to hold prices down, which is good for the retailer, while having negative correlations among some suppliers may also be good because it allows the retailer to diversify the default risk.

The following proposition describes an equilibrium for the N supplier model where all suppliers participate.

Proposition 4. *If $sp_{01...1} - c_1 > sp_{101..1} - c_2 > ... > sp_{11..10} - c_N \geq 0$, then there exists an N-supplier equilibrium with prices $(K_1^*, K_2^*, ..., K_N^*) = (sp_{01...1}, sp_{101..1}, sp_{11..10})$ and corresponding order quantities $(z_1^*, z_2^*, ..., z_N^*) = (D, D, ..., D)$.*

Using expressions for the equilibrium prices and order quantities from Proposition 4, one can derive the equilibrium profits for the retailer, the suppliers the system and the coordinated system:

$$R^* = sD(1 - p_{11...1} - \sum_{i=1}^{N} \bar{p}_i), \tag{15a}$$

$$S_i^* = (s\bar{p}_i - c_i)\, D, \text{ for } i = 1, ..., N \tag{15b}$$

$$U^* = C^* = sD(1 - p_{11...1}) - \sum_{i=1}^{N} c_i D, \tag{15c}$$

where $\bar{p}_i) = P[\delta_i = 0, \delta_j = 1, j \neq i] = p_{1...101...1}$, with 0 at index i, is the probability of all suppliers except i defaulting.

As we have seen already, if correlation between any pair of suppliers increases, equilibrium prices and profits of those suppliers decrease, while equilibrium prices and profits of other suppliers may increase. The system benefits from negative correlation among suppliers. However, the effect of default correlations on equilibrium retailer's profits is not clear.

While one could, in principle, compute equilibria for the N-supplier model, where fewer than N suppliers are participating, the task of identifying conditions that would guarantee an equilibrium becomes tedious as N increases. Instead, we will consider some special cases that clarify the role of competition and default correlation.

First, note that the retailer can only benefit from more suppliers in the market. More suppliers means more competition and lower prices. Specifically, if an entering supplier changes the equilibrium from an N-supplier equilibrium to an $N+1$-supplier equilibrium, the prices from the existing suppliers drop from $sp_{1...101...1*}$ to $sp_{1...101...11}$.

Second, suppose that suppliers can be divided into M groups, each group containing at least two suppliers, with supplier defaults within each group perfectly correlated. Then competition within a group will keep prices fixed at the second lowest production cost within each group and the retailer will be facing a problem with M suppliers and fixed prices. Thus, whereas for a few suppliers, the retailer prefers the case where default correlations are high, as the number of suppliers in the economy grows, competition in each of the M groups helps to fix their prices and the problem is transformed to the one where traditional diversification arguments are applicable.

5. Stochastic Demand

So far we have considered the case where supply is uncertain and demand is certain. We now extend the analysis to the case where both supply and demand are uncertain. In particular, in this section we assume that the demand is a random variable following an absolutely continuous distribution with distribution function $G(z)$, and density $g(z)$. The generalized failure rate (GFR) is defined as

$$h(z) = zg(z)/\overline{G}(z),$$

where $\overline{G}(z) = 1 - G(z)$. The retailer's and suppliers' optimization problem are given by equations (3) and (4). We first consider a single supplier model which will furnish us with the benchmark expressions for the subsequent analysis on many suppliers.

5.1 One supplier

Assume there exists a single supplier with default probability $p_1 = \pi$. The following Lemma describes the solution of the retailer's problem when the wholesale price is fixed, which is a slight modification of a newsvendor problem.

Lemma 1. *Given wholesale price K, the optimal order quantity, z, for the problem in (3) satisfies:*

$$s(1-\pi)\overline{G}(z) = K \tag{16}$$

Next, consider the problem where the wholesale price is set by the supplier. This problem is equivalent to the problem studied in Lariviere and Porteus (2001) with sales revenues given by $s(1-\pi)$. Observe that using the one-to-one relationship between wholesale prices and the optimal order quantity implied by (16), one can transform problem (4) to an equivalent problem where the supplier selects the order quantity:

$$\max_z \left[s(1-\pi)\overline{G}(z) - c\right] z. \tag{17}$$

The first order condition for (17) is

$$s(1-\pi)\left[\overline{G}(z^*) - g(z^*)z^*\right] = c. \tag{18}$$

The following lemma, which follows from Theorem 1 in Lariviere and Porteus (2001), describes the equilibrium solution of the Stackelberg game between the supplier and the retailer.

Lemma 2. *If the demand distribution has finite mean, support on $[a, b)$, and increasing generalized failure rate $h(z)$, then equation (18) has a unique solution z^* and the supplier optimal order quantity is either z^* or a.*

The following Proposition describes the effect of the default probability π on the equilibrium quantities

Proposition 5. *Under the assumptions of Lemma 2, the equilibrium order quantity, z^*, supplier's profit, S^*, and retailer's profit, R^* are decreasing in the default probability π.*

5.2 Two suppliers

The following proposition summarizes the solution of the retailer's problem with stochastic demand.

Proposition 6. *Given wholesale prices (K_1, K_2), the optimal order quantities, (z_1, z_2), for the problem in (3) satisfy the following:*

$$\text{If } \begin{cases} K_2 > \frac{p_{00}}{1-\pi_1} K_1 + s p_{10}, \\ K_1 \leq s(1-\pi_1) \end{cases} \quad \text{then } \begin{cases} s(1-\pi_1)\overline{G}(z_1) = K_1 \\ z_2 = 0. \end{cases} \tag{19a}$$

$$\text{If } \begin{cases} K_1 > \frac{p_{00}}{1-\pi_2} K_2 + s p_{01}, \\ K_2 \leq s(1-\pi_2) \end{cases} \quad \text{then } \begin{cases} z_1 = 0 \\ s(1-\pi_2)\overline{G}(z_2) = K_2. \end{cases} \tag{19b}$$

$$\text{If } \begin{cases} K_1 \leq \frac{p_{00}}{1-\pi_2} K_2 + s p_{01}, \\ K_2 \leq \frac{p_{00}}{1-\pi_1} K_1 + s p_{10} \end{cases} \quad \text{then } \begin{cases} s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] = K_1 \\ s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K_2. \end{cases} \tag{19c}$$

$$\text{Otherwise} \quad \begin{cases} z_1 = 0 \\ z_2 = 0. \end{cases} \tag{19d}$$

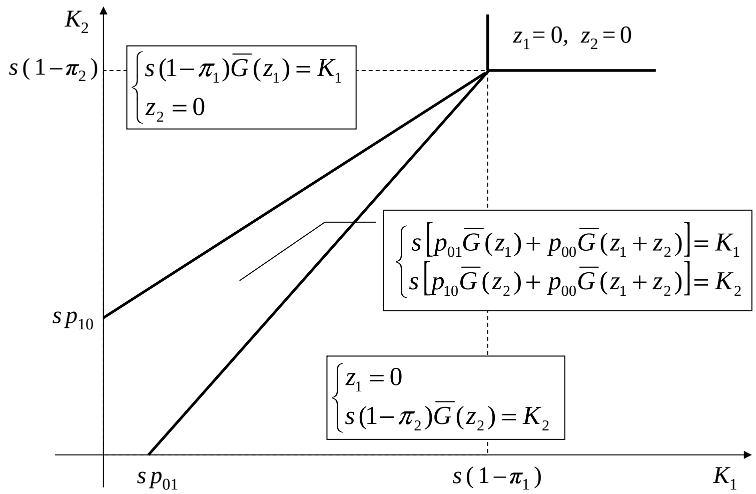
Proof. See Appendix ∎

When both wholesale prices are too high, the retailer's response is to place no orders. If wholesale prices are reasonable and the difference between them is significant (as in (19a) and (19b)), the retailer only places an order with one supplier. Finally, if the difference between wholesale prices is small (as in (19c)), the retailer orders from both suppliers. The threshold levels depend on the joint probability of two defaults, the marginal default probabilities, and on the expected retail price. The actual order amounts placed by the retailer also depend on the demand distribution.

