MARKET VOLATILITY, MARKET FRICTIONS, AND THE CROSS-SECTION OF STOCK RETURNS*

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Abstract

We study a 3-factor asset-pricing model with market returns, (innovations in) market volatility, and (innovations in) market friction volatility. Market friction volatility is defined as the volatility of the difference between observed asset prices and fundamental values. Both volatility components are extracted nonparametrically from a single time-series of high-frequency Standard and Poor's depository receipts' (SPIDERS) trade prices.

We find that market volatility and market friction volatility are negatively priced in the cross-section of daily and monthly 25 size- and value-sorted Fama-French portfolios. In our sample, the performance of a 3-factor model with market return, market volatility, and market friction volatility is similar to the performance of the Fama-French 3-factor model when pricing these portfolios.

JEL Classification: G12
Keywords: Market volatility, market liquidity, cross-section of stock returns

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1 Introduction

We investigate the extent to which market volatility and market friction volatility explain documented cross-sectional asset price anomalies. We define market friction volatility as the volatility of the difference between observed asset prices and fundamental values.

Our hypothesis is that an asset's exposure to both volatility factors can lead to non-diversifiable risk which might be priced in equilibrium. We test this hypothesis using a 2-factor model consisting of the return on the market portfolio and either innovations in market volatility or innovations in market friction volatility, and a 3-factor model consisting of the return on the market portfolio and innovations in both volatility measures. We use the 25 size- and value-sorted portfolios of Fama and French (1992, 1993) as the relevant test assets. We consider the Standard & Poor's depository receipts (SPDR or SPIDERS) as a proxy for the market portfolio. Both volatility measures are obtained nonparametrically by employing high-frequency SPIDERS transaction data and the identification procedure proposed in recent work by Bandi and Russell (2003, 2006). Our cross-sectional tests are conducted at monthly and daily frequencies.

At *monthly* frequencies, when augmenting the classical CAPM with innovations in either volatility measure (i.e., in a 2-factor model), we find that both volatility measures are negatively priced in the cross-section of stock returns. This finding is intuitive. Both volatility measures are expected to increase in unfavorable economic times. Stocks that co-vary positively with them provide a hedge, and therefore should have higher prices and lower expected returns. In the context of a 3-factor model comprising market returns and both volatility measures, we find that both volatility measures remain significantly negatively priced in the cross-section of size- and value-sorted portfolios. In our sample, we show that a 3-factor model with market returns, market volatility, and market friction volatility yields similar pricing errors as the Fama-French 3 factor model when pricing the cross-section of the 25 Fama-French portfolios.

Market volatility and market friction volatility are also negatively priced in the cross-section of *daily* stock returns. Interestingly, in a 3-factor model comprising both volatility measures and daily market returns, market friction volatility appears to largely subsume the information contained in market volatility.

In order to examine the cross-sectional pricing implications of our volatility factors, we employ recent advances in nonparametric volatility estimation by virtue of high-frequency asset price data. The usefulness of high-frequency asset price data in cross-sectional asset pricing tests with market volatility and frictions is twofold.

First, high-frequency data have been shown to deliver "more accurate" equilibrium volatility estimates (see, e.g., Andersen et al., 2003, and Barndorff-Nielsen and Shephard, 2002). While it is customary to compute (daily and monthly) realized variance estimates by summing up squared continuously-compounded intra-daily returns sampled at fixed intervals (say, 5 or 20 minutes), in this work we sample continuously-compounded returns optimally using the mean squared error - based approach suggested by Bandi and Russell (2003, 2006). Interestingly, we show that optimally-sampled realized variances deliver more statistically significant volatility factor loadings than realized variances sampled every 5 or 20 minutes. In addition, optimally-sampled realized variances yield a substantially larger spread in the volatility loadings across value- and size-sorted portfolios.

Second, high-frequency data provide important information about market frictions (see, e.g., Stoll, 2000). Frictions are real (i.e., liquidity related) and informational (i.e., related to adverse selection induced by asymmetric information). Average bid-ask spreads and “effective spreads” are widely used friction measures. Bandi and Russell (2005) suggest an alternative measure, named "full-information transaction cost" or FITC, based on moments of high-frequency asset price data. The FITCs measure deviations of transaction prices from fundamental values and have been shown to be cross-sectionally more correlated with liquidity and asymmetric information proxies such as size, turn-over, Easley and O’Hara’s probability of informed trading (PIN), and analysts forecasts, then average bid-ask spreads and “effective spreads.” Following Bandi and Russell (2005, 2006), in this paper we use (innovations in) the standard deviation of the difference between high-frequency SPIDERS transaction prices and fundamental values as our aggregate friction proxy. Simple arbitrage arguments suggest that, when aggregate frictions are small, SPIDERS’ prices should track closely the value of the underlying basket of securities. Conversely, large differences signal pervasive market frictions. We show that innovations in our friction proxy are highly correlated with aggregate liquidity and asymmetric information measures.

Our findings relate to existing results in the literature. On the one hand, Ang et al. (2006), Adrian and Rosenberg (2006), and Moise (2006) show that market volatility is a distress proxy carrying a negative price of risk. On the other hand, aggregate (il-)liquidity has been documented to be a negatively-priced systematic risk factor (Acharya and Pedersen, 2005, Pástor and Stambaugh, 2003, and Sadka, 2003, among others).[1] We present an asset-pricing model that explicitly incorporates *both* components after their extraction from a single time-series of high-frequency SPIDERS

[1] The cross-sectional relation between expected stock returns and idiosyncratic, rather than systematic, liquidity has been investigated in numerous papers, including Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Datar et al. (1998), and Elaswarapu (1997). Amihud (2002), Jones (2002), and Fujimoto (2003), among others, study the time-series properties of market excess returns and market liquidity. The effect of idiosyncratic asymmetric information in cross-sectional asset pricing is discussed in Easley et al. (2002). Amihud et al. (2006) and Cochrane (2005) provide insightful discussions of the current state of the literature on liquidity and asset prices.

transaction prices.

Our 3-factor model with market returns, market volatility, and market friction volatility relates to, but is different from, the 3-factor model (with market returns and two volatility components) of Adrian and Rosenberg (2006). Adrian and Rosenberg (2006) propose a factor model for market volatility with a persistent component correlated with the business cycle and a transitory component interpreted as a market skewness and kurtosis proxy. Both components are found to be priced in the cross-section of stock returns with a significantly negative price of risk. As in Ang et al. (2006), Adrian and Rosenberg (2006), and Moise (2006) our first component is a market volatility proxy. Our second component, which is novel, relates to aggregate market frictions. The former is highly persistent, as the first volatility factor in Adrian and Rosenberg (2006). The second component is less persistent, as the second factor of Adrian and Rosenberg (2006), but is not a transitory factor of market volatility. Rather, it is an aggregate friction factor correlated with macro (il-)liquidity events.

The paper is structured as follows. In Section 2 we present the data and the assumed SPIDERS price formation mechanism. Section 3 analyses the volatility estimates. In Section 4 we discuss the asset pricing tests. In Section 5 we present the empirical results at monthly and daily frequencies. Section 6 contains robustness checks. In Section 7 we look at pricing errors. Section 8 concludes. Tables and Figures are in the Appendix.

2 Data

We use high-frequency transaction price data on the Standard & Poor's depository receipts (SPDR or SPIDERS) to construct the two volatility measures, i.e., market volatility and market friction volatility. SPIDERS represent shares in a trust that owns stocks in the same proportion as that found in the S&P500 index. They trade like a stock (with the ticker symbol SPY on the Amex) at approximately one-tenth of the level of S&P500, and are used by large institutions and traders either as bets on the overall direction of the market or as a means of passive management.

SPIDERS are exchange traded funds (ETFs).[2] They can be redeemed for the underlying portfolio of assets *at the end of the trading day.* Equivalently, investors have the right to obtain newly issued SPIDERS shares from the fund company in exchange for a basket of securities that mirrors the SPIDERS' portfolio. This implies that SPIDERS, like other ETFs, must trade at a value that

[2] A growing academic literature focuses on ETFs. Among other issues, the existing work studies the dynamics of price deviations from net asset value (Engle and Sarkar, 2002), compares the return from holding ETFs (specifically, SPIDERS) to the return from holding the underlying index (Elton et al., 2002), analyzes the tax implications of ETFs (Poterba and Shoven, 2002), investigates price discovery (Hasbrouck, 2002) and competition (Boehmer and Boehmer, 2002) in the ETF market.

is near net asset value (NAV). If they traded above their NAV, arbitrageurs would purchase the basket of underlying securities for a lower price and force the fund company to issue new shares. Conversely, if they traded below their NAV, arbitrageurs would buy shares and redeem them for the underlying portfolio of securities (see Cherkes et al., 2006, for further discussions and comparisons between ETFs and closed-end funds). Similarly, SPIDERS' values will not deviate much from fundamental values[3] *during the day* either, since the future convergence of prices would open up the possibility for simple, immediate investment opportunities. Assume trading prices are higher than fundamental values. An arbitrageur could buy the underlying basket of security, sell SPIDERS short,[4] wait for price convergence, and unwind the position for an initial profit.

As Elton et al. (2002) and Engle and Sarkar (2002) point out, the process of share deletion/creation acts as an extremely effective mechanism in keeping prices close to NAV and assuring that potential differences disappear quickly. Conversely, since arbitrages require acquisition of the underlying basket of securities, the extent of deviations from NAV signals *pervasive* market frictions rendering arbitrages harder to implement. This basic intuition justifies our approach. The next subsection discusses identification of the volatility of these deviations using high-frequency SPIDERS data. Rather than focusing on deviations of trade prices from NAV or IOPV, we assume that the fundamental value of the basket of securities, which is only approximated by the NAV at close and by the IOPV during the day, is *unobservable*. This property is important (see Engle and Sarkar, 2002). The NAV is evaluated at the closing transaction price of each of the assets. On the one hand, each closing transaction price could be higher or lower than the individual fundamental value. On the other hand, the closing transaction could occur earlier in the day, especially for less frequently-traded stocks.[5] Similarly, the IOPVs can also be stale since they are posted at equispaced intervals of 15 seconds. In what follows, we simply call the unobserved reference value of the underlying portfolio of securities "fundamental value."

We employ high-frequency transaction prices on SPIDERS obtained from the Trade and Quote (TAQ) database in CRSP for the period February 1993 - March 2005. We use the entire consolidated market. The cross-sectional asset pricing tests employ daily and monthly return data on the 25 size- and value-sorted Fama-French portfolios over the same period.

We now describe the assumed high-frequency SPIDERS price formation mechanism and our procedure to separate market volatility from market friction volatility.

[3]The NAV is computed at market close. During the day, an estimated value of the portfolio called Indicative Optimized Portfolio Value (IOPV) is posted. The IOPV is provided every 15 seconds using the most recent transaction price of each component of the portfolio.

[4]Contrary to individual stocks, SPIDERS can be short-sold on a down-tick.

[5]In addition, SPIDERS continue to trade 15 minutes after the NYSE closes. This is another source of error for the posted NAV.

2.1 High-frequency price formation and construction of the volatility estimates

The logarithmic SPIDERS transaction price prevailing at the end of a trading day t of length h is expressed as

$$\widetilde{p}_{th} = p_{th} + \psi_{th} \qquad t = 1, 2, ..., T.$$

Every observed transaction price $\widetilde{p}_{th}$ contains two components: a fundamental price p_{th} and a market friction ψ_{th}. Each trading day can be divided into M sub-periods. The j-th intra-daily continuously-compounded observed return between day $t-1$ and day t is then defined as

$$\widetilde{r}_{j,t} = \widetilde{p}_{(t-1)h+j\delta} - \widetilde{p}_{(t-1)h+(j-1)\delta} \qquad j = 1, 2, ..., M,$$

where $\delta = h/M$ is the interval over which the intra-daily returns are computed. Thus, similarly to the observed price process, the observed return process comprises a fundamental return component, $\widetilde{r}_{j,t} = p_{(t-1)h+j\delta} - p_{(t-1)h+(j-1)\delta}$, as well as a friction component, $\nu_{j,t} = \psi_{(t-1)h+j\delta} - \psi_{(t-1)h+(j-1)\delta}$, i.e.,

$$\widetilde{r}_{j,t} = r_{j,t} + \nu_{j,t} \qquad t = 1, 2, ..., T, \quad j = 1, 2, ..., M.$$

As in much recent work (see the discussions in Bandi and Russell, 2006b, and Barndorff-Nielsen and Shephard, 2006), we assume that the fundamental return series is a stochastic volatility martingale difference sequence driven by Brownian shocks, i.e.,

$$r_{j,t} = \int_{(t-1)h+(j-1)\delta}^{(t-1)h+j\delta} \sigma_s dW_s,$$

where σ_s is a càdlàg spot volatility process bounded away from zero.[6] The frictions in the price process ψ are independent of the fundamental prices and i.i.d. with a bounded fourth moment. In consequence, the return frictions ν follow an $MA(1)$ model with a negative first-order autocovariance equal to $-2E(\psi^2)$. Since the fundamental returns are uncorrelated, this structure carries over to the observed high-frequency returns. The empirical autocorrelation structure of the high-frequency SPIDERS returns strongly supports this simple specification (see Fig. 1).[7]

The frictions are modeled as having a stochastic order of magnitude $O_p(1)$. Price discreteness, as well as the existence of different prices for buyers and sellers, for instance, justify this assumption (see Bandi and Russell, 2006). The fundamental return process is assumed to have an

[6] Spot volatility can display jumps, diurnal effects, high-persistence (possibly of the long memory type), and nonstationarities.

