Contracting with Asymmetric Information in Supply Chains

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April 27, 2007

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ABSTRACT

When a retailer holds no private information, a powerful supplier can use several contract types to extract for herself the first-best channel profit, leaving the retailer nothing but his reservation profit. In the case where the retailer holds private information on the probability distribution of market demand, most well analyzed contracts do not allow the supplier to achieve for herself the first-best channel profit. In equilibrium, the retailer is able to extract an information rent above his reservation profit, and the overall channel may deviate from its first-best solution. Such a result seems to conform with the general notion that when two parties deal with each other, the less informed party cannot avoid paying an information rent to the informed party. This paper shows that using judicially designed wholesale price contracts involving a buyback policy, a supplier can theoretically avoid paying any information rent to the privately informed retailer, and, at the same time, can extract all the first-best channel profit.

1 Introduction

Consider a powerful supplier who sells to a newsvendor retailer. When there is no information asymmetry, the supplier can use one of several simple contracts to extract for herself all the first-best channel profit, leaving the retailer nothing but his reservation profit. Examples of such contracts include a two-part tariff contract consisting of a wholesale price plus a lump-sum transfer (Moorthy 1987), and a wholesale price contract with a return/buyback policy for over stocked units (Pasternack 1985). For more examples of such contracts, see Cachon (2003).

What if the retailer does hold private information, for example, about the level or the probability distribution of market demand? Then, it can easily be demonstrated that none of those contracts mentioned above allows the supplier to achieve for herself the first-best channel profit: in equilibrium, the retailer gets an information rent above his reservation and the channel overall may not even be at its first-best. Such a result seems to conform with the general notion in game theory that when two parties deal with each other under information asymmetry, the less informed party needs to pay to induce information revelation. To the contrary, we show in this paper that by using judicially designed contracts, a supplier can indeed avoid paying any information rent to the privately informed retailer, and at the same time, extract all the first-best channel profit.

The particular contract form we consider in this paper consists of three parameters, a wholesale price, a buyback price and a lump-sum transfer. Retailer's private information is about the state of the market demand. We will first present a discrete model with two demand states to illustrate the basic idea. To demonstrate the generality of the model, we then extend the results to a setting where there is a continuum of demand states.

In many real-world supplier-retailer channels, the retailer is often endowed with superior information about the market condition or demand, due to his being closer to the market and having direct contact with customers. The supplier then faces the problem of designing an efficient contract to elicit the retailer's private information which could be critical for making informed decisions about her production capacity or quantity. Such a fundamental problem has recently attracted attention among researchers. For example, Arya and Mittendorf (2004) consider a supplier-retailer channel where the retailer holds private information about the value that final customers put on the product. In this setting, they consider a supplier who uses a return policy added to a quantity and transfer price contract to elicit retailer's information. While the resulting contract can improve supplier's profitability, it cannot eliminate the information rent the supplier has to pay, nor can it achieve the first-best solution for the channel.

Closest to our work is the recent paper by Taylor and Xiao (2006) who independently develop a result that is close to ours in nature. They consider a setting where a supplier sells to a newsvendor retailer who can choose to privately learn about market demand by exerting costly forecasting effort. The supplier offers contracts to influence the retailer's forecasting decision (i.e., whether or not to expend effort to forecast), and to elicit the retailer's information if forecasting is performed. They show that by using contracts with three parameters, namely, a quantity, a return price and a lump-sum transfer, the supplier can induce the retailer to choose the first-best decision for the channel, and at the same time, eliminate the information rent paid to the retailer.[1]

Under Taylor and Xiao's model setting, the supplier, in designing her menu of contracts, faces two levels of problem. First, whether to "endow" the retailer with private demand information (i.e., whether to induce the retailer to perform a forecast), and second, if this is done, and the retailer obtains private information, how to induce a truthful revelation of this information. In our paper, on the other hand, the retailer is exogenously endowed with private information, as is typical in an adverse selection setting, and thus, the supplier faces only the problem of inducing a truthful revelation of information. Aside from this difference in model settings, our work makes contributions in two respects. First, the form of our contracts has a linear wholesale price, allowing the retailer to choose the order quantity, while Taylor and Xiao in their contracts directly specify the order quantity for the retailer. Contracts specifying a linear wholesale price are more widely used in practice than contracts specifying a direct quantity. From a supplier's point of view, on the other hand, specifying a wholesale price and leaving the retailer with the right to choose quantity is, in general, a more restrictive policy, and hence a less efficient mechanism than specifying the order quantity for the retailer, especially under a setting with asymmetric information. The following question then arrises: how much efficiency does a supplier have to lose when using contracts involving a wholesale price rather than a direct order quantity? We show in this paper that when a return/buyback policy is added, a supplier can design wholesale price contracts so as to make her loss in efficiency approach arbitrarily close to zero. Second, we prove rigorously that the model extends to any number of demand states, while Taylor and Xiao consider only two-state models. This is an important extension since it indicates that the optimality of the policy is not linked to the number of free parameters relative to the number of states or possible types of retailers.