Figure 2 provides a graphical representation of the retailer's response function described in Proposition 6.

When one of the suppliers is priced out (equations (19a) and (19b)) the order quantity with the other supplier turns out to be exactly what one would obtain using the single supplier model. Thus, the default correlation structure plays no role in determining the single order quantity.

Figure 2: Retailer's order quantities as function of wholesale prices $K_i, i = 1, 2$. Stochastic demand.



When both suppliers participate [equation (19c)], in general, one has to solve the system of equations that determine the order quantities numerically. However, there are a few important special cases for which analytical results are available.

First, consider the case of perfectly negatively correlated defaults ($p_{00} = p_{11} = 0$, $p_{01} = \pi_2 = 1 - \pi_1$, $p_{10} = \pi_1 = 1 - \pi_2$). In Figure 2, the wedge-shaped region of the shared retailer's business stretches to fill the entire rectangle $[0, s(1-\pi_1)] \times [0, s(1-\pi_2)]$. Because $p_{00} = 0$, the system of equations (19c) becomes separable with optimal order quantities, $z_i, i = 1, 2$, depending only on the corresponding values of $K_i, i = 1, 2$. Thus, the retailer orders from each supplier as if that supplier was the only one available.

Second, consider the case of perfectly positively correlated supplier defaults ($p_{01} = p_{10} = 0$, $p_{00} = 1 - \pi$). The wedge-shaped region of the shared retailer's business in Figure 2 shrinks to a line $K_2 = K_1$ and the supplier who charges a lower price is the only one who receives orders.

Last, consider the case when suppliers are identical and the problem is symmetric ($\pi_1 = \pi_2$, $p_{01} = p_{10}$, $K_1 = K_2$). In this case the retailer will order identical amounts from each supplier ($z_1 = z_2 = z$). For this case, the following proposition describes how the optimal total order quantity depends on the default correlation.

Proposition 7. *Suppose that the suppliers are identical, $\pi_1 = \pi_2 = \pi$, the problem is symmetric, $p_{01} = p_{10}$, and wholesale prices are fixed, $K_1 = K_2 = K$. Then, the optimal order quantity to each supplier, z^*, and the optimal total order quantity, $2z^*$, are non-increasing in p_{00}.*

Proof. See Appendix ■

Note that the problem where the retailer controls wholesale prices could be readily solved using results presented here. The retailer's optimal policy would be to lower wholesale prices to production costs ($K_i = c_i$) and then order according to Proposition 6 (or Proposition 1 if demand, D, is deterministic). However, the problem with the suppliers determining wholesale prices, which is the focus of this paper, is significantly more complicated.

The suppliers maximize their discounted expected profits given by equation (4). Since the retailer will not use supplier i if the wholesale price, K_i, exceeds the expected benefit, $(1-\pi_i)s, i = 1, 2$, we consider pricing policies restricted in the rectangle $[0, (1-\pi_1)s] \times [0, (1-\pi_2)s]$.

Unfortunately, it is difficult to prove analytically that there exists a pure-strategy equilibrium for the game between suppliers. The game is not supermodular, therefore, the results in Topkis (1998) cannot be applied. It is also difficult to produce parsimonious conditions that would ensure quasi-concavity and continuity of the suppliers' profit functions (even though we can verify these properties numerically for particular values of problem parameters). Therefore, we cannot invoke results from Debreu (1952). The following lemma furnishes us with conditions that would ensure that the supplier profit functions are continuous. Define z_k^{mon} to be an equilibrium solution when supplier k is a monopolist (equation (18))

Lemma 3. *Suppose that $p_{10} > 0$ and $p_{01} > 0$. If for all $(z_1, z_2) \in [0, z_1^{mon}] \times [0, z_2^{mon}]$,*

$$p_{01}p_{10}g(z_1)g(z_2) + p_{00}g(z_1 + z_2)\left[p_{01}g(z_1) + p_{10}g(z_2)\right] > 0, \tag{20}$$

then, for any supplier i, the optimal order quantity $z_i(K_i, K_{-i})$ is a continuous function of K_i for a fixed wholesale price of the other supplier K_{-i}.

Proof. See Appendix ■

Condition (20) is the requirement for the determinant of the Jacobian corresponding to the system (19c) to be non-zero. For any demand distribution whose density function does not vanish on the interval $[0, z^{mon}]$ (e.g. normal, exponential, gamma), condition (20) is satisfied and the conclusions of Lemma 3 hold.

The following proposition provides necessary conditions for the equilibrium where both suppliers receive orders.

Proposition 8. *If there exists a pure-strategy equilibrium in the game between suppliers in which both suppliers receive orders, then*

(i) *If supplier 1 controls wholesale price K_1 while the wholesale price K_2 is fixed, then the equilibrium order quantities (z_1^*, z_2^*) satisfy*

$$\begin{cases} p_{00}\overline{G}(z_1+z_2) + p_{01}\overline{G}(z_1) - p_{00}g(z_1+z_2)z_1 - p_{01}g(z_1)z_1 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_1}{p_{00}g(z_1+z_2)+p_{10}g(z_2)} = \frac{c_1}{s}, \\ p_{00}\overline{G}(z_1+z_2) + p_{10}\overline{G}(z_2) = K_2. \end{cases} \tag{21}$$

The equilibrium price K_1^* is

$$K_1^* = p_{00}\overline{G}(z_1^*+z_2^*) + p_{01}\overline{G}(z_1^*) \tag{22}$$

(ii) *If both suppliers control wholesale prices, then the equilibrium order quantities, (z_1^*, z_2^*), satisfy*

$$\begin{cases} p_{00}\overline{G}(z_1+z_2) + p_{01}\overline{G}(z_1) - p_{00}g(z_1+z_2)z_1 - p_{01}g(z_1)z_1 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_1}{p_{00}g(z_1+z_2)+p_{10}g(z_2)} = \frac{c_1}{s}, \\ p_{00}\overline{G}(z_1+z_2) + p_{10}\overline{G}(z_2) - p_{00}g(z_1+z_2)z_2 - p_{10}g(z_2)z_2 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_2}{p_{00}g(z_1+z_2)+p_{01}g(z_1)} = \frac{c_2}{s}. \end{cases} \tag{23}$$

The equilibrium wholesale prices are

$$\begin{cases} K_1^* = s\left[p_{01}\overline{G}(z_1^*) + p_{00}\overline{G}(z_1^*+z_2^*)\right], \\ K_2^* = s\left[p_{10}\overline{G}(z_2^*) + p_{00}\overline{G}(z_1^*+z_2^*)\right]. \end{cases} \tag{24}$$

Proof. See Appendix ∎

Corollary 1. *If the suppliers are identical ($\pi_1 = \pi_2 = \pi$, $c_1 = c_2 = c$, $p_{10} = p_{01}$) and if there exists a symmetric pure-strategy equilibrium, then the equilibrium order quantities, $z_1^* = z_2^* = z^*$, satisfy*

$$p_{00}\overline{G}(2z) + p_{01}\overline{G}(z) - p_{00}g(2z)z - p_{01}g(z)z + \frac{p_{00}^2 g^2(2z)z}{p_{00}g(2z) + p_{01}g(z)} = \frac{c}{s}, \tag{25}$$

and the common equilibrium wholesale price is:

$$K_1^* = K_2^* = s\left[p_{01}\overline{G}(z^*) + p_{00}\overline{G}(2z^*)\right]. \tag{26}$$

Proposition 8 and Corollary 1 suggest that as long as there exists a two supplier equilibrium and system of equations (19c) is invertible we can first solve for the equilibrium order quantities and then compute corresponding equilibrium prices.