[7] We could allow for fairly unrestricted dependence in the frictions along similar lines as in Bandi and Russell (2005). As shown, this extension seems unimportant in our data.

order of magnitude $O_p(\sqrt{\delta})$ over any sub-interval of length δ, thereby implying that the size of the fundamental price changes decreases with the sampling interval. This assumption, which is standard in continuous-time asset pricing, represents slow accumulation (and processing) of information leading to negligible fundamental price updates over small time intervals.

Using this framework, Bandi and Russell (2003, 2006) show that sample moments of high-frequency return data, sampled at the highest frequencies at which information arrives, identify the friction moments. Availability of high sampling frequencies is represented here by letting the distance between observations δ go to zero asymptotically or, equivalently, by letting the number of observations M go to infinity for every trading day. In the second moment case, one obtains

$$\frac{\sum_{j=1}^{M} \widetilde{r}_{j,t}^{2}}{M} \underset{M\to\infty}{\overset{p}{\to}} E_t(\nu^2) = 2E_t(\psi^2). \tag{1}$$

The result in Eq. (1) hinges on the fact that the friction process dominates the fair return process at high frequencies. When taking sample moments of the observed return data, the fundamental return component washes out asymptotically since its stochastic order $O_p(\sqrt{\delta})$ is smaller than the stochastic order of the frictions, $O_p(1)$. Hence, the moments of the observed returns consistently estimate the friction moments at high frequencies.[8] In what follows, we identify the variance of the SPIDERS friction return component for each trading day in our sample by computing the estimator in Eq. (1) using intra-daily returns. We expect the consistency result in Eq. (1) to be fairly accurate since we employ a large number of high-frequency returns per trading day; the average number of intra-daily returns is about 3,000.

We now turn to the daily variance of the fundamental return process, i.e., $V_t = \int_{t-1}^{t} \sigma_s^2 ds$. In the absence of frictions, the sum of the squared intra-daily returns $\sum_{j=1}^{M} \widetilde{r}_{j,t}^{2}$ (realized variance) estimates V_t consistently (see, e.g., Revuz and Yor, 1994). The presence of frictions leads to an important bias-variance trade-off. High sampling frequencies determine substantial noise accumulation and biased estimates. Low sampling frequencies lead to (fairly) unbiased but highly volatile estimates. Bandi and Russell (2003, 2006) provide a simple rule-of-thumb to optimize this trade-off and choose the optimal sampling frequency δ (or, equivalently, the optimal number of observations M) for every horizon of interest. For each day in our sample, given the assumed friction structure, the (approximate) optimal number of observations M_t^* is defined as

[8] Interestingly, this consistency result would not be affected by the presence of infrequent news arrivals leading to discrete changes in the fundamental price process. In other words, we could easily allow for the presence of a Poisson jump component in the fundamental price (see Bandi and Russell, 2005). The estimator, and its consistency properties, would not change.

$$M_t^* \approx \left(\frac{\widehat{Q}_t}{\widehat{\alpha}_t} \right)^{1/3}, \quad (2)$$

where $\widehat{Q}_t$ is equal to $\frac{\widetilde{M}}{3} \sum_{j=1}^{\widetilde{M}} \widetilde{r}_{j,t}^4$, the quarticity estimator of Barndorff-Nielsen and Shephard (2002) with returns sampled every 15 minutes, and $\widehat{\alpha}_t$ is equal to $\left(\frac{\sum_{j=1}^{M} \widetilde{r}_{j,t}^2}{M} \right)^2$, the friction-in-returns second moment estimator raised to the second power. The optimal number of observations M_t^* can be interpreted as a signal-to-noise ratio. The higher the signal coming from the underlying fundamental price process ($\widehat{Q}_t$ estimates $\int_{t-1}^{t} \sigma_s^4 ds$) relative to the size of the frictions (as represented by $\widehat{\alpha}_t$), the higher the optimal number of high-frequency observations to be used in realized variance estimation.

Fig. 2 (Panel a) reports a histogram of the optimal sampling intervals and descriptive statistics. The average interval is about 29 minutes, the median value is about 14 minutes. Fig. 2 (Panel b) presents a time-series plot of the optimal sampling intervals. The intervals display an obvious downward trend. This trend is due to friction second moment estimates being relatively higher in the first part of the sample (see Fig. 4). According to the ratio in Eq. (2), in order to achieve friction reduction, a higher relative friction component should lead to a smaller optimal number of return observations and, thus, a lower optimal sampling frequency.

Since realized variance is computed over a 6-hour period (from 10 a.m. to 4 p.m.) and the cross-section of stock returns is evaluated at daily frequencies (at the minimum), we correct the volatility estimates for the lack of overnight returns by multiplying them by the constant factor

$$\zeta = \frac{\frac{1}{T} \sum_{t=1}^{T} R_t^2}{\frac{1}{T} \sum_{t=1}^{T} \widehat{V}_t},$$

where R_t and $\widehat{V}_t = \sum_{j=1}^{M_t^*} \widetilde{r}_{j,t}^2$ are the daily SPIDERS return and the (6-hour) optimally-sampled realized variance estimates over day t. This correction ensures that the average of the corrected variance estimates coincides with the variance of the daily returns (see Fleming et al., 2001, 2003).

In sum, $\frac{\sum_{j=1}^{M} \widetilde{r}_{j,t}^2}{M}$ (with M equal to the number of observations in each day) and $\zeta \sum_{j=1}^{M_t^*} \widetilde{r}_{j,t}^2$ (with M_t^* optimally defined as in Eq. (2)) give us daily estimates of the friction variance and daily estimates of the fundamental price variance, respectively.

When using monthly return data, we further average the friction variances across days in a specific month k to obtain a monthly measure:

$$\widehat{E}_k(\nu^2) = \frac{1}{\#Days} \sum_{t=1}^{\#Days} \frac{\sum_{j=1}^{M} \widetilde{r}_{j,t,k}^2}{M}. \quad (3)$$

As for the fundamental price variances, we sum the daily realized variances across days in a month to obtain the corresponding monthly value:

$$\widehat{V}_k = \sum_{t=1}^{\#Days} \zeta \sum_{j=1}^{M_t^*} \widetilde{r}_{j,t,k}^2. \tag{4}$$

Finally, we compute innovations in volatility. In the friction case we have

$$IFV_k = FV_k - FV_{k-1} = \sqrt{\widehat{E}_k(\nu^2)} - \sqrt{\widehat{E}_{k-1}(\nu^2)}.$$

In the fundamental price volatility case, we have

$$IV_k = \sqrt{\widehat{V}_k} - \sqrt{\widehat{V}_{k-1}}.$$

Our hypothesis is that *innovations* in the SPIDERS friction second moments are highly correlated with *innovations* in aggregate market frictions (liquidity, for instance) and hence provide a good proxy for them. In other words, even though SPIDERS trade just like any other stock, we expect innovations in SPIDERS frictions to be a much less noisy measure of innovations in the overall market frictions than innovations in the individual stocks' frictions. This hypothesis is justified by the very nature of ETFs and simple no arbitrage considerations. As Gastineau (2001) and Spence (2002), among others, point out, the liquidity properties of an ETF should reflect the liquidity properties of the underlying portfolio of securities.

3 A look at the volatility estimates

3.1 *Market volatility*

Fig. 3 contains 6-hour realized variance estimates obtained by sampling returns optimally for each day in our sample (Panel a), realized variance estimates constructed using 20-minute intervals (Panel b), and realized variance estimates constructed using 5-minute intervals (Panel c). The 5- and 20-minute frequencies are widely used in the literature to avoid microstructure-induced contaminations. As expected, the optimally-sampled realized variance estimates appear better behaved than the variances obtained by sampling at fixed, ad-hoc, intervals. We focus on optimally-sampled realized variances in what follows.

Fig. 5 plots the monthly realized variance estimates constructed using sums of daily realized variance estimates (as in Eq. (4) above) and the monthly variance estimates constructed by summing squared daily returns. The correlation between the two variance series is 75%.

3.2 *Market friction volatility*

Classical market microstructure frictions are imputed to liquidity and asymmetric information (Stoll, 2000). At the macro level, systematic liquidity risk has received much attention in recent work (see Acharya and Pedersen, 2005, and Pástor and Stambaugh, 2003, among others). Fig. 6 plots the monthly friction volatility estimates, FV. The graph suggests a general decline in market frictions which mirrors a well-documented decline in average bid-ask spreads with spikes corresponding to known (il-)liquidity events, such as the Asian crisis (October 1997), the LTCM collapse and Russian debt default (October/November 1998), the 9/11 terrorist attack, and so on.

For comparison, in Fig. 7 we report IFV and innovations in the Pástor and Stambaugh liquidity measure over the same period.[9] Pástor and Stambaugh's measure is a price reversal measure. The idea is that less liquid stocks should have larger price reversals following signed order flow than more liquid stocks. For stock i in month k, liquidity is defined as the least-squares γ estimate from the regression

$$r^e_{i,t+1,k} = \theta_{i,t} + \phi_{i,k} r_{i,t,k} + \gamma_{i,k} sign(r^e_{i,t,k}) v_{i,t,k} + \varepsilon_{i,t+1,k},$$

where r is a stock return, r^e is an excess stock return, and v is dollar volume. Pástor and Stambaugh expect γ to be negative in general (the price impacts of trades get reversed in the future) and larger in magnitude for less liquid stocks. To construct innovations in aggregate liquidity, they scale the differences in the monthly liquidity measures by relative market size at k and average the differences across stocks with data available in consecutive months, i.e.,

$$\Delta\widehat{\gamma}_k = \left(\frac{m_k}{m_1}\right) \frac{1}{N_k} \sum_{i=1}^{N_k} \left(\gamma_{i,k} - \gamma_{i,k-1}\right).$$

Subsequently, they run the regression

$$\Delta\widehat{\gamma}_k = a + b\Delta\widehat{\gamma}_{k-1} + c\left(\frac{m_{k-1}}{m_k}\right)\widehat{\gamma}_{k-1} + u_k$$

Finally, innovations in aggregate (il-)liquidity are measured by $PS_k = \frac{u_k}{100}$. The correlation between IFV and PS is -0.23 (positive innovations signal possible illiquidity events in our case, negative innovations signal illiquidity events in the case of Pástor and Stambaugh's measure). The correlation between IFV and market returns is -0.28, i.e., increases in illiquidity are often associated with market downturns (the correlation between market returns and Pástor and Stambaugh's illiquidity proxy is 0.23). Subsection 3.3. expands on this finding. The correlation between IFV (PS) and IV

[9] Data on Pástor and Stambaugh's liquidity measure are downloaded from CRSP.

is 0.57 (-0.31). In our period our proxy is more highly correlated with *SMB* (i.e., the difference in returns between small and large firms) and *HML* (i.e., the difference in returns between high and low book-to-market stocks) than *PS* (-0.14 and 0.14 versus -0.01 and -0.02). *PS* is more highly correlated with the momentum factor than *IFV* (-0.14 versus -0.01). See Table IX.

Alternative aggregate liquidity measures have been proposed. Amihud (2002) uses the so-called "illiquidity ratio." For each stock i and each month k, he computes

$$\frac{1}{\#Days}\sum_{t=1}^{\#Days}\frac{|r_{i,t,k}|}{v_{i,t,k}}.$$

An aggregate measure can then be defined by averaging across stocks for each month:

$$ILL_k = \frac{1}{N_k}\sum_{i=1}^{N_t}\left(\frac{1}{\#Days}\sum_{t=1}^{\#Days}\frac{|r_{i,t,k}|}{v_{i,t,k}}\right).$$

As earlier, this measure can be re-scaled by $\left(\frac{m_k}{m_1}\right)$. Amihud's "illiquidity ratio" looks directly at price impacts. Periods of illiquidity are periods during which small volumes determine large price moves. Finally, "share turnover" is sometimes used to quantify aggregate liquidity. For each stock i and each month k, compute

$$\frac{1}{\#Days}\sum_{t=1}^{\#Days} turn_{i,t,k},$$

where *turn* denotes the ratio between the number of shares transacted and the number of shares outstanding. Aggregate liquidity is then measured by

$$T_k = \frac{1}{N_k}\sum_{i=1}^{N_t}\left(\frac{1}{\#Days}\sum_{t=1}^{\#Days} turn_{i,t,k}\right).$$

Again, this measure can be re-scaled to achieve stationarity. Eckbo and Norli (2002), for example, scale it by a factor $\frac{c_1}{c_t}$ with c_t defined as the 24-month moving average of market turnover (between month $k-24$ and month $k-1$) and c_1 defined as market turnover in the first month in the sample. Fujimoto (2003) contains a thorough discussion of these measures.