Notice that, using (24), equation (23) can be rewritten as follows:

$$\begin{cases} K_1^* - c_1 = sz_1\left[p_{00}g(z_1+z_2) + p_{01}g(z_1) - \frac{p_{00}^2 g^2(z_1+z_2)}{p_{00}g(z_1+z_2)+p_{10}g(z_2)}\right] \geq 0, \\ K_2^* - c_2 = sz_2\left[p_{00}g(z_1+z_2) + p_{10}g(z_2) - \frac{p_{00}^2 g^2(z_1+z_2)}{p_{00}g(z_1+z_2)+p_{01}g(z_1)}\right] \geq 0. \end{cases} \tag{27}$$

Therefore, the values for suggested equilibrium prices are feasible for the suppliers.

We now consider two special cases. First, if the supplier defaults are perfectly negatively correlated ($p_{00} = 0$, $p_{01} = \pi_2 = 1 - \pi_1$, $p_{10} = \pi_1 = 1 - \pi_2$), then the retailer orders from each supplier as she was the only one available, and the problem (23) decomposes into two single supplier games where:

$$\begin{cases} p_{01}\overline{G}(z_1) - p_{01}g(z_1)z_1 = \frac{c_1}{s}, \\ p_{10}\overline{G}(z_2) - p_{10}g(z_2)z_2 = \frac{c_2}{s}. \end{cases} \tag{28}$$

Hence, in this case, each supplier charges her monopolist price, the retailer responds with the monopolist order quantity and each supplier earns monopolist profits.

Second, if the supplier defaults are perfectly positively correlated and suppliers are identical ($p_{01} = p_{10} = 0$, $p_{00} = 1 - \pi_1 = 1 - \pi_2$, $\pi_1 = \pi_2 = \pi$, $c_1 = c_2 = c$), then system (23) is simplified into

$$(1 - \pi)\overline{G}(z_1 + z_2) = \frac{c}{s}, \tag{29}$$

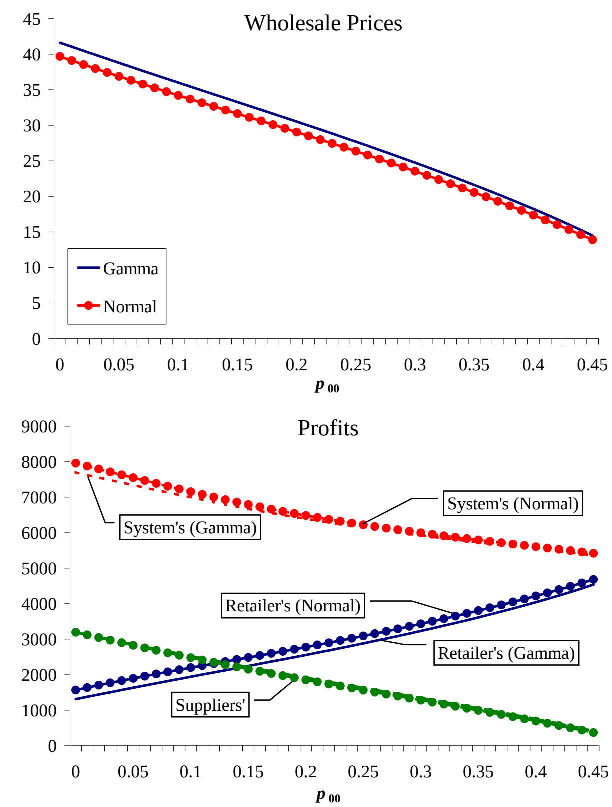
which describes the order quantity in a model with a single supplier and fixed wholesale prices. In particular, the suppliers' problem turns into a classical Bertrand competition game , where the winner is the supplier with the lowest production cost. In this case, because production costs are assumed to be equal, the orders can be arbitrarily allocated between suppliers who earn zero expected profits.

Thus, comparing cases with perfectly positive and perfectly negative default correlation, we observe that, similar to the deterministic demand case (Section 4.1), the suppliers prefer negative default correlation.

To study the supply chain performance at the intermediate values of the default correlation, we resort to numerical analysis. In the numerical examples we would like to capture the entire range of default correlation from perfect negative to perfect positive. To achieve this, as discussed in Section 3, we must set $\pi_1 = \pi_2 = \pi = \frac{1}{2}$. Although, in practice, one would expect higher survival probabilities than $\frac{1}{2}$ (hence, in practice defaults cannot be perfectly negatively correlated), the insights obtained from these numerical examples remain valid for more realistic parameter values. Because the graphs for more realistic values of π look similar, we do not present them in the paper. Instead, in the following example we explore the effect of the survival probability ($\overline{\pi} = 1 - \pi$) on equilibrium quantities.

Example 1. Consider a model with two identical suppliers. First, we will compare results for two demand distributions: Normal and Gamma. We assume that in both cases the demand distribution has a mean of 150 units and a standard deviation of 50 units. The expected retail price is $s = 100$, the common per unit cost of production $c_1 = c_2 = c = 10$, and the marginal probabilities of default $\pi_1 = \pi_2 = \frac{1}{2}$.

Figure 3: Equilibrium prices, equilibrium order quantities, and equilibrium profits. Comparison between Gamma and Normal distributions of demand. Demand means are 150, standard deviations are 50. Other parameters: $s = 100$, $c_1 = c_2 = c = 10$, $\pi_1 = \pi_2 = \frac{1}{2}$



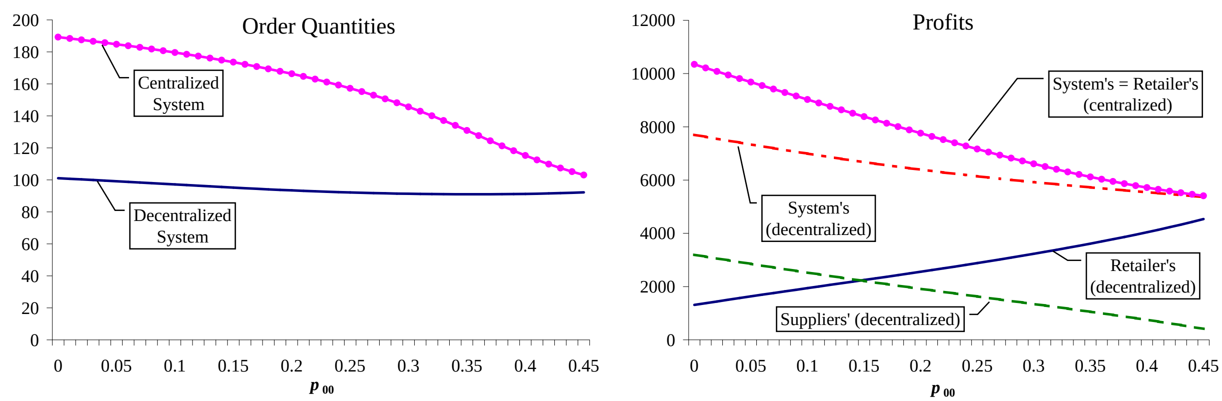
The top left panel of Figure 3 shows that the equilibrium wholesale price decreases with correlation. In particular as p_{00} increases the wholesale price drops from a high near 40 to a low near 10[2]. The top right panel shows the order quantities. Notice that the relationship is not monotonic. The bottom left panel shows the behavior of profits for the retailer, for the suppliers and for the overall system. Just as in the deterministic demand case, the equilibrium retailer's profit is increasing in p_{00}, while the equilibrium supplier and system profits are decreasing in p_{00}. Similar behavior was observed with other distribution parameters and for other distributions (e.g. exponential).

Next, let's contrast ordering decisions, profits of the centralized system with those of the decentralized system. For this case we only present the results for the gamma distribution. This paper deals with the decentralized supply chains. In the framework of our discussion one could think that the centralized system is the system where the retailer buys parts from the suppliers at prices $K_i = c_i, i = 1, 2$. As the left panel of Figure 4 demonstrates, the order quantities in the

[2] In practice we would not expect to observe such low values of the wholesale prices, however, these values are typical for the Bertrand competition models as we discuss in Section 7.

centralized system are greater than order quantities in the decentralized system. This relationship is to be expected because the decision maker (which we can also call the retailer) in the centralized system pays lower prices ($c_i, i = 1, 2$) for the inputs than what the retailer pays ($K_i^*, i = 1, 2$) in the decentralized system (see inequalities (27)). The right panel of Figure 4 shows that, because the decentralized system is not coordinated, the overall system profits could be improved if the coordination is imposed (e.g. with the coordinating contracts). Observe that, as the correlation between supplier defaults increases, the loss of profits in the supply chain due to the lack of coordination decreases. With perfect positive correlation between supplier defaults, the entire system profit is given to the retailer and the decentralized system is coordinated. Despite improvements in coordination, however, the total decentralized system profits still decline as the supplier defaults become more correlated.

Figure 4: Centralized vs. decentralized systems. Left panel: order quantities. Right panel: profits. The distribution of demand is Gamma with mean of 150 and standard deviation of 50. Other parameters: $s = 100$, $c_1 = c_2 = c = 10$, $\pi_1 = \pi_2 = \frac{1}{2}$



Next, let's consider the effect of survival probability on the equilibrium solution. A priori, based on what we know about the behavior of the single supplier system (see Proposition 5) we would expect that an increasing survival probability would benefit the retailer, the suppliers, and the system. This intuition turns out to be correct for the extreme value of the default correlation. If supplier defaults are perfectly positively correlated, then, as we discussed earlier (see discussion for equation (29)), the suppliers will earn zero profits, and the retailer will capture the entire system profits. One can prove that the retailer's (system's) profit is increasing in the suppliers' survival probability. If supplier defaults are perfectly negatively correlated, then the problem decomposes into two single supplier games (see discussion for equation (28)). For each of the single supplier games, it follows from Proposition 5 that the retailer's the supplier's and the system's profits are increasing in the supplier survival probability.

Surprisingly, the benefits from an increasing survival probability might not be there for the suppliers for the intermediate values of the default correlation. Assume that supplier defaults are independent. On the top left panel of Figure 5 we observe that the equilibrium wholesale prices (K^*) are non-monotone in the survival probability ($\overline{\pi} = 1 - \pi$). Low equilibrium wholesale prices for high levels of the survival probability can be attributed to the fierce competition between two very reliable suppliers. Low equilibrium wholesale prices for low levels of the survival probabilities can be explained by the lack of interest from the retailer in doing business with the highly unreliable suppliers. For moderate levels of the survival probabilities, the wholesale prices are higher because the retailer values the diversification benefits that ordering from both suppliers provides. As one can see on the bottom left panel of Figure 5, the system and the retailer benefit from the increase in the supplier survival probabilities. However, the suppliers' profits are non-monotone. It is tempting to conjecture that, in the framework of our model, had the suppliers controlled their survival probabilities, they would have chosen some intermediate values. Unfortunately, we cannot prove or disprove this conjecture without introducing the survival probability decisions into the model (adding another layer of complexity) and then computing the equilibrium.

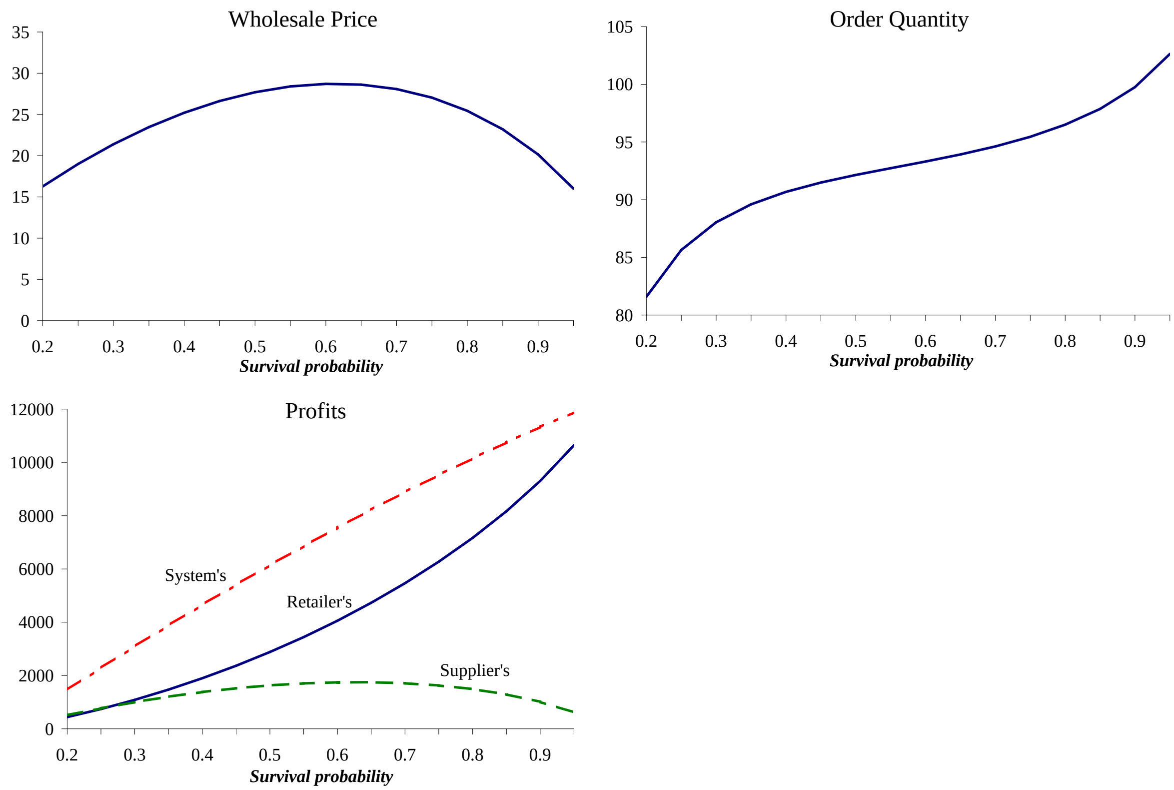
Finally, let's study the effect of the demand uncertainty. Figure 6 shows how the demand uncertainty affects the equilibrium results. When the standard deviation is small, the equilibrium under stochastic demand is close to that for the deterministic case. As the standard deviation increases, the shape of the equilibrium solution changes. The first panel of the figure shows that relative to the deterministic case, the wholesale equilibrium prices may be higher or lower, depending on the nature of default correlation. The sensitivity of order quantities to default correlation also depends on demand uncertainty and is shown in the second panel. The third and fourth panels show the sensitivity of profits for the supplier and retailer respectively, as default correlation increases. If supplier defaults are highly positively correlated, i.e. p_{00} is high, the suppliers benefit from an increasing standard deviation of demand. On the other hand if supplier defaults are highly negatively correlated, p_{00} is low, the suppliers prefer a lower standard deviation. For very low default correlations, the retailer benefits from increased demand uncertainty. Finally, observe that increasing standard deviation of the demand dampens the effects of the default correlation.