The correlations between IFV and PS, innovations in ILL ($IILL$), and innovations in T (IT) are reported in Table X. Using data between April 93 and December 2002,[10] we find that IFV, $IILL$, and IT correlate with each other. Their correlations are economically meaningful and statistically significant. The correlations between IFV and PS, (un-scaled) $IILL_k$, and (un-scaled) IT are

[10] We thank Akiko Fujimoto for providing the ILL and T data.

-0.224, 0.235, and 0.281, respectively. The correlations between IFV, (scaled) $IILL$, and (scaled) IT are 0.210 and 0.258. In our sample, PS is uncorrelated with $IILL$ and IT. The correlation between PS and un-scaled $IILL$ (scaled $IILL$) is -0.062 (-0.017). The correlation between PS and un-scaled IT (scaled IT) is -0.02 (0.002).

It is interesting to notice that the correlation between IFV and IT is positive. Positive changes in turnover are often associated with informed trading, rather than simply with increases in liquidity. It is reasonable to expect more informed trading to take place in the presence of larger deviations between observed prices and fundamental values. In agreement with the cross-sectional results of Bandi and Russell (2005), IFV appears to contain an important asymmetric information component.

3.3 Market frictions and excess returns

As in Amihud (2002) using ILL, we examine the effect of our friction measure on expected stock market returns. Amihud (2002) fits an AR(1) model for ILL. Subsequently, he regresses excess market returns on lagged ILL and unexpected contemporaneous ILL (i.e., the residuals from the ILL autoregression). He finds that expected market returns are an increasing function of expected illiquidity and a decreasing function of unexpected illiquidity. Using monthly data, we apply the same procedure to our friction proxy. A first-order autoregression of $\log FV$ on $\log FV_{-1}$ gives

$$\begin{aligned} \log FV &= -\underset{(-4.85)}{2.08} + \underset{(11.89)}{0.711} \log FV_{-1} + residual \\ R^2 &= 50.6\%, \qquad Durbin - Watson = 2.55. \end{aligned}$$

A regression of MKT (the monthly excess return on the market) on $\log FV_{-1}$ and the residuals from the previous autoregression gives

$$\begin{aligned} MKT &= \underset{(1.56)}{9.19} + \underset{(1.45)}{1.19} \log FV_{-1} - \underset{(-2.49)}{2.91}\, residual \\ R^2 &= 5.7\%, \qquad Durbin - Watson = 1.93. \end{aligned}$$

The autoregression mirrors Amihud's results. He finds an autoregressive parameter equal to 0.768 (t-statistic of 5.89) with an R^2 around 50%. The outcome of the risk-return regression yields results that are qualitatively similar to those in Amihud (2002). However, the statistical significance of the positive coefficient on $\log FV_{-1}$ is lower, indicating that unexpected frictions have a more significant (negative) effect on realized stock market returns than expected frictions. Using several aggregate liquidity measures, including PS and T, Fujimoto (2003) finds a similar result.

We now run the same regression using excess market returns on the Fama-French size-sorted portfolios (see Table XI). The coefficient on $\log FV_{-1}$ is statistically insignificant across portfolios (its value and t-statistic are constantly around 1.35 and 1.6, respectively). The coefficient on the residuals, on the other hand, has a negative sign as in the market case and decreases monotonically (in absolute value) going from small to large stocks. Similarly, the statistical significance of the coefficient decreases monotonically (the associated t-statistics for portfolios in the 2nd, 4th, 6th, 8th, and 10th size deciles are $-3.86, -3.42, -3.00, -2.64$, and, -1.83, respectively). Not surprisingly, smaller stocks have more exposure to unexpected market friction risk. In the next sections we show that this exposure is priced in equilibrium.

4 Cross-sectional asset pricing

We consider a classical intertemporal asset-pricing model as in Merton (1973). Denote excess returns on a generic asset i by R_i^e and excess returns on the market by R_m^e. Assume existence of p state variables F_s. Equilibrium expected excess returns are expressed as linear combinations of the beta of the asset returns with the market return, $\beta_i^m = \frac{Cov\left(R_{i,t}^e, R_{m,t}^e\right)}{Var\left(R_{m,t}^e\right)}$, and the betas of the asset returns with the state variables, $\beta_i^s = \frac{Cov\left(R_{i,t}^e, F_{s,t}\right)}{Var(F_{s,t})}$, namely

$$E(R_i^e) = \lambda_m \beta_i^m + \sum_{s=1}^{p} \lambda_s \beta_i^s. \tag{5}$$

The lambdas have the usual interpretation in terms of prices of risk. Specifically, λ_m is the price of market risk and λ_s is the price of risk associated with the generic s factor.

We use the Fama-French 25 size- and value-sorted portfolios as test assets and focus on 2- and 3-factor models. In the 2-factor model case, $p = 1$ and the two factors are MKT and either volatility measure (IV or IFV). In the 3-factor model case, $p = 2$ and the three factors are MKT and both IV and IFV.

The estimation method consists of two steps. In the first step, we estimate the betas (the factor loadings) for each portfolio by fitting a linear regression model with AR(1)-GARCH(1,1) errors to the time-series of each portfolio's excess returns as in Moise (2006). The model is:

$$
\begin{aligned}
R_{i,t}^{e} &= \alpha_i + \beta_i^m R_{m,t}^e + \sum_{s=1}^{p} \beta_i^s F_{s,t} + \eta_{i,t} \qquad i = 1, ..., 25, \quad t = 1, ..., T \\
\eta_{i,t} &= \varepsilon_{i,t} - \gamma_i \eta_{i,t-1}, \\
\varepsilon_{i,t} &= \sqrt{h_{i,t}} e_{i,t}, \\
h_{i,t} &= \omega_i + \phi_i h_{i,t-1} + \theta_i \varepsilon_{i,t-1}^2, \\
e_{i,t} &\sim N(0,1).
\end{aligned}
$$

Given the factor loadings, the prices of risk are estimated in the second step by regressing cross-sectionally the portfolios' average excess returns on the factor loadings as implied by Eq. (5).

5 The price of volatility risk

5.1 *Monthly frequencies*

Figs. 8 and 9 plot the monthly average excess returns of the 25 Fama-French portfolios as well as the volatility factor loadings obtained from 2-factor models (*MKT* and *IV*, and *MKT* and *IFV*). Fig. 10 gives the same plots in the 3-factor model case (*MKT*, *IV*, and *IFV*). The average excess returns have a familiar pattern: they largely increase in the growth-value dimension (i.e., going from low book-to-market to high book-to-market stocks) and decrease in the size dimension (i.e., going from small stocks to large stocks). In the 2-factor models and in the 3-factor model, both the *IV* and the *IFV* factor loadings decrease with value and increase with size, albeit sometimes not mononotically. The inverse relation between size and factor loadings is generally stronger than the inverse relation between value and factor loadings.

Since the relation between excess returns and factor loadings is largely negative across size- and value-sorted portfolio, volatility risk might be priced with a negative sign. We test this implication by regressing average excess returns on factor loadings as described in the previous section. Table I contains the results. In a 2-factor model with *MKT* and *IV*, both variables have significant risk prices. The corresponding t-statistics are 4 and −2.54. The estimated (yearly) small-minus-big *IV* risk premia $\left(\widehat{\lambda}_{IV}\left(\widehat{\beta}_{Small}^{IV} - \widehat{\beta}_{Big}^{IV}\right)\right)$ for stocks in the 2nd, 3rd, and 4th book-to-market quantiles are 2.8%, 1%, and 3.12% (Table VI, Panel a). In a 2-factor model with *MKT* and *IFV*, the prices of risk are again statistically significant with t-statistics equal to 4.05 and −3.45. The estimated (yearly) small-minus-big *IFV* risk premia $\left(\widehat{\lambda}_{IV}\left(\widehat{\beta}_{Small}^{IFV} - \widehat{\beta}_{Big}^{IFV}\right)\right)$ for stocks in the 2nd, 3rd, and 4th book-to-market quantile are equal to 6.8%, 4.8%, and 8.28% (Table VI, Panel b). When combining *MKT*, *IV*, and *IFV* in a 3-factor model, we again find that all prices of risk are statistically significant with t-statistics equal to 4.05, -2.22, and -2.69, respectively. Across the 2nd, 3rd, and 4th book-to-market quintiles, the IV and IFV small-minus-big risk premia are now equal to 1.9%, 0.22%, and 0.94%, and 2.15%, 2%, and 5.52%, respectively (Table VI, Panel c). In sum, in our sample market friction volatility appears to carry a price of risk that is more statistically significant than the price of risk of market volatility. The corresponding risk premia are also larger.

5.2 *Daily frequencies*

We start with a 2-factor model with MKT and IV. Table III shows the volatility loadings computed using optimally-sampled realized variance, realized variance sampled every 5 minutes, and realized variance sampled every 20 minutes. The "optimally-sampled" loadings appear to have more spread in both the size and value dimension. In addition, they are generally more statistically significant than the alternative loadings. These findings provide further economic and statistical justification for using optimally-sampled realized variances in place of realized variances based on fixed, ad-hoc, intervals. As said, all pricing tests are based on optimally-sampled realized variances.

As in the monthly case, the portfolios excess returns largely increase in the growth-value dimension and decrease in the size dimension. Interestingly, contrary to the monthly case, the IV factor loadings increase fairly monotonically with both value and size (see Table III and Fig. 11). The size effect appears stronger, confirming the relation between size and volatility loadings documented by Moise (2006). Table V contains the prices of risk. The t-statistics associated with the market price of risk and the volatility risk are equal to 1.51 and -2.64. Despite the positive relation between value and volatility loadings, and in light of the stronger negative relation between size and volatility loadings, we find a negative price of volatility risk at daily frequency. The associated small-minus-big yearly risk premia are equal to 11%, 6.8%, and 9.6% (Table VII).

We now turn to a 2-factor model with MKT and IFV. The IFV factor loadings increase with size and decrease with value, albeit not monotonically (Fig. 12). The resulting effect is a strongly significant IFV price of risk in the daily cross-section of Fama-French portfolios. The t-statistic for the friction volatility price of risk is equal to -3.54 whereas the market price of risk carries a t-statistic equal to 1.08.

When combining MKT, IV and IFV in a 3-factor model, IFV largely subsumes the information contained in IV (the corresponding factor loadings are displayed in Fig. 13). IV becomes insignificant (t-statistic of -0.78) while the t-statistic on IFV becomes -3.82. The small-minus-big yearly IFV risk premia associated with the 2nd, 3rd, and 4th value quintile are equal to 6.4%, 9.7%, and 4.2% (Table VII).

In sum, our findings point to the importance of the role played by market frictions in pricing the cross-section of monthly and daily Fama-French portfolio returns. In our sample, market volatility plays a stronger role at the monthly frequency than at the daily frequency.

6 Robustness checks

This section evaluates the robustness of our results to the use of alternative volatility and friction measures at the monthly frequency. Specifically, we use (innovations in) monthly variances obtained by summing squared daily returns as a proxy for *IV* and *PS* as a proxy for *IFV*. When replacing optimally-sampled monthly realized variances with monthly variances constructed using daily data, our findings[11] do not change. As expected, *PS* plays a similar role as our friction measure (see Table I). On the one hand, in our sample our proxy appears to be more statistically significant than *PS* (when controlling for either market volatility or friction volatility, *PS* becomes insignificant); on the other hand, the size of the pricing errors is fairly stable across different models. We do not attribute too much importance to the superior statistical significance of *IFV* in comparison with *PS*. While this finding might be sample-specific, the similar performance of the two measures reinforces the importance of market frictions in explaining cross-sectional asset prices above and beyond the information contained in systematic market volatility.

Thus far, the analysis has been conducted in the context of a model with constant prices of risk (c.f., Eq. (5)). We now allow for time-varying risk prices. Following Fama-McBeth (1973), we estimate the cross-sectional regression implied by Eq. (5) for each month, obtain monthly estimates of the prices of risk, and assess the statistical significance of the resulting average risk prices. The corresponding results are reported in Table II. The statistical significance of the prices of risk is now lower regardless of the model used. In a two factor model with *MKT* and *IV*, or in a 3-factor model with *MKT*, *SMB*, and *HML*, our findings are qualitatively very similar to the findings obtained by Adrian and Rosenberg (2006) using the same cross-section of size and value-sorted portfolios over a longer time horizon (1963/7 - 2003/12 - c.f., Table 4 in Adrian and Rosenberg, 2006). Importantly, the statistical significance of both *IFV* and *PS* is higher than the statistical significance of the market volatility proxy, *IV*.

7 Pricing Errors

Fig. 14 and Fig. 15 display the monthly and daily pricing errors, by plotting realized mean excess returns versus predicted mean excess returns, for three models, i.e., the CAPM, the Fama-French 3-factor model, and a 3-factor model with *MKT* and two volatility components, *IV* and *IFV*.

[11] Not reported, but available from the authors upon requests.

Consider the monthly pricing errors, first. As is well-known, the Fama-French 3-factor model strongly dominates the CAPM. Interestingly, the 3-factor model with volatility components compares favorably with the Fama-French 3-factor model when pricing the cross-section of size- and value-sorted Fama-French portfolios. Not surprisingly, the main exceptions are growth portfolios, i.e., the portfolios in the first value quintile and first, second, and third size quintile (see Fig. 14). The growth portfolios have realized mean excess returns which increase across size quintiles (see Fig. 10, for instance). The corresponding IV and IFV loadings increase, too. The combination of negative prices of risk and increasing loadings leads to a divergence between realized mean excess returns and excess returns implied by the model: the realized excess returns on these portfolios are lower than the mean returns implied by the model.