5.3 Three or more suppliers

Similarly to the model with two suppliers, one can derive expressions for the optimal retailer's order quantities if wholesale prices are fixed and the expressions for the equilibrium prices and order quantities for a model with both supply and demand uncertainty and more than 2 suppliers competing for business.[3]

[3]These expressions are not included in the paper, but are available upon request.

Figure 5: Dependence of equilibrium prices, equilibrium order quantities, and equilibrium profits on survival probability. Identical suppliers with independent defaults. Demand distribution is gamma with mean of 150 and standard deviation of 50. Other parameters: $s = 100$, $c_1 = c_2 = c = 10$, $\pi_1 = \pi_2 = \frac{1}{2}$



The following example illustrates the role of competition for the problem with more than two suppliers and stochastic demand.

Example 2. (Three supplier, stochastic demand model with two positively correlated suppliers)
Consider a model with $N = 3$ competing identical suppliers where defaults of suppliers 2 and 3 are perfectly positively correlated. The competition between these two suppliers will drive their prices to the highest production cost ($K_2 = K_3 = c$) and the model can be converted to the $N = 2$ supplier model described in part (i) of Proposition 8. The demand follows Gamma distribution with mean 150. The values of the other parameters are $s = 100, c_1 = c_2 = c_3 = c = 10, \pi_1 = \pi_2 = \pi_3 = \frac{1}{2}$.

Figure 7 shows that when the standard deviation of demand is small the retailer's profit does not change very much in the default correlation between supplier 1 and suppliers 2 and 3. However, for large values of the standard deviation the retailer's profit decreases in the supplier default correlation.

Figure 6: Effect of the standard deviation of demand on the equilibrium prices, quantities, and profits. Two identical suppliers. Demand follows Gamma distribution with mean 150. Other parameters: $s = 100$, $c_1 = c_2 = c = 10$, $\pi_1 = \pi_2 = \frac{1}{2}$
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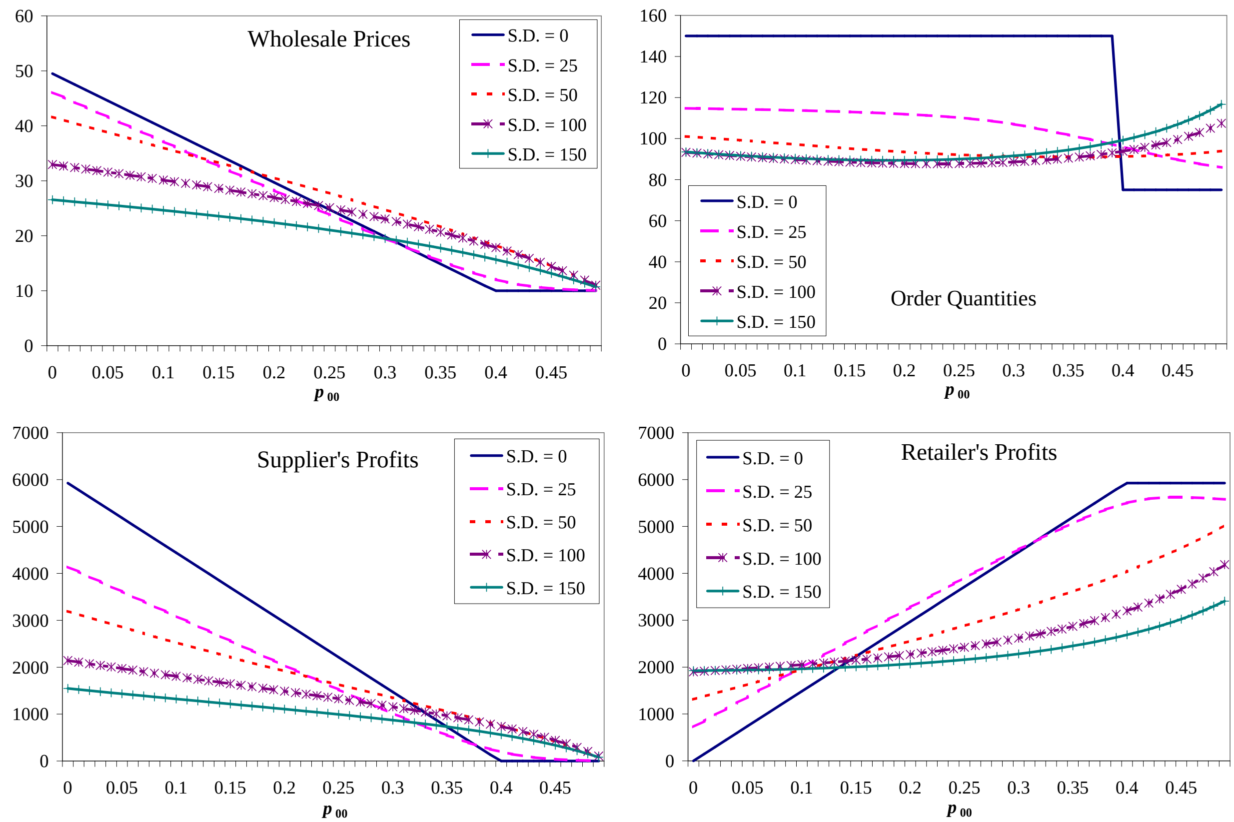

As we have discussed in the section on deterministic demand, in general, if one can identify groups of suppliers with perfectly positively correlated defaults, the competition within each group will fix the wholesale prices and the problem will be reduced to a simpler one with fewer suppliers and exogenous prices. Thus, competition, which was the driver of the counterintuitive results in the model with two suppliers, will force multi-supplier models into the framework of fixed wholesale prices where our portfolio-selection-problem–trained intuition can be applied.

6. The Timing of Payments and the Wholesale Prices

In the presence of supply risk, the timing of retailer-to-supplier payments is important. In this section we discuss how the timing of payments affects equilibrium wholesale prices and the consequences of our assumption of up-front payments.

Figure 7: Effect of the standard deviation of demand on the equilibrium prices, quantities, and profits. Three suppliers. Defaults of supplier 2 and 3 are perfectly positively correlated. Demand follows Gamma distribution with mean 150. Other parameters: $s = 100$, $c_1 = c_2 = c_3 = c = 10$, $\pi_1 = \pi_2 = \pi_3 = \frac{1}{2}$. Because of the competition, the wholesale prices of suppliers 2 and 3 are $K_2 = K_3 = c$. Profits of supplier 2 and 3 are zero.