The pricing errors are similar at daily levels. While the contribution of IV is less substantial at daily frequencies (c.f., Table V), our friction measure is more likely to be noisier at daily than at monthly frequencies. It seems therefore premature to downplay the role played by market variance at frequencies lower than monthly. As in the monthly case, a 3-factor model with MKT, IV, and IFV performs similarly to the Fama-French 3 factor model. As earlier, the main exceptions are portfolios in the first value quintile.

8 Conclusions

This paper studies a 3-factor asset-pricing model with market returns, (innovations in) market volatility, and (innovations in) market friction volatility. We define market friction volatility as the volatility of the difference between observed asset prices and fundamental values. Both volatility components are extracted nonparametrically from a single time-series of high-frequency Standard and Poor's depository receipts' (SPIDERS) trade prices. We show that our friction proxy correlates with macro illiquidity and asymmetric information proxies.

We find that market volatility and market friction volatility are negatively priced in the cross-section of daily and monthly 25 size- and value-sorted Fama-French portfolios. In our sample, the performance of a 3-factor model with market return, market volatility, and market friction volatility is similar to the performance of the Fama-French 3-factor model when pricing these portfolios.

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Appendix: Figures and Tables

Figure 1. Autocorrelation function of the observed SPDR returns

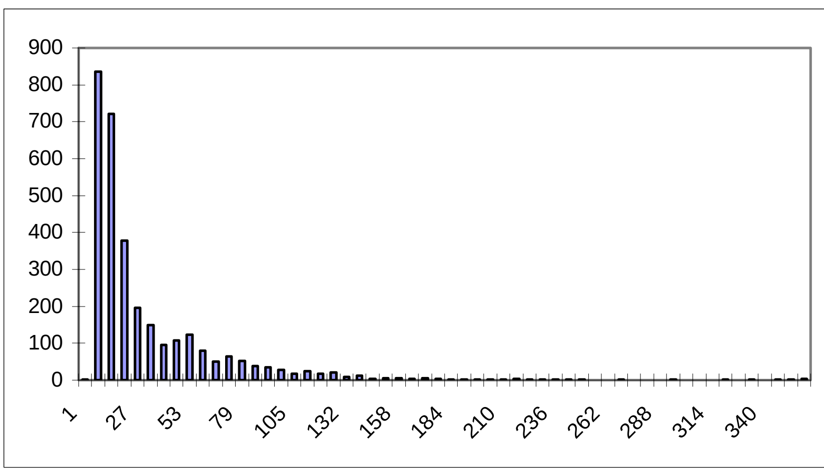
High-frequency transaction prices on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. The data are collected from the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific, and Philadelphia.

```
Lag   Covariance   Correlation   -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1     Std Error

  0   3.26278E-7       1.00000   |                    |********************|           0
  1   -7.1485E-8       -.21909   |                ****| .                  |    0.044811
  2   -1.7989E-9       -.00551   |                  . | .                  |    0.046913
  3   -4.2528E-9       -.01303   |                  . | .                  |    0.046914
  4   -3.2565E-8       -.09981   |                  **| .                  |    0.046921
  5   3.18552E-9       0.00976   |                  . | .                  |    0.047346
  6   1.33231E-8       0.04083   |                  . |*.                  |    0.047350
  7   -2.3546E-8       -.07216   |                  .*| .                  |    0.047420
  8   3.62875E-9       0.01112   |                  . | .                  |    0.047640
  9   -2.1113E-8       -.06471   |                  .*| .                  |    0.047646
 10   3.64909E-9       0.01118   |                  . | .                  |    0.047822
 11   5.31233E-9       0.01628   |                  . | .                  |    0.047827
 12   1.66286E-8       0.05096   |                  . |*.                  |    0.047838
 13   -2.4186E-9       -.00741   |                  . | .                  |    0.047947
 14     -9.81E-9       -.03007   |                  .*| .                  |    0.047949
 15    1.6344E-8       0.05009   |                  . |*.                  |    0.047987
 16   -3.012E-10       -.00092   |                  . | .                  |    0.048092
 17   -4.2426E-9       -.01300   |                  . | .                  |    0.048092
 18   -3.0794E-9       -.00944   |                  . | .                  |    0.048099
 19   2.92423E-9       0.00896   |                  . | .                  |    0.048103
 20   -1.3661E-8       -.04187   |                  .*| .                  |    0.048106
 21   1.65097E-8       0.05060   |                  . |*.                  |    0.048179
 22   1.56147E-8       0.04786   |                  . |*.                  |    0.048286
 23   8.77186E-9       0.02688   |                  . |*.                  |    0.048381
 24   1.17495E-8       0.03601   |                  . |*.                  |    0.048411

"." marks two standard errors
```

Figure 2. Optimal sampling frequencies

Panel (a) plots a histogram of daily (MSE-based) optimal sampling frequencies for the realized variance estimator $\sum_{j=1}^{M^*} \tilde{r}_j^2$ constructed using SPDR return data. The optimal frequencies are estimated as δ=1/M*, where $M^* = \left(\hat{Q}/\hat{\alpha}\right)^{1/3}$, with the numerator representing the 15-minutes quarticity estimator $\hat{Q} = \left(\sum_{j=1}^{\tilde{M}} \tilde{r}_j^4\right)\tilde{M}/3$ and the denominator being defined as $\hat{\alpha} = \left(\sum_{j=1}^{\tilde{M}} \tilde{r}_j^2 / M\right)^2$. $\widetilde{M}$ and M denote the number of 15-minute SPDR returns and the number of observed SPDR returns over the trading day. On average, M=3,000. Panel (b) plots the (daily) time series plot of the optimal intervals. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected for the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia.

Panel (a)



Mean	28.59
Median	13.99
Std. Dev.	36.87
Kurtosis	17.97
Skewness	3.37
Minimum	1.00
Maximum	360.00

Panel (b)

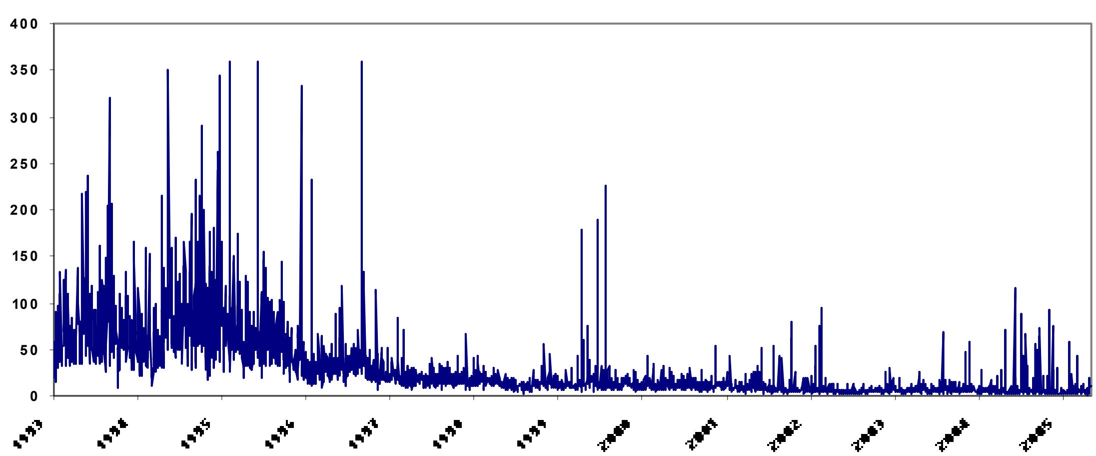
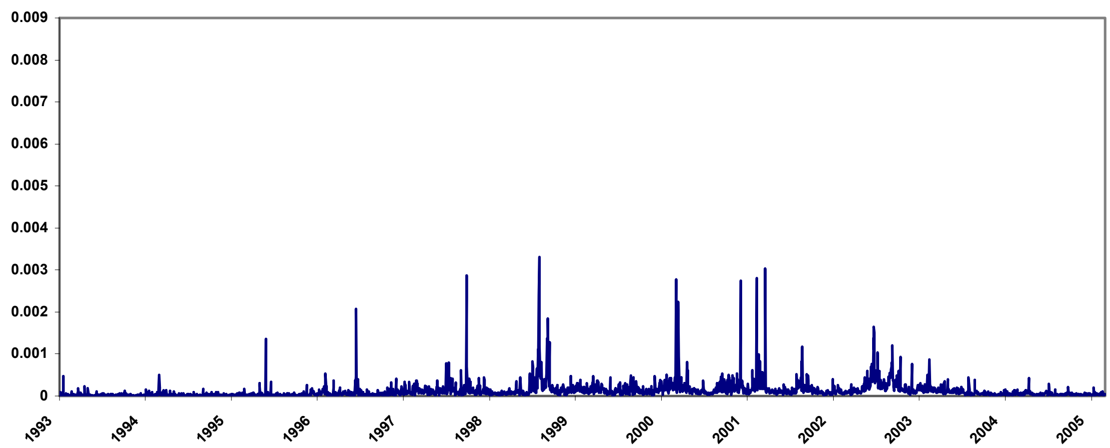


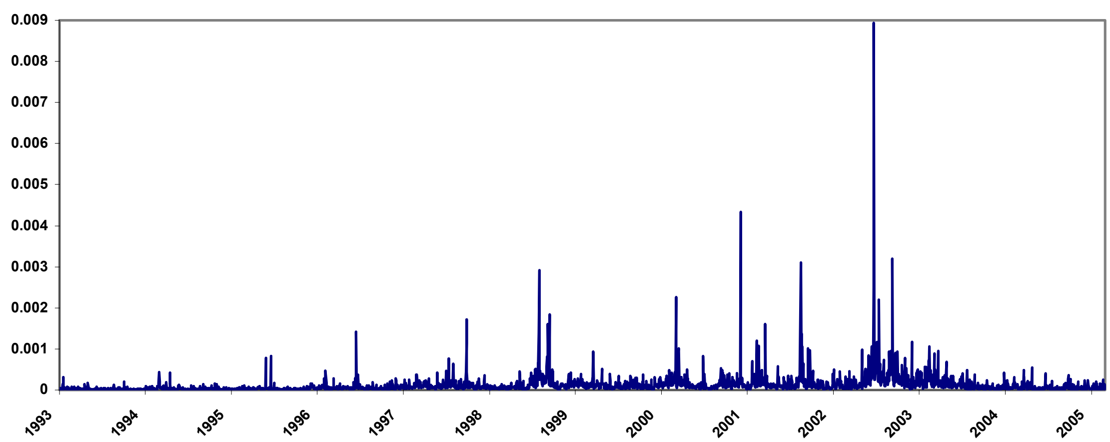
Figure 3. Daily realized variance estimates

Panel (a) plots the time series of daily realized variance estimates constructed using optimally-sampled SPDR returns. Panel (b) plots the time series of daily realized variance estimates constructed using 20-minute SPDR returns. Panel (c) plots the time series of daily realized variance estimates constructed using 5-minute SPDR returns. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected for the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia.

Panel (a) Optimally-sampled realized variances



Panel (b) 20-minute realized variances



Panel (c) 5-minute realized variance

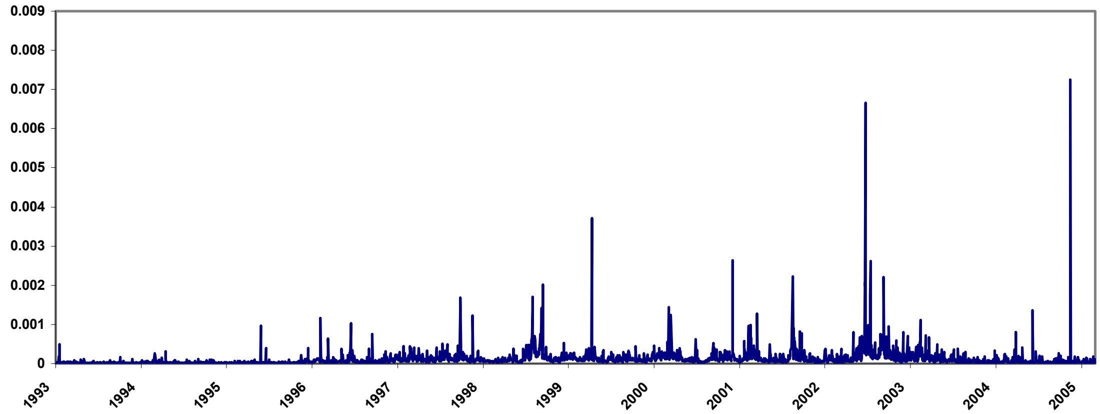


Figure 4. Friction second moment estimates

The figure plots the daily time series of friction second moment estimates constructed using SPDR returns. For each day in our sample, the estimates are obtained by computing un-centered second moments of the SPDR high-frequency tick-by-tick returns. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected for the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia.

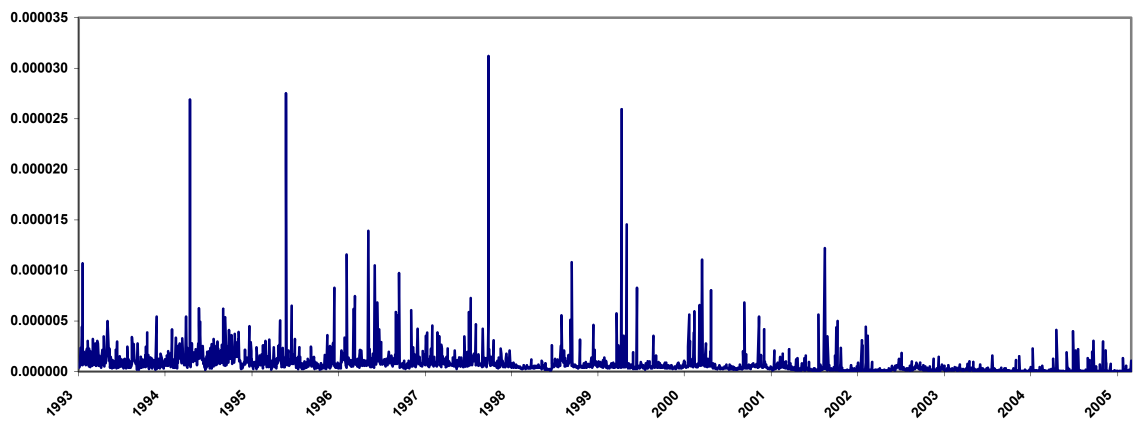


Figure 5. Monthly variances

We plot monthly variances computed using sums of daily optimally-sampled realized variance estimates over the month (var-hi freq) and monthly variances computed by summing up squared daily S&P 500 returns (var-low fre). The daily optimally-sampled realized variances sum up squared optimally-sampled intra-daily SPDR returns. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected for the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia.

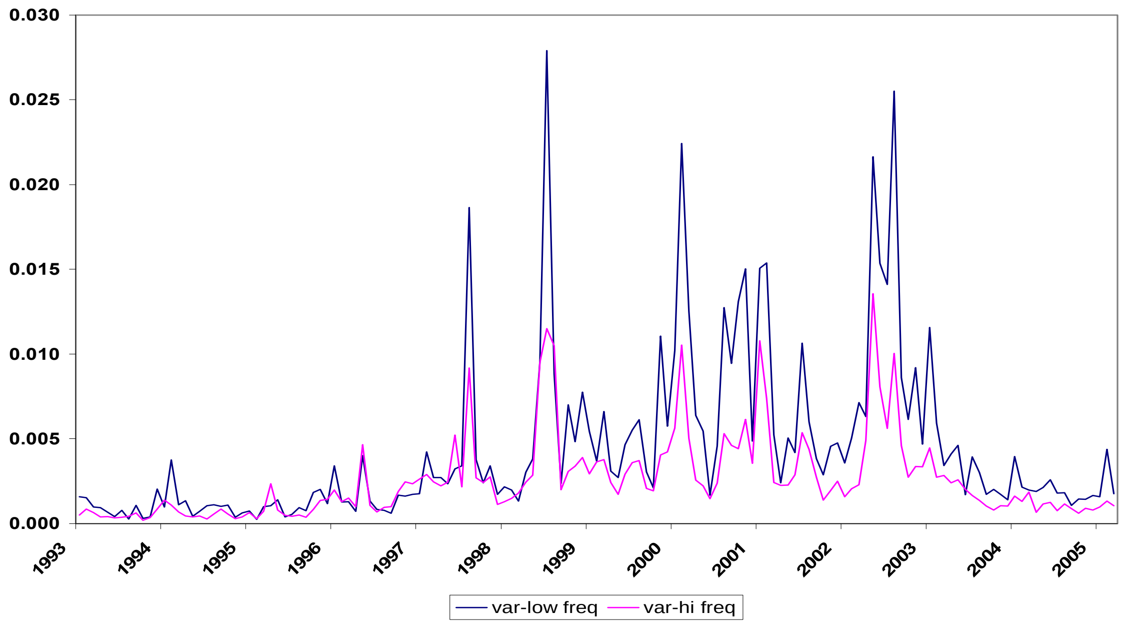


Figure 6. Monthly friction volatility

This figure plots the time series of monthly friction volatilities obtained from high-frequency SPDR returns. For each day in our sample, we compute un-centered second moments of the SPDR high-frequency tick-by-tick returns. The monthly friction volatility estimates are calculated by taking the square root of the averages of the daily estimates over the month. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected from the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia. The months highlighted below are:

May 1994: rate hike (The Credit Union Accountant)

October/November 1994: significant liquidity shortage in the market associated with exacerbated volatile market conditions and the Peso crisis (The Financial Post);

June 1995: Fed announced carrying out weekend system repos since the market needed extra liquidity (AFX News);

July 1996: weaker-than-expected employment report (Investors Chronicle);

October 1997: Asian crisis;

October/November 1998: LTCM crisis and the Russian debt default;

May 1999: Uncertainty over the direction of monetary policy in Argentina and anticipation of a rate hike by US Fed (Emerging Markets Debt Report);

April 2000: big increase in oil price (Financial Director);

September 2001: the 9/11 attack;

March 2002: worries about the Iraq war (Financial Times);

August 2004: worries about oil prices and global liquidity (Business Line).

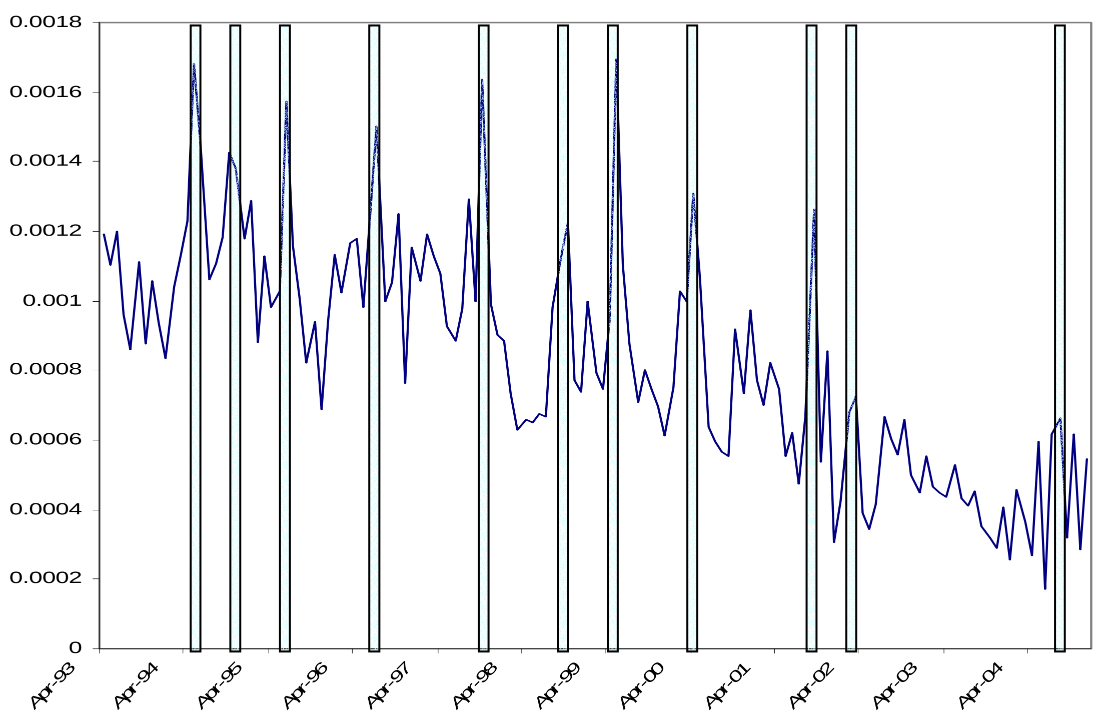
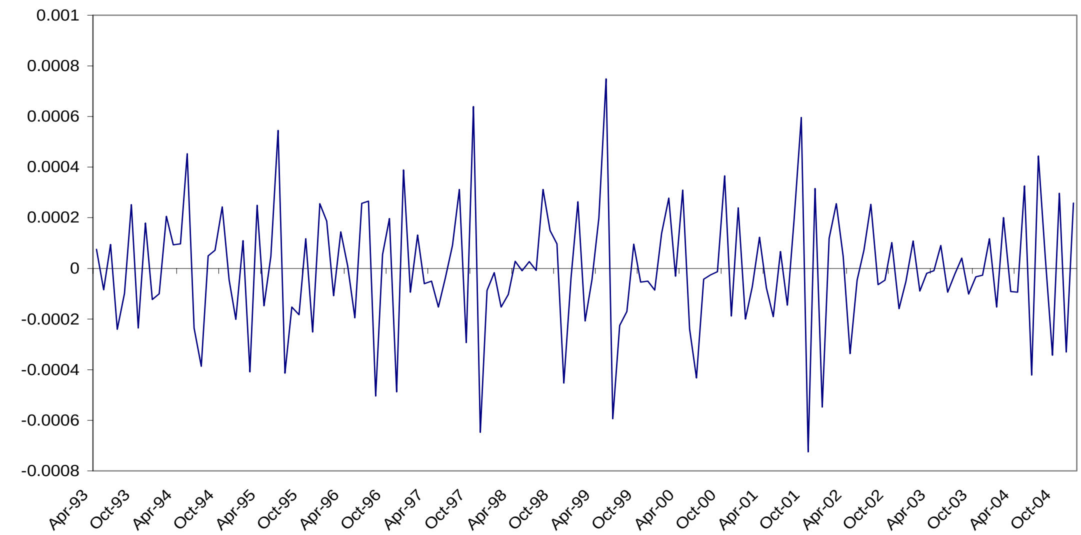


Figure 7. Two friction proxies

Panel (a) plots innovations in the monthly time series of friction volatilities constructed using SPDR returns. For each day in our sample, we compute un-centered second moments of the SPDR high-frequency tick-by-tick returns. The monthly friction volatility estimates are calculated by taking the square root of the averages of the daily estimates over the month. High-frequency transaction price data on the Standard & Poor's depository receipts (SPDRs) are downloaded from the Trade and Quote (TAQ) database in CRSP for the period February 1993 – March 2005. Trade prices on SPDRs are collected from the consolidated market, which comprises the following exchanges: AMEX, CBOE, NASDAQ, NYSE, Boston, Cincinnati, Midwest, Pacific and Philadelphia. Panel (b) plots innovations in Pastor and Stambaugh's (2003) liquidity measure over the same period. Pastor and Stambaugh's measure is downloaded from CRSP.

Panel (a) Innovations in monthly friction volatility



Panel (b) Innovations in the liquidity factor of Pastor and Stambaugh (2003)

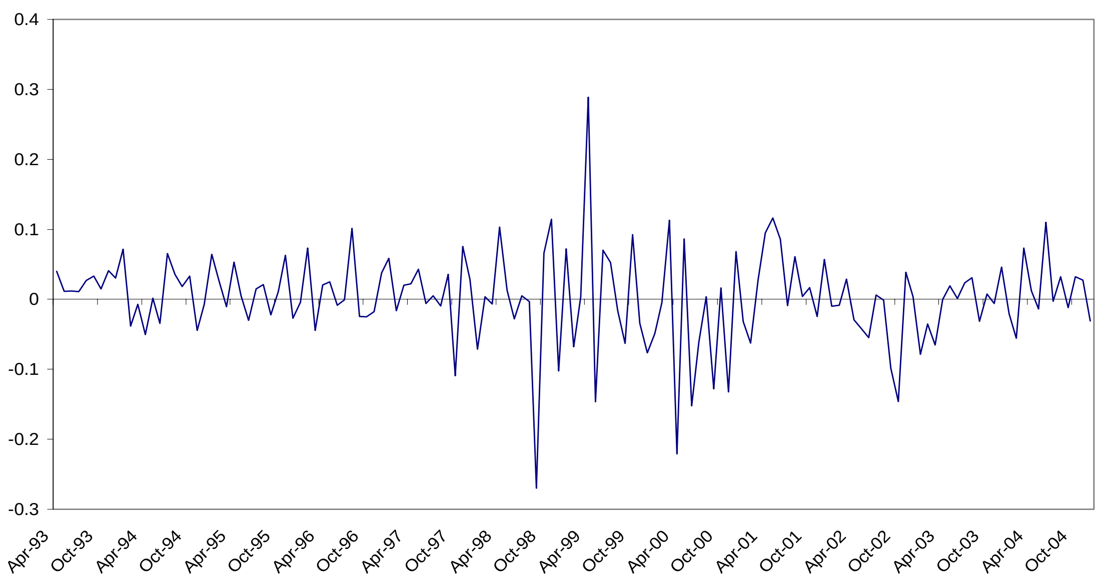


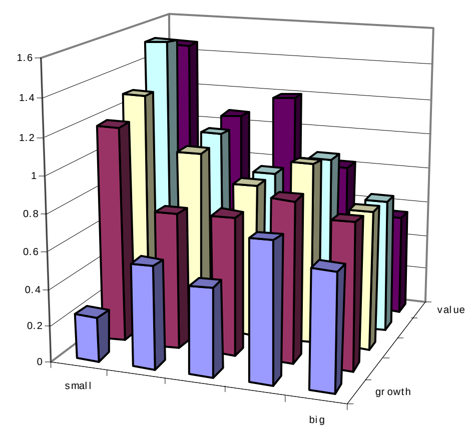
Figure 8. Monthly average returns and optimally-sampled volatility loadings