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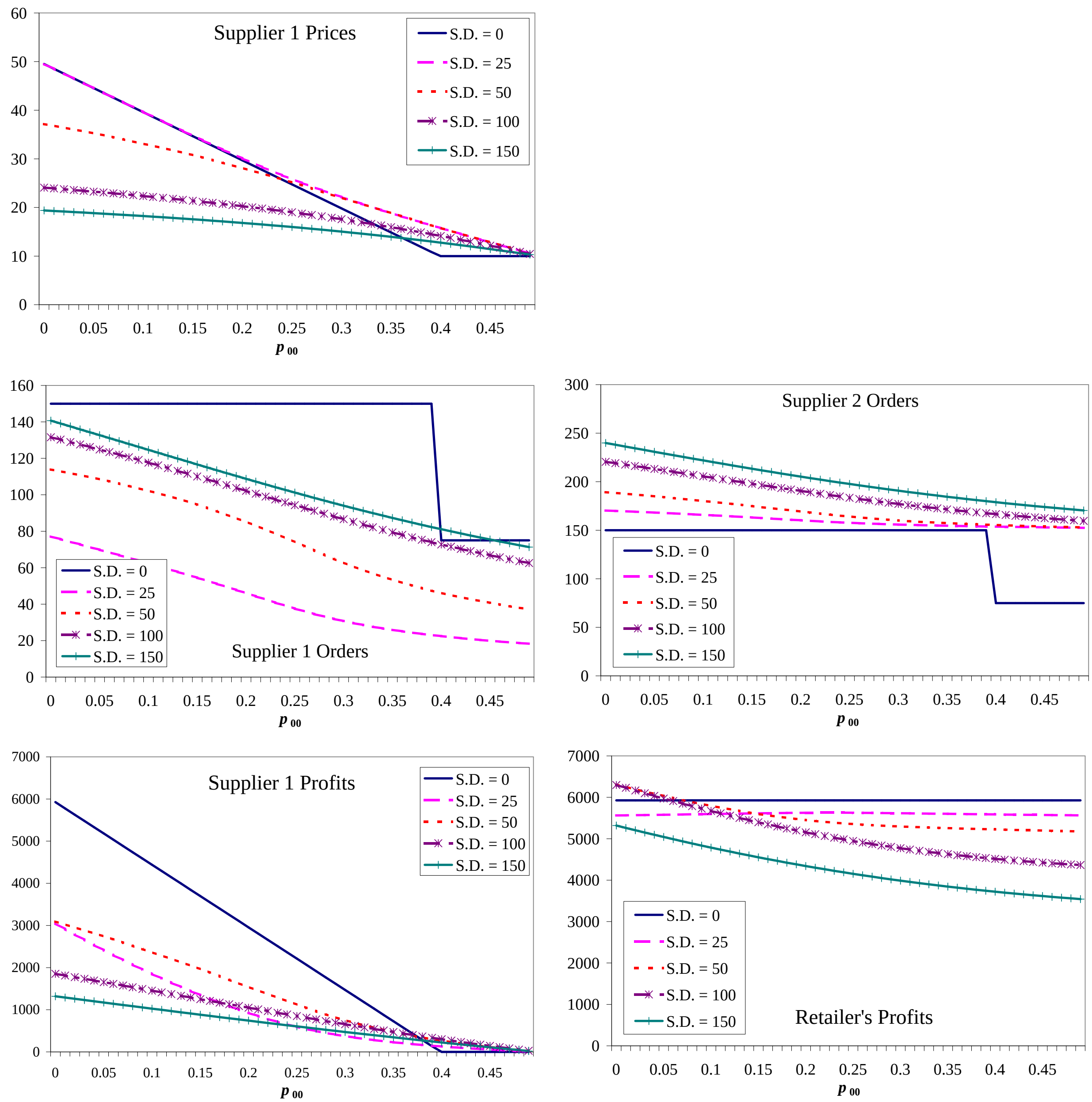


To reduce supplier risk exposure, the retailer would prefer to pay after the product has been delivered, whereas the suppliers would prefer to receive full payments before production has begun.

Many payment schemes exist. For example, a supplier may announce a policy of the form $\phi_i = \{\alpha_i, w_i^F, w_i^D\}$ where w_i^F is the per unit *up-front* wholesale price, w_i^D is the per unit *on-delivery* price, and $0 \leq \alpha_i \leq 1$ is the proportion of the units for which the retailer must pay up-front. Alternatively, a policy may call for the up-front payment of a certain percent of the total expected cost with the balance due on-delivery. Both of these policies are examples of linear pricing policies.

For any linear policies $\{\phi_i\}_{i=1}^N$ from N suppliers, the retailer's discounted expected profit is

$$R(z_1, z_2, ..., z_N) = sE\left\{\min\left[D, \sum_{i=1}^{N}(1-\delta_i)z_i\right]\right\} - \sum_{i=1}^{N} K_i(\phi_i)z_i. \tag{30}$$

after orders of size z_i, $i = 1, ..., N$ are placed with the suppliers. Note that the retailer's discounted expected profit depends on the suppliers' policies $\phi_i, i = 1, 2, ..., N$ only through $K_i = K_i(\phi_i), i = 1, 2, ..., N$. Therefore, the retailer responds with the same order quantity to any policy ϕ such that $K_i(\phi) = K$. In particular, the retailer is indifferent between up-front payment ($\alpha = 1$) and on-delivery payment ($\alpha = 0$) provided that w_i^F and w_i^D satisfy the following equation:

$$E[1-\delta_i]w_i^D = w_i^F. \tag{31}$$

Because the suppliers can choose arbitrary values for w_i^F and w_i^D, equation (31) need not hold and the retailer may favor either the up-front or the on-delivery payment policy. This feature could potentially complicate our analysis because the tradeoffs between competition and diversification effects in the supply chain can depend on the payment schemes. Fortunately, the following proposition[4] shows that *in equilibrium*, for all linear policies, the timing of payments does not matter.

Proposition 9. *In equilibrium, the retailer, the suppliers and the supply chain are indifferent between methods of payment as long as the policies are in the linear family. In particular, in equilibrium all parties are indifferent between up-front and on-delivery payments and relationship (31) holds.*

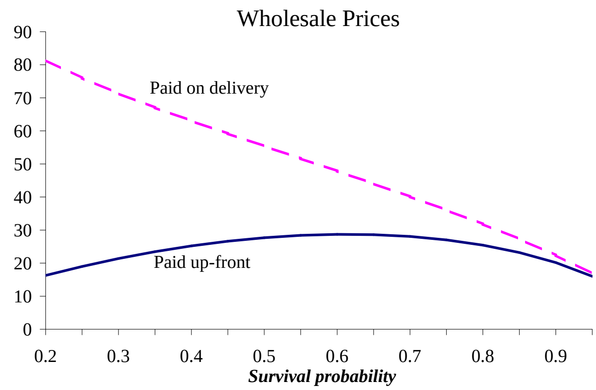
A corollary to Proposition 9 and relationship (31) is the formula for the equilibrium on-delivery prices

$$K_i^D = \frac{K_i^F}{E[1-\delta_i]} = \frac{K_i^F}{1-\pi_i}. \tag{32}$$

Using this formula, for the data from the numerical example 1 we computed the equilibrium on-delivery prices (see Figure 8). From Figure 8 we observe that the on-delivery wholesale prices could be significantly higher than the corresponding up-front wholesale prices. Thus, a partial reason why we observe low values for equilibrium wholesale prices in the numerical examples in this paper could be that we chose to work with the up-front rather than on-delivery prices.

[4]The proof of this proposition follows from an observation that the supplier profits also depend on their policies ϕ only through $K(\phi)$.

Figure 8: Equilibrium wholesale prices if paid up-front and if paid on delivery as functions of survival probabilities. Two identical suppliers. Demand follows Gamma distribution with mean of 150 and standard deviation of 50. Other parameters: $s = 100$, $c_1 = c_2 = c = 10$, $\pi_1 = \pi_2 = \frac{1}{2}$.