Panel (a) plots monthly average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panel (b) plots the volatility factor loadings (with a minus sign) associated with innovations in optimally-sampled realized variance (IV) from the regression:

$$\begin{cases} R^{e}_{i,t} = \alpha_i + \beta_i^{m} R^{e}_{m,t} + \beta_i^{IV} IV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T, where $R^{e}_{i,t}$ denotes excess returns on portfolio i and $R^{e}_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average monthly excess returns



Panel (b) Volatility loadings $-\hat{\beta}_i^{IV}$

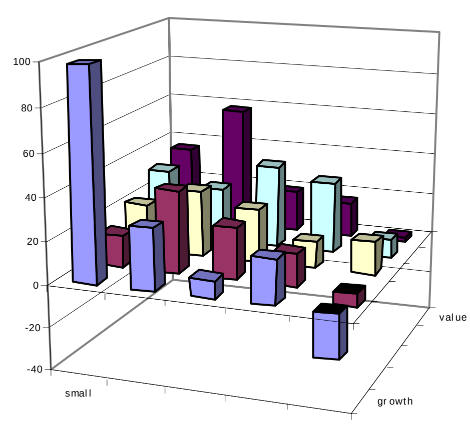


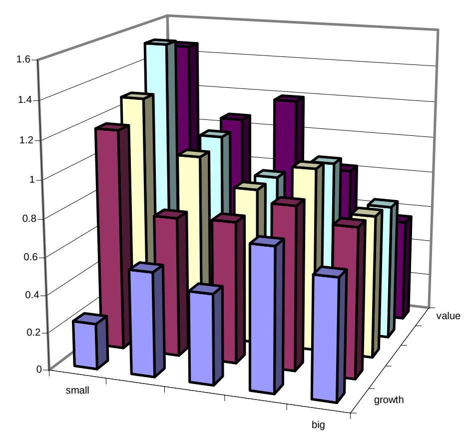
Figure 9. Monthly average returns and friction volatility loadings

Panel (a) plots monthly average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panel (b) plots the friction volatility factor loadings (with a minus sign) associated with innovations in friction volatility (IFV) from the regression:

$$\begin{cases} R^e_{i,t} = \alpha_i + \beta^m_i R^e_{m,t} + \beta^{IFV}_i IFV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T, where $R^e_{i,t}$ denotes excess returns on portfolio i and $R^e_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average monthly excess returns



Panel (b) Friction volatility loadings $-\hat{\beta}^{IFV}_i$

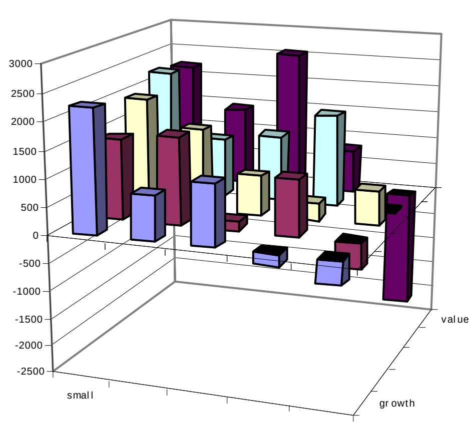


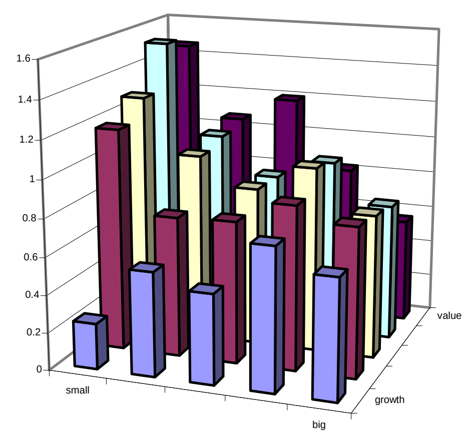
Figure 10. Monthly average returns and volatility loadings

Panel (a) plots monthly average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panels (b) and (c) plot market volatility factor loadings (with a minus sign) associated with innovations in market volatility (IV) and friction volatility factor loadings (with a minus sign) associated with innovations in friction volatility (IFV) from the regression

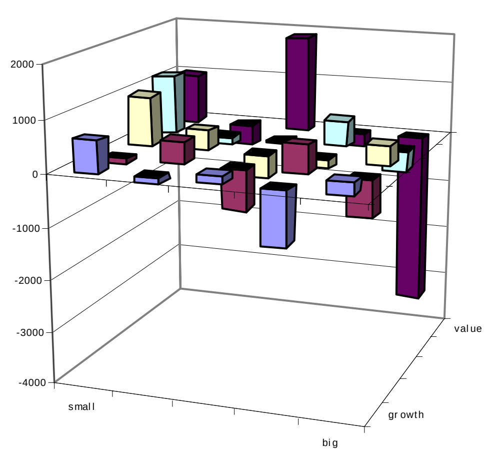
$$\begin{cases} R^{e}_{i,t} = \alpha_i + \beta^{m}_{i} R^{e}_{m,t} + \beta^{IV}_{i} IV_t + \beta^{IFV}_{i} IFV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T, where $R^{e}_{i,t}$ denotes excess returns on portfolio i and $R^{e}_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average monthly excess returns



Panel b) Friction volatility loadings $-\hat{\beta}^{IFV}_{i}$



Panel (c) Volatility loadings $-\hat{\beta}_i^{IV}$

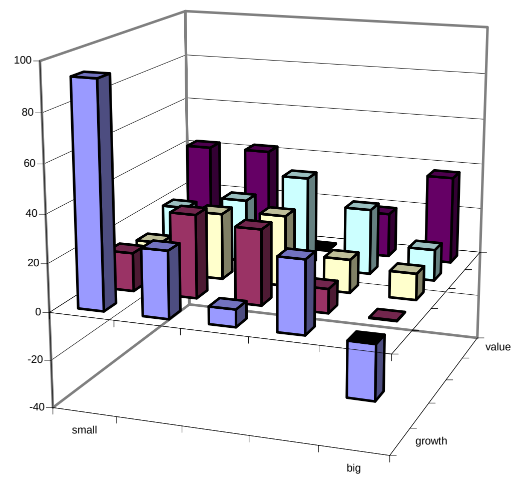


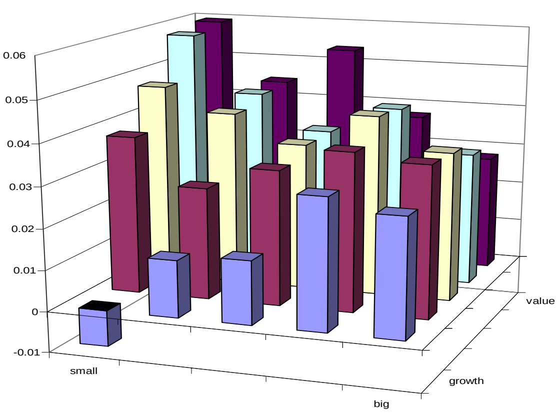
Figure 11. Daily average returns and optimally-sampled volatility loadings

Panel (a) plots daily average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panel (b) plots the volatility factor loadings (with a minus sign) associated with innovations in optimally-sampled realized variance (IV) from the regression

$$\begin{cases} R^{e}_{i,t} = \alpha_i + \beta^{m}_{i} R^{e}_{m,t} + \beta^{IV}_{i} IV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i} \varepsilon^{2}_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T, where $R^{e}_{i,t}$ denotes excess returns on portfolio i and $R^{e}_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average daily excess returns



Panel (b) Volatility loadings $-\hat{\beta}^{IV}_{i}$

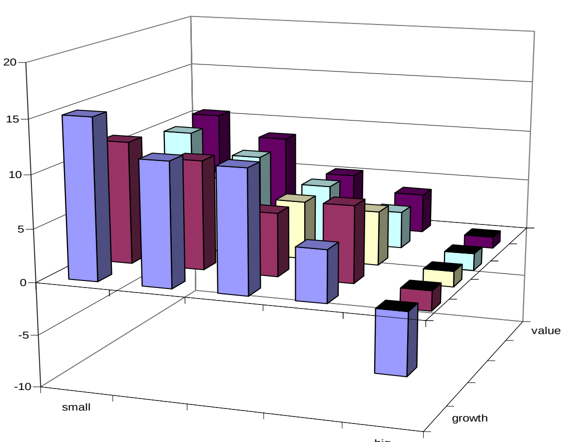


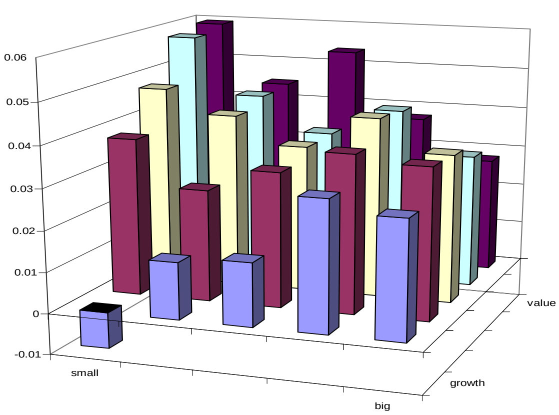
Figure 12. Daily average returns and friction volatility loadings

Panel (a) plots daily average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panel (b) plots friction volatility factor loadings (with a minus sign) associated with innovations in friction volatility (IFV) from the regression

$$\begin{cases} R^e_{i,t} = \alpha_i + \beta_i^m R^e_{m,t} + \beta_i^{IFV} IFV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}} e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

$i=1,\ldots 25$, $t=1,\ldots T$, where $R^e_{i,t}$ denotes excess returns on portfolio i and $R^e_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average daily excess returns



Panel (b) Friction volatility loadings $-\hat{\beta}_i^{IFV}$

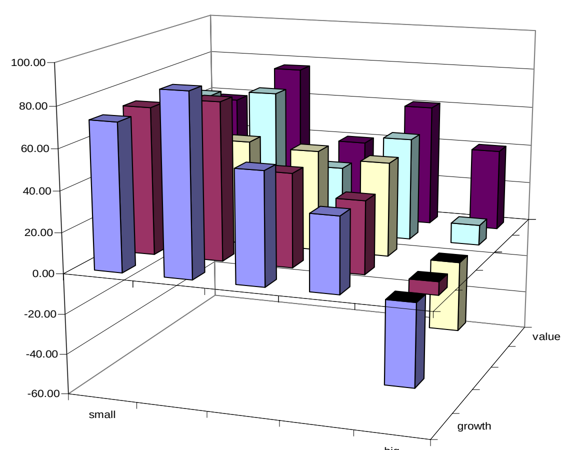


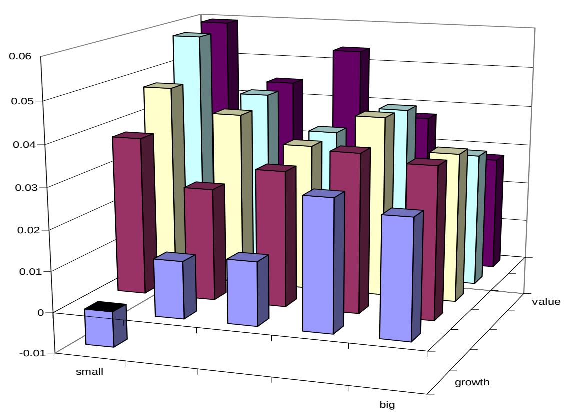
Figure 13. Daily average returns and volatility loadings

Panel (a) plots daily average excess returns for the 25 Fama-French size- and value-sorted portfolio. The return data is collected for the period February 1993 – March 2005. Panels (b) and (c) plot market volatility factor loadings (with a minus sign) associated with innovations in market volatility (IV) and friction volatility factor loadings (with a minus sign) associated with innovations in friction volatility (IFV) from the regression

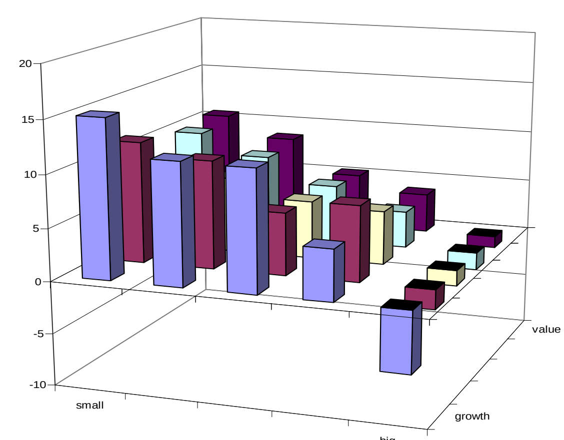
$$\begin{cases} R^e_{i,t} = \alpha_i + \beta^m_i R^e_{m,t} + \beta^{IV}_i IV_t + \beta^{IFV}_i IFV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

$i=1,\ldots 25$, $t=1,\ldots T$, where $R^e_{i,t}$ denotes excess returns on portfolio i and $R^e_{m,t}$ denotes excess returns on the market portfolio.

Panel (a) Average daily excess returns



Panel (b) Friction volatility loadings $-\hat{\beta}^{IFV}_i$



Panel (c) Volatility Loadings $\hat{\beta}_i^{IV}$

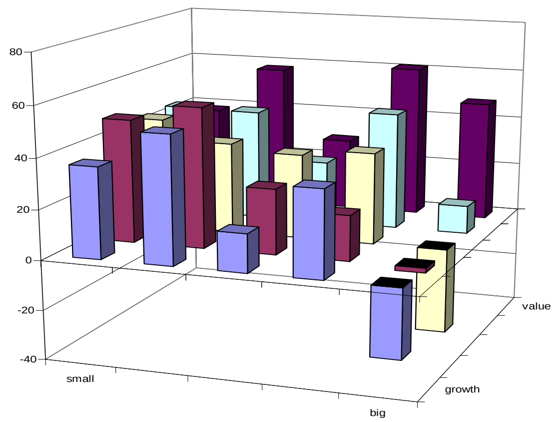
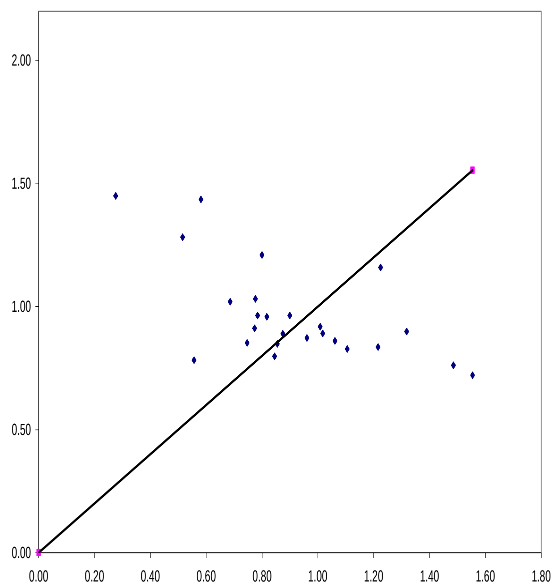


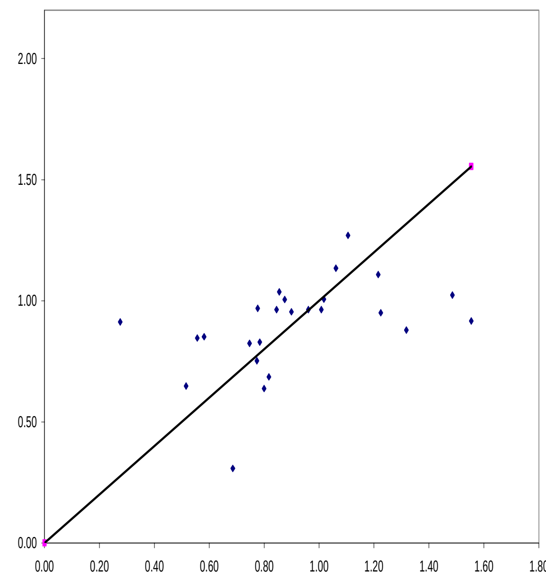
Figure 14. Predicted versus realized monthly average excess returns – 25 Fama-French portfolios

Predicted mean excess returns are plotted on the vertical axis, mean realized excess returns are plotted on the horizontal axis. We use 3 asset pricing models: the CAPM, a 3-factor model consisting of market excess return, innovations in market volatility (IV) and innovations in friction volatility (IFV), and the Fama-French 3 factor model, FF-3. In Panel (c) SiVj stands for the portfolio in the ith size quintile and in the jth value quintile.

Panel (a) CAPM



Panel (b) FF-3 Factor model (R_m^e, SMB, HML)



Panel (c) 3-Factor model (R_m^e, IV, IFV)

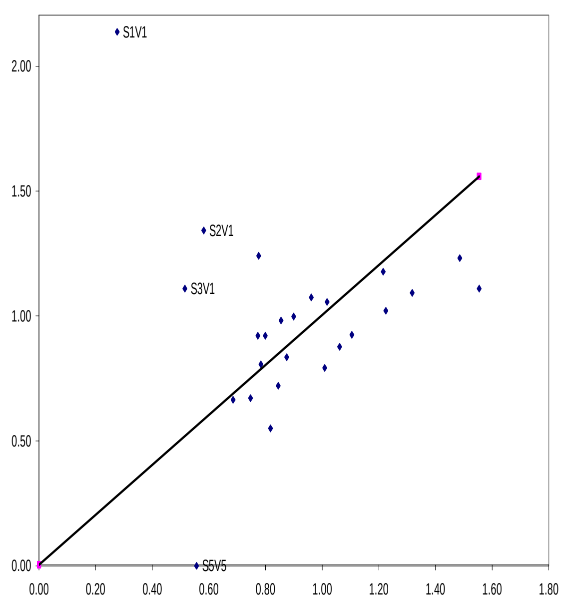
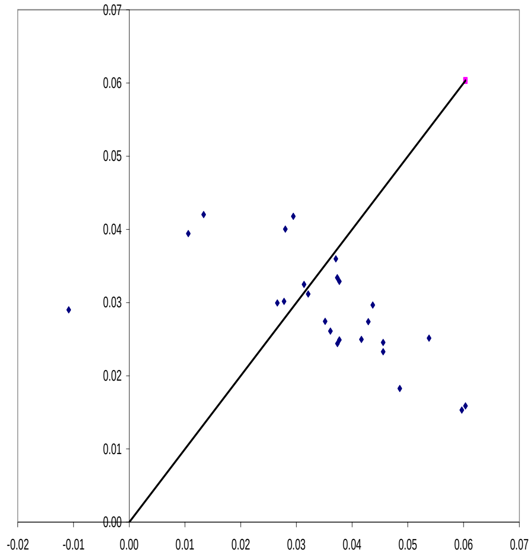


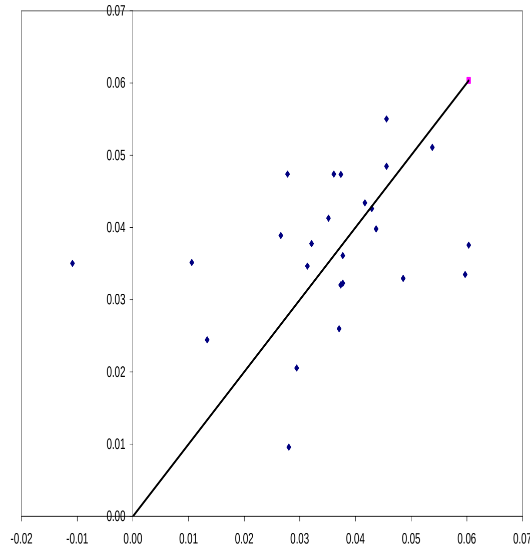
Figure 15. Predicted versus realized daily average excess returns – 25 Fama-French portfolios

Predicted mean excess returns are plotted on the vertical axis, mean realized excess returns are plotted on the horizontal axis. We use 3 asset pricing models: the CAPM, a 3-factor model consisting of market excess return, innovations in market volatility (IV) and innovations in friction volatility (IFV), and the Fama-French 3 factor model, FF-3. In Panel (c) SiVj stands for the portfolio in the ith size quintile and in the jth value quintile.

Panel (a) CAPM



Panel (b) FF-3 Factor model (R_m^e, SMB, HML)



Panel (c) 3-Factor model (R_m^e, IV, IFV)

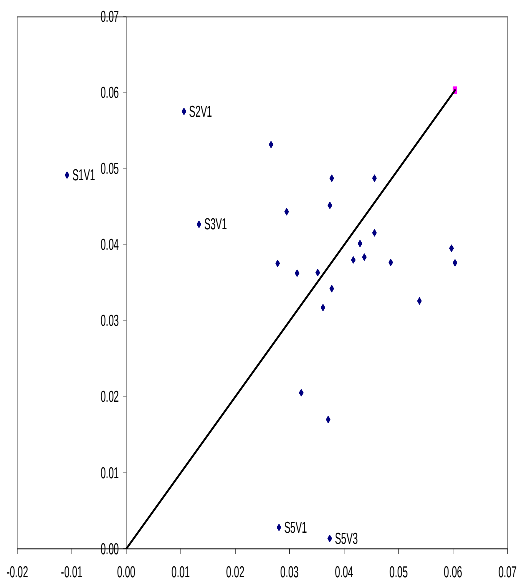


Table I. Constant prices of risk – monthly data

We report the estimated factors' risk prices from the asset pricing model:

$$E(R^e_{i,t+1}) = \lambda_m\beta^m_i + \lambda_{IV}\beta^{IV}_i + \lambda_{IFV}\beta^{IFV}_i + \lambda_{PS}\beta^{PS}_i + \lambda_{SMB}\beta^{SMB}_i + \lambda_{HML}\beta^{HML}_i + \lambda_{MOM}\beta^{MOM}_i$$

The left-hand side variable is the vector of mean excess returns on the Fama-French 25 portfolios, the betas represent factors' risk loadings estimated from the corresponding time-series model. IV denotes innovations in monthly optimally-sampled realized volatility, IFV denotes innovations in friction volatility, and PS denotes innovations in the liquidity factor of Pastor and Stambaugh (2003). SMB, HML, and MOM refer to the classical size, value, and momentum factors. The time period is February 1993 – March 2005.

$\hat{\lambda}_m$	$\hat{\lambda}_{IV}$	$\hat{\lambda}_{IFV}$	$\hat{\lambda}_{PS}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	J_T	p	df
0.7397	-0.0111						9.9368	0.9917	23
(4.00)	-(2.54)								
0.6999		-0.0003					7.3829	0.9992	23
(4.05)		-(3.45)							
0.8724			0.0519				6.7968	0.9996	23
(4.97)			(2.23)						
0.7691	-0.0100	-0.0003					9.6164	0.9895	22
(4.05)	-(2.22)	-(2.69)							
0.6698	-0.0118		0.0375				8.6923	0.9948	22
(3.63)	-(2.81)		(1.55)						
0.6839		-0.0003	0.0282				8.7111	0.9947	22
(3.91)		-(2.90)	(1.12)						
0.6963	-0.0110	-0.0002	0.0322				9.3396	0.9863	21
(3.62)	-(2.57)	-(2.39)	(1.23)						
0.5913				0.2754	0.3740		4.6856	0.9999	22
(2.69)				(0.80)	(1.71)				

Table II. Time-varying prices of risk (Fama-McBeth, 1973) – monthly data

We report the estimated factors' risk prices from the asset pricing model:

$$E(R^{e}_{i,t+1}) = \lambda_{m}\beta^{m}_{i} + \lambda_{IV}\beta^{IV}_{i} + \lambda_{IFV}\beta^{IFV}_{i} + \lambda_{PS}\beta^{PS}_{i} + \lambda_{SMB}\beta^{SMB}_{i} + \lambda_{HML}\beta^{HML}_{i} + \lambda_{MOM}\beta^{MOM}_{i}$$

The left-hand side variable is the vector of excess returns on the Fama-French 25 portfolios, the betas represent factors' risk loadings estimated from the corresponding time-series model. The model is re-estimated every month following Fama and McBeth (1973). The resulting risk-prices are then averaged. IV denotes innovations in monthly optimally-sampled realized volatility, IFV denotes innovations in friction volatility, and PS denotes innovations in the liquidity factor of Pastor and Stambaugh (2003). SMB, HML, and MOM refer to the classical size, value, and momentum factors. The time period is February 1993 – March 2005.

$\hat{\lambda}_{m}$	$\hat{\lambda}_{IV}$	$\hat{\lambda}_{IFV}$	$\hat{\lambda}_{PS}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	MSE	R^2	AdjR^2
0.8305	-0.0024						8.884	0.80	0.78
(2.27)	-(0.53)								
0.6779		-0.0002					8.5499	0.84	0.83
(1.82)		-(2.10)							
0.7374			0.0637				9.016	0.84	0.83
(1.81)			(2.49)						
0.8674	-0.0008	-0.0002					8.757	0.82	0.79
(2.36)	-(0.20)	-(1.78)							
0.7214	-0.0049		0.0477				7.948	0.82	0.80
(1.95)	-(1.13)		(1.87)						
0.6444		-0.0002	0.0563				6.605	0.86	0.84
(1.72)		-(1.66)	(1.76)						
0.7546	-0.0037	-0.0002	0.0423				7.4437	0.83	0.79
(2.00)	-(1.03)	-(1.63)	(1.46)						
0.5442				0.3214	0.4252		3.181	0.92	0.91
(1.45)				(0.90)	(1.30)				

Table III. Estimates of the volatility loadings

We consider daily return data on the 25 size- and value-sorted Fama-French portfolios. The time period is February 1993 – March 2005. The time series of portfolios excess returns ($R^e_{i,t}$) is regressed on the excess market returns ($R^e_{m,t}$) and innovations in volatility (IV_t) using a model in which the errors follow an AR(1)-IGARCH(1,1) specification:

$$\begin{cases} R^e_{i,t} = \alpha_i + \beta_i^m R^e_{m,t} + \beta_i^{IV} IV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T. The volatility measures used are: optimally-sampled realized volatility, 5-minutes realized volatility, and 20-minutes realized volatility. We estimate the model by maximum likelihood. The t-statistics are reported in parentheses.