7. Conclusion

As supply chains have become more competitive the flow of materials has become more efficient and inventories have become leaner. As a result, supply disruptions in any part of the chain can be very costly. Indeed, because of possibilities of supply disruptions due to weather, terrorism, firm specific manufacturing failures, or financial defaults, significant attention has recently been focused on contingency measures, backup agreements, and alternative sources of supply. A growing literature has emerged that address many of the consequences of supply chain risk. We contribute to this literature by focusing on the effect of supplier competition in a market where the retailer is considering diversification as a strategy to reduce supply chain risk. We believe that this paper is one of the first to address supply-chain management questions in a model where competition between suppliers affects equilibrium wholesale prices, in a way that depends on the degree of default correlation.

Using a simple one-period model of a supply chain with one retailer and multiple risky suppliers, this paper studies questions of supplier selection, pricing and ordering policies among firms. In our model, the suppliers compete for business from the retailer, and are, collectively, Stackelberg leaders in a game with the retailer.

With more than one supplier, the retailer may decide to hedge default risk by splitting orders. If the wholesale prices were *exogenously* fixed, then, as one would expect, the negative correlation between default events yields higher diversification benefits to the retailer. However, in our competitive environment, the wholesale prices are determined *endogenously* by the suppliers. In general, as default correlations decreases, competition between the suppliers decrease, and the equilibrium wholesale prices charged by the suppliers increase. As a result, default correlation plays an enormous role on the trade-offs faced by retailers, who on the one hand want to minimize disruption risk, but on the other hand want low competitive prices.

For the two supplier problem with deterministic demand, the results are very stark. Retailers would prefer high codependence in the default correlations! That is, the price competition effects, induced by high correlations, more then offset the diversification benefits.

When there are more than two suppliers, it is possible for the retailer to obtain diversification benefits and low wholesale prices at the same time. For example, if two competing suppliers are highly correlated, with the third supplier being negatively correlated with the others, then the retailer has the best of both worlds. Specifically, the firms that are highly codependent will compete away their profits and charge low wholesale prices. At the same time, the retailer may be able to use the third supplier to hedge against disruption risk. Obviously, if a fourth supplier is introduced that has defaults highly correlated with the third, then the retailer will be even better off, and diversification benefits may be obtained without any drawbacks.

When considering models with multiple risky suppliers, where there is both random supply as well as random demand, the analysis becomes more difficult, but the overall direction of the results remain unchanged. That is, the benefits to the retailer due to the lower wholesale prices outweigh the losses due to the weaker diversification, with the exact tradeoff depending on the codependence structure of default correlations.

Thus, contrary to initial intuition about the advantages of diversification, positive default correlation benefits the retailer. At the same time, a negative default correlation benefits the suppliers and the channel as a whole. Therefore, the preferences of the retailer and the channel for default correlation are misaligned.

The economic consequences of our results are as follows. Once the suppliers are chosen, any actions they can take to reduce their correlation will be advantageous for them. For example, they may attempt to sell to different customers, use different production technologies, procure from different raw materials sources, and reduce exposures to common country specific risks or common catastrophic events. As their default correlations decrease, the equilibrium prices that they can charge should rise.

This paper also presents the comparison between centralized and decentralized supply chains, the studies of the effects of the supplier survival probabilities and the volatility of customer demand on the equilibrium solution, and the investigation of the importance of the timing of payments from the retailer to the suppliers.

In our analysis we have made several simplifying assumptions. For example, we assumed that the default and demand processes are independent, the default distribution of suppliers does not depend on the order quantities, the production lead times for both suppliers are equal, the retailer cannot default. However, even in our simple case, including supply risk considerations into operational planning significantly affects ordering and pricing decisions in a supply chain and alters the nature of competition among firms. One could extend this model by introducing default processes for the retailer and customers. With appropriate assumptions (e.g. customer defaults are independent of firms defaults, demand and default random variables are independent), the analysis of the more general model leads to the same insights on the role of the supplier default risk as the analysis of the simpler model does. Three very important assumptions in this paper are the Bertrand competition among the suppliers, the perfect substitutability of the products offered by the suppliers, and the symmetric information. These assumptions allow us to focus on two important factors in the procurement process: competition among suppliers and risk management through outsourcing. Although these assumptions are common in the contracting literature, they ignore the multi-criteria nature of the procurement practice and predict unrealistically low profit margins for the suppliers and high margin for the retailer. Thus, numerical examples in this paper probably overstate the magnitude of profits to the retailer and understate the profits of the suppliers. However, the insights we gleaned from the analysis and examples in this paper about the tradeoffs between competition and diversification would remain true in more realistic settings. We leave an important and interesting problem of studying the effects of asymmetric information on supply risk management to future research.

Appendix. Proofs.

Proof of Proposition 1.

Observe that, by the proposition hypothesis, it is not optimal for the retailer to order amounts from the suppliers that add up to a quantity lower than D. Therefore, we restrict the search for the optimal order quantities to $z_1^* + z_2^* \geq D$ and $z_i^* \leq D, i = 1, 2$. Retailer's profit:

$$R(z_1, z_2) = (sp_{01} - K_1)z_1 + (sp_{10} - K_2)z_2 + sp_{00}D.$$

The three cases now follow easily:

If $K_1 \leq sp_{01}$ and $K_2 \leq sp_{10}$, then order quantities (D, D), maximize retailer's profits.

If $K_1 \leq sp_{01}$ and $K_2 > sp_{10}$, then the optimal order quantities are $(D, 0)$.

If $K_1 > sp_{01}$ and $K_2 \leq sp_{10}$, then the optimal order quantities are $(0, D)$.

Suppose $K_1 > sp_{01}$ and $K_2 > sp_{10}$. Then the retailer would like to order as little as possible from the suppliers subject to the constraint $z_1 + z_2 \geq D$. Therefore, the retailer will order D from one of the suppliers and 0 from the other unless he is indifferent between the two (which occurs when $K_2 = K_1 + s(\pi_1 - \pi_2)$). ■

Proof of Proposition 3.

(i) For any choice of prices (K_1, K_2, K_3) to be equilibrium prices for the retailer's orders (D, D, D), the retailer must prefer these orders to any other combination. By imposing conditions

$$\begin{aligned}
&R(D, D, D) > R(D, D, 0), \quad R(D, D, D) > R(D, 0, D), \quad R(D, D, D) > R(0, D, D), \\
&R(D, D, D) > R(D, 0, 0), \quad R(D, D, D) > R(0, D, 0), \quad R(D, D, D) > R(0, 0, D),
\end{aligned} \tag{33}$$

where $R(z_1, z_2, z_3)$ is the retailer's profit, we specify a set of values for prices (K_1, K_2, K_3) that would ensure that (D, D, D) is the preferred retailer's response. For example, $R(D, D, D) > R(D, D, 0)$ is equivalent to

$$\begin{aligned}
&s(p_{000} + p_{001} + p_{010} + p_{011} + p_{100} + p_{101} + p_{110})D - K_1D - K_2D - K_3D > \\
&s(p_{000} + p_{001} + p_{010} + p_{011} + p_{100} + p_{101})D - K_1D - K_2D
\end{aligned}$$

or after simplification

$$sp_{110} > K_3$$

Conditions (33) are equivalent to

$$sp_{011} > K_1, \quad sp_{101} > K_2, \quad sp_{110} > K_3. \tag{34}$$

From the supplier's problem (4) and the fact that the supplier who unilaterally violates (34) will receive no order, it follows that $(K_1^*, K_2^*, K_3^*) = (sp_{011}, sp_{101}, sp_{110})$.