	Book-to-Market Equity (BE/ME) Quintiles									
Size Quintiles	Low	2	3	4	High	Low	2	3	4	High
					$\hat{\beta}_i^{IV}$ – Optimally Sampled					
Small	-15.14	-11.60	-6.84	-9.97	-10.56	-(8.06)	-(6.87)	-(4.74)	-(8.12)	-(8.85)
2	-11.66	-10.33	-7.20	-8.00	-8.56	-(5.04)	-(5.59)	-(4.83)	-(4.52)	-(5.39)
3	-11.56	-5.96	-5.51	-5.56	-5.24	-(5.57)	-(3.96)	-(3.73)	-(3.89)	-(4.00)
4	-4.87	-7.24	-5.11	-3.52	-3.79	-(2.75)	-(6.10)	-(3.89)	-(2.75)	-(2.38)
Big	5.91	1.90	1.48	1.67	1.11	(6.96)	(1.61)	(1.08)	(1.08)	(0.57)
					$\hat{\beta}_i^{IV}$ – IV20					
Small	-12.92	-10.20	-5.08	-7.50	-8.04	-(6.91)	-(6.28)	-(4.04)	-(6.68)	-(7.59)
2	-10.14	-8.01	-3.60	-4.87	-7.46	-(5.33)	-(4.99)	-(2.45)	-(2.90)	-(4.75)
3	-7.48	-3.94	-3.51	-3.50	-7.01	-(4.39)	-(2.84)	-(2.63)	-(2.47)	-(5.38)
4	-3.06	-3.51	-5.81	-3.43	-5.40	-(2.21)	-(3.34)	-(5.14)	-(3.01)	-(4.13)
Big	4.09	-0.19	1.68	1.98	0.13	(5.27)	-(0.19)	(1.37)	(1.56)	(0.08)
					$\hat{\beta}_i^{IV}$ – IV5					
Small	-11.83	-10.31	-5.16	-7.44	-8.52	-(6.12)	-(5.86)	-(3.64)	-(5.91)	-(6.79)
2	-9.36	-8.98	-4.47	-6.77	-8.48	-(4.55)	-(5.03)	-(2.61)	-(3.54)	-(4.75)
3	-7.03	-3.53	-3.59	-4.63	-8.29	-(3.60)	-(2.32)	-(2.32)	-(2.84)	-(5.43)
4	-2.97	-3.40	-6.51	-5.69	-5.40	-(1.88)	-(2.83)	-(4.64)	-(4.25)	-(3.36)
Big	4.32	0.13	0.48	0.35	-0.83	(4.91)	(0.11)	(0.35)	(0.25)	-(0.46)

Table IV. Estimates of the friction volatility loadings

We consider daily return data on the 25 size- and value-sorted Fama-French portfolios. The time period is February 1993 – March 2005. The time series of portfolios excess returns ($R^e_{i,t}$) is regressed on the excess market returns ($R^e_{m,t}$), innovations in volatility (IV_t), and innovations in friction volatility (IFV) using a model in which the error follows an AR(1)-IGARCH(1,1) specification:

$$\begin{cases} R^e_{i,t} = \alpha_i + \beta^m_i R^e_{m,t} + \beta^{IV}_i IV_t + \beta^{IFV}_i IFV_t + \eta_{i,t} \\ \eta_{i,t} = \varepsilon_{i,t} - \gamma_i \eta_{i,t-1} \\ \varepsilon_{i,t} = \sqrt{h_{i,t}}\, e_{i,t} \\ h_{i,t} = \omega_i + \theta_{1,i}\varepsilon^2_{i,t-1} + \phi_{1,i} h_{i,t-1} \\ e_{i,t} \sim N(0,1) \end{cases}$$

i=1,… 25, t=1,…T. We estimate the model by maximum likelihood. The t-statistics are reported in parentheses.

Size Quintiles	Book-to-Market Equity (BE/ME) Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	$\hat{\beta}^{IFV}_i$									
Small	-36.43	-49.29	-44.38	-44.66	-38.48	-(1.73)	-(2.66)	-(2.79)	-(3.43)	-(3.01)
2	-50.91	-56.03	-36.38	-44.29	-57.81	-(2.44)	-(3.22)	-(2.60)	-(2.78)	-(3.92)
3	-15.29	-26.09	-33.79	-25.03	-28.88	-(0.79)	-(1.88)	-(2.79)	-(1.83)	-(2.07)
4	-34.67	-18.10	-36.43	-46.99	-61.04	-(2.07)	-(1.76)	-(3.16)	-(4.37)	-(4.80)
Big	27.79	1.88	33.90	-11.15	-48.11	(3.40)	(0.16)	(2.43)	-(0.82)	-(3.05)

Table V. Constant prices of risk – daily data

We report the estimated factors' risk prices from the asset pricing model:

$$E(R^{e}_{i,t+1}) = \lambda_{m}\beta^{m}_{i} + \lambda_{IV}\beta^{IV}_{i} + \lambda_{IFV}\beta^{IFV}_{i} + \lambda_{SMB}\beta^{SMB}_{i} + \lambda_{HML}\beta^{HML}_{i} + \lambda_{MOM}\beta^{MOM}_{i}$$

The left-hand side variable is the vector of mean excess returns on the Fama-French 25 portfolios, the betas represent factors' risk loadings estimated from the corresponding time-series model. IV denotes innovations in daily optimally-sampled realized volatility and IFV denotes innovations in friction volatility. SMB, HML, and MOM refer to the classical size, value, and momentum factors. The period February 1993 – March 2005.

$\hat{\lambda}_{m}$	$\hat{\lambda}_{IV}$	$\hat{\lambda}_{IFV}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	J_T	p	df
0.0198	-0.0033					31.977	0.1006	23
(1.51)	-(2.64)							
0.0141		-0.0006				22.8503	0.4695	23
(1.08)		-(3.54)						
0.0201	-0.0011	-0.0005				23.8349	0.9863	22
(1.54)	-(0.78)	-(3.82)						
0.0239			0.0099	0.0240		6.7489	0.9992	22
(1.80)			(0.55)	(2.88)				

Table VI. Monthly risk premia

The differences in risk premia between small stocks and big stocks are estimated as the product between the factors' risk prices and the differences in the factors' loadings between small and big stocks. IV represents innovations in optimally-sampled volatility. IFV represents innovations in market friction volatility. Results are reported in percentages, per month.

Panel (a) The model is: $E(R^{e}_{i,t+1}) = \lambda_m \beta^{m}_{i} + \lambda_{IV} \beta^{IV}_{i}$

	Book-to-Market Equity (BE/ME) Quintiles				
	Low	**2**	**3**	**4**	**High**
Average Excess Returns	0.5339	0.8708	0.9759	1.0131	1.0215
$\hat{\lambda}_{IV}(\hat{\beta}^{IV}_{Small} - \hat{\beta}^{IV}_{Big})$	1.3230	0.2359	0.0791	0.2616	0.3863

Panel (b) The model is: $E(R^{e}_{i,t+1}) = \lambda_m \beta^{m}_{i} + \lambda_{IFV} \beta^{IFV}_{i}$

	Book-to-Market Equity (BE/ME) Quintiles				
	Low	**2**	**3**	**4**	**High**
Average Excess Returns	0.5339	0.8708	0.9759	1.0131	1.0215
$\hat{\lambda}_{IFV}(\hat{\beta}^{IFV}_{Small} - \hat{\beta}^{IFV}_{Big})$	0.7979	0.5788	0.4019	0.6941	1.2971

Panel (c) The model is $E(R^{e}_{i,t+1}) = \lambda_m \beta^{m}_{i} + \lambda_{IV} \beta^{IV}_{i} + \lambda_{IFV} \beta^{IFV}_{i}$

	Book-to-Market Equity (BE/ME) Quintiles				
	Low	**2**	**3**	**4**	**High**
Average Excess Returns	0.5339	0.8708	0.9759	1.0131	1.0215
$\hat{\lambda}_{IV}(\hat{\beta}^{IV}_{Small} - \hat{\beta}^{IV}_{Big})$	1.1584	0.1580	0.0187	0.0797	0.0461
$\hat{\lambda}_{IFV}(\hat{\beta}^{IFV}_{Small} - \hat{\beta}^{IFV}_{Big})$	0.1071	0.1793	0.1730	0.4612	1.2812

Table VII. Daily risk premia

The differences in risk premia between small stocks and big stocks are estimated as the product between the factors' risk prices and the differences in the factors' loadings between small and big stocks. IV represents innovations in optimally-sampled volatility. IFV represents innovations in market friction volatility. Results are reported in percentages, per day.

Panel (a) The model is: $E(R^{e}_{i,t+1}) = \lambda_m \beta^{m}_{i} + \lambda_{IV} \beta^{IV}_{i}$

	Book-to-Market Equity (BE/ME) Quintiles				
	Low	**2**	**3**	**4**	**High**
Average Excess Returns	-0.0373	0.0017	0.0117	0.0258	0.0316
$\hat{\lambda}_{IV}(\hat{\beta}^{IV}_{Small} - \hat{\beta}^{IV}_{Big})$	0.0695	0.0446	0.0275	0.0384	0.0385

Panel (b) The model is: $E(R^{e}_{i,t+1}) = \lambda_m \beta^{m}_{i} + \lambda_{IV} \beta^{IV}_{i} + \lambda_{IFV} \beta^{IFV}_{i}$

	Book-to-Market Equity (BE/ME) Quintiles				
	Low	**2**	**3**	**4**	**High**
Average Excess Returns	-0.0373	0.0017	0.0117	0.0258	0.0316
$\hat{\lambda}_{IFV}(\hat{\beta}^{IFV}_{Small} - \hat{\beta}^{IFV}_{Big})$	0.0321	0.0256	0.0391	0.0168	-0.0048

Table VIII. Cross-correlation matrix of the daily factors

IV represents innovations in optimally-sampled volatility, IFV denotes innovations in friction volatility. XMKT, SMB, HML, and MOM refer to excess market returns, size, book-to-market, and momentum factors.

	XMKT	IV	IFV	SMB	HML	MOM
XMKT	1.00					
IV	-0.22	1.00				
IFV	-0.07	0.33	1.00			
SMB	-0.13	-0.07	-0.05	1.00		
HML	-0.65	0.15	-0.03	-0.21	1.00	
MOM	-0.07	-0.05	-0.02	0.14	-0.01	1.00

Table IX. Cross-correlation matrix of the monthly factors

IV represents innovations in optimally-sampled volatility, IFV denotes innovations in friction volatility. XMKT, SMB, HML, and MOM refer to excess market returns, size, book-to-market, and momentum factors. PS denotes innovations in Pastor and Stambaugh's (2003) illiquidity measure.

	XMKT	IV	IFV	PS	SMB	HML	MOM
XMKT	1.00						
IV	-0.41	1.00					
IFV	-0.28	0.57	1.00				
PS	0.23	-0.31	-0.23	1.00			
SMB	0.19	-0.19	-0.14	-0.01	1.00		
HML	-0.54	0.13	0.14	-0.02	-0.51	1.00	
MOM	-0.21	0.04	-0.01	-0.14	0.18	-0.07	1.00

Table X. Cross-correlation matrix of the friction proxies

Panel (a). IFV denotes innovations in friction volatility. IILL, PS, and IT denote innovations in Amihud's (2002) illiquidity measure, innovations in Pastor and Stambaugh's (2003) measure, and innovations in aggregate turnover. IILL and IT are un-scaled.

	IFV	PS	IILL	IT
IFV	1.00			
PS	-0.224	1.00		
IILL	0.235	-0.062	1.00	
IT	0.281	-0.02	-0.121	1.00

Panel (b). IFV denotes innovations in friction volatility. IILL, PS, and IT denote innovations in Amihud's (2002) illiquidity measure, innovations in Pastor and Stambaugh's (2003) measure, and innovations in aggregate turnover. IILL and IT are scaled.

	IFV	PS	IILL	IT
IFV	1.00			
PS	-0.224	1.00		
IILL	0.210	-0.017	1.00	
IT	0.258	0.002	-0.189	1.00

Table XI. Time series regression of excess returns on expected and unexpected frictions

We run a regression of excess returns on lagged logFV and the residuals from an AR(1) regression of logFV on lagged logFV, namely

$$R_{i,t} = \alpha^{i} + \beta^{i} \log FV_{t-1} + \lambda^{i} \text{residual}_{t}$$

where

$$\text{residual}_{t} = \log FV_{t} - \hat{\delta}_0 - \hat{\delta}_1 \log FV_{t-1} \text{ .}$$

We report results for the market (mkt) and the 2nd, 4th, 6th , 8th, and 10th size-decile portfolios.

	mkt	2nd	4th	6th	8th	10th
	1.19	1.28	1.48	1.36	1.39	1.48
β	(1.45)	(1.06)	(1.31)	(1.31)	(1.39)	(1.63)
	-2.91	-6.703	-5.525	-4.454	-3.795	-2.3
λ	(-2.49)	(-3.86)	(-3.42)	(-3.00)	(-2.64)	(-1.83)