(ii) Similarly to (i) we specify a set of prices that satisfy

$$\begin{aligned}
&R(D,D,0) > R(D,D,D), \quad R(D,D,0) > R(D,0,0), \quad R(D,D,0) > R(0,D,0),\\
&R(D,D,0) > R(0,0,D), \quad R(D,D,0) > R(0,D,D), \quad R(D,D,0) > R(D,0,D),
\end{aligned} \tag{35}$$

which imposes the following constraints on the prices:

$$\begin{aligned}
&K_3 > sp_{110}\\
&K_2 < sp_{10*}\\
&K_1 < sp_{01*}\\
&sp_{001} + sp_{011} + sp_{101} - K_1 - K_2 > sp_{110} - K_3\\
&sp_{011} - K_1 > sp_{110} - K_3\\
&sp_{101} - K_2 > sp_{110} - K_3
\end{aligned}$$

Because supplier 3 is priced out in this equilibrium, $K_3^* = c_3$. To have other equilibrium prices $K_1^* = sp_{01*}$ and $K_2^* = sp_{10*}$, the following conditions on the default distributions and c_3 must hold:

$$\begin{aligned}
&c_3 > sp_{*10}\\
&c_3 > sp_{1*0}\\
&c_3 > sp_{*10} + sp_{100} - sp_{001}
\end{aligned}$$

(iii) For $(D,0,0)$ to be the best retailer's response, it must satisfy

$$\begin{aligned}
&R(D,0,0) > R(D,D,0), \quad R(D,0,0) > R(D,0,D), \quad R(D,0,0) > R(D,D,D),\\
&R(D,0,0) > R(0,D,0), \quad R(D,0,0) > R(0,0,D), \quad R(D,0,0) > R(0,D,D),
\end{aligned} \tag{36}$$

which is equivalent to

$$\begin{aligned}
&K_3 > sp_{1*0}\\
&K_2 > sp_{10*}\\
&K_1 < K_2 + K_3 + sp_{011} - sp_{100} - sp_{101} - sp_{110}\\
&sp_{01*} - K_1 > sp_{10*} - K_2\\
&sp_{0*1} - K_1 > sp_{1*0} - K_3\\
&sp_{10*} - K_2 + p_{110} - K_3 < 0
\end{aligned}$$

In order to consider an equilibrium where suppliers 2 and 3 are priced out, we take $K_2^* = c_2$ and $K_3^* = c_3$. Then, the supplier 1 can charge at most

$$\min(sp_{01*} + c_2 - sp_{10*}, sp_{0*1} + c_3 - sp_{1*0}, c_2 - sp_{101} + c_3 - sp_{110} + sp_{011} - sp_{100}).$$

If supplier 1 charges a higher price, she will lose retailer's business. To ensure that this price is feasible for supplier 1 we require that

$$c_1 < \min\left(sp_{01*} + c_2 - sp_{10*}, sp_{0*1} + c_3 - sp_{1*0}, c_2 - sp_{101} + c_3 - sp_{110} + sp_{011} - sp_{100}\right). \quad \blacksquare$$

Proof of Proposition 6.

From the first order conditions, $(z_1, 0)$ is the optimal retailer's response if

$$\left.\frac{\partial R}{\partial z_1}\right|_{z_2=0} = 0 \quad \text{and} \quad \left.\frac{\partial R}{\partial z_2}\right|_{z_2=0} \leq 0,$$

or equivalently,

$$\begin{cases} s(1-\pi_1)\overline{G}(z_1) = K_1 \\ s\left[p_{10} + p_{00}\overline{G}(z_1)\right] \leq K_2. \end{cases}$$

Equivalently, $s(1-\pi_1)\overline{G}(z_1) = K_1$ and $K_2 \geq \frac{p_{00}}{1-\pi_1}K_1 + sp_{10}$. The proof for the remaining cases is similar. $\blacksquare$

Proof of Lemma 3.

Without loss of generality, assume that $i = 1$. By the inverse function theorem, the solution of the system (19c) is unique and differentiable. The conclusion follows from an observation that the system of equations (19c) is equivalent to the system (19a) when $K_1 = \frac{(1-\pi_1)}{p_{00}}\left[K_2 - sp_{01}\right]$ and is equivalent to the system (19b) when $K_1 = \frac{p_{00}}{1-\pi_2}K_2 + sp_{10}$. $\blacksquare$

Proof of Proposition 7.

Knowing that the solution will be symmetric $z_1 = z_2 = z$ we can rewrite retailer's objective function as follows:

$$R(z) = 2(1-\pi-p_{00})E\min(D, z) + p_{00}E\min(D, 2z) - 2Kz$$

Taking cross-partial derivative of this expression

$$\frac{\partial^2 R}{\partial p_{00}\partial z} = 2\left[\overline{G}(2z) - \overline{G}(z)\right] \leq 0$$

Hence, $R(z)$ is sub-modular in (p_{00}, z). Therefore, the optimal order quantity z^* is non-increasing in p_{00}. $\blacksquare$

Proof of Proposition 8.

If supplier 1 controls wholesale price K_1 while the wholesale price K_2 is fixed and system (19c) is invertible, then supplier 1 problem can be written as

$$\max_{z_1, z_2}\left\{s\left[p_{01}\overline{G}(z_1) + p_{00}\overline{G}(z_1+z_2)\right] - c\right\}z_1,$$

$$\text{s.t.} \quad s\left[p_{10}\overline{G}(z_2) + p_{00}\overline{G}(z_1+z_2)\right] = K_2.$$

The first order conditions that the solution of this problem must satisfy are

$$\begin{cases} p_{00}\overline{G}(z_1+z_2) + p_{01}\overline{G}(z_1) - p_{00}g(z_1+z_2)z_1 - p_{01}g(z_1)z_1 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_1}{p_{00}g(z_1+z_2)+p_{10}g(z_2)} = \frac{c_1}{s}, \\ p_{00}\overline{G}(z_1+z_2) + p_{10}\overline{G}(z_2) = K_2. \end{cases}$$

From system (19c), the equilibrium price K_1 is

$$K_1 = p_{00}\overline{G}(z_1+z_2) + p_{01}\overline{G}(z_1)$$

This represents the best response of supplier 2 to the wholesale price K_2. Similarly, one can derive a system of equations which represents the best response of supplier 2 to the wholesale price K_1. The equilibrium point must satisfy

$$\begin{cases} p_{00}\overline{G}(z_1+z_2) + p_{01}\overline{G}(z_1) - p_{00}g(z_1+z_2)z_1 - p_{01}g(z_1)z_1 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_1}{p_{00}g(z_1+z_2)+p_{10}g(z_2)} = \frac{c_1}{s}, \\ p_{00}\overline{G}(z_1+z_2) + p_{10}\overline{G}(z_2) - p_{00}g(z_1+z_2)z_2 - p_{10}g(z_2)z_2 + \\ \qquad\qquad\qquad\qquad + \frac{p_{00}^2 g^2(z_1+z_2)z_2}{p_{00}g(z_1+z_2)+p_{01}g(z_1)} = \frac{c_2}{s}. \end{cases}$$

The equilibrium wholesale prices are

$$\begin{cases} K_1^* = s\left[p_{01}\overline{G}(z_1^*) + p_{00}\overline{G}(z_1^*+z_2^*)\right], \\ K_2^* = s\left[p_{10}\overline{G}(z_2^*) + p_{00}\overline{G}(z_1^*+z_2^*)\right]. \end{cases}$$

■

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