There is a broader supply chain literature studying contracting problems with asymmetric information under settings different from ours. These problems can in general be classified into two categories: screening problems where the uninformed party offers contracts to the informed to induce information revelation, and signaling problems in which the informed party offers contracts to the uninformed to signal the true information. Our work here belongs to the first category. Also, under this category, Corbett and deGroote (2000), Corbett (2001), Corbett and Tang (1999) and Ha (2001) consider supplier-retailer channels where one party holds private information about his costs (e.g., inventory ordering or holding costs). Lariviere (2002) compares return policy and quantity flexibility contracts in a setting similar to Taylor and Xiao, but where the information asymmetry is about retailer's forecasting costs. Burnetas, Gilbert and Smith (2005) study quantity discount contracts for a supplier to induce retailer's demand information, while Porteus and Whang (1999) consider minimum order quantity requirement contracts. None of these papers, however, identifies contract that allows a supplier or the uninformed party to extract the first-best channel profit. For the category of signaling problems, Cachon and Lariviere (2001) and Ozer

[1]Note, Taylor and Xiao consider another contract form that incorporates a sales rebate policy, rather than the return policy. They show that using such a contract, the supplier can neither avoid paying information rent nor achieve first-best channel profit.

and Wei (2006) study channels where an informed manufacturer (retailer) offers contracts to her supplier to secure production capacity. Readers interested in more details about this literature are directed to the two excellent review papers by Cachon (2003) and Chen (2003).

The main innovation of our work is to investigate contracts that combine wholesale pricing commitments, transfer payments, and return option features , simultaneously, in such a way so that the less informed party can extract information at essentially zero cost. Most contracting relationships in supply chains have focused on two of these three features. And in the absence of information asymmetry this has proved fruitful. For example, two-part tariff contracts have a commitment price and a fixed transfer fee with no option features. Buyback contracts typically have fixed wholesale prices with put options allowing the retailer to return unsold goods, but there is no fixed transfer fee. Several other policies have been investigated that provide retailers with flexibility for managing demand using call options, for example, that allows the retailer to order more quantities at predetermined prices, after demand has materialized, but typically these policies have not involved transfer payments as well. Our policies involve all three components. Our results indicate that these types of contracting relationships should be found in supply chains where there are large asymmetries of information, uncertainty in demand, and where the less informed supplier has significant market power, and can offer menus of choices to retailers who possess superior information.

The paper proceeds as follows. In section 2 we establish our basic model where there are two possible types of retailer and demonstrate how the information rent and loss in system efficiency can simultaneously be made arbitrarily small. The implications of this policy are explored. One wonders if this policy is a knife-edge result, depending critically on the assumption of there only being two possible types of retailer. Our main results are in section 3, where we consider a continuum of types of retailers, and show that the three parameter policies are still viable policies for the supplier to extract the full benefits of the supply chain and attain the first-best solution. Section 4 summarizes the findings and offers indications of the types of supply chains where these policies are most likely to be found useful.

2 A Model with Two Demand States

Consider a supplier who produces a product at a constant unit cost of c dollars, and contracts with a retailer for selling the product to the market. The retailer, as a price-taker, sells the product at a constant price for r dollars per unit.

Market demand for the product is uncertain and, depending on the market condition, can occur in two states: a low state L and a high state H. In particular, in the low state of the market, demand follows a probability distribution of $F_L(x)$ for $x \geq 0$. Demand in high state is *stochastically higher* than that in the low state, such that $F_L(x) \geq F_H(x)$ for all $x \geq 0$. Let $f_i(x)$ denote the density function of $F_i(x)$, for $i = L, H$.

Being closer to the market and having direct contacts with consumers, the retailer has a better knowledge about the market condition than the supplier. Specifically, the retailer knows for sure which of the two demand states will occur, while the supplier has only a subjective assessment about the likelihood of the two states. Let p be the probability that the supplier believes that the demand is in low state, and let $(1-p)$ be the probability that the demand is in the high state.

The goal of the supplier is to design a menu of contracts so as to maximize her own expected profit. To this end, from the revelation principal, see Myerson (1979), for example, the supplier can restrict her attention to a direct revelation mechanism that induces the retailer to truthfully report the market state through his choice of contracts from the menu. Hereafter, corresponding to the demand state L (H), we designate the retailer as a type L (H) retailer.

If the supply chain was centralized there would be no asymmetric information and the only decision to make would be how much to produce. For demand type i, $i = H, L$, it is easy to show that the optimal quantity $Q_i^c = \overline{F}_i^{-1}(\frac{c}{r})$, which is the first-best solution. In a decentralized system, if the supplier can induce the type i retailer choose the quantity Q_i^c, we say the channel is coordinated.

The policy that we consider for the supplier consists of three parameters: a per unit wholesale price, w, charged to the retailer for buying product, a buyback price, b, charged to the supplier for buying back each product that is unsold, and an additional fixed fee, T, for the retailer to pay. The supplier offers the retailer two contracts, denoted by (w_L, b_L, T_L) and (w_H, b_H, T_H), corresponding to the two demand states or retailer types, whose specific values are determined through maximizing supplier's expected profit function subject to retailer's individual rationality (IR) and incentive compatibility (IC) constraints. The IR constraints ensure that the retailer accepts a contract by meeting his reservation expected profit; and the IC constraints induce the retailer to choose the specific contract designated to his true type.

The retailer after selecting one of the two contracts, pays the fixed fee and decides on the number of products Q to buy. Then, depending on the realization of market demand, the retailer return any unsold products back to the supplier for the credit at the buyback price specified in the contract. Figure 1 specifies the sequences of events.

Figure 1: Here

Note that the values of parameters in any offered contract must satisfy a set of *strict* inequalities such that

$$b_i < w_i < r. \tag{1}$$

Otherwise, it cannot be a viable contract, since if $b_i \geq w_i$, the retailer simply chooses an infinite order quantity, regardless of his demand condition, and if $b_i < w_i \geq r$, the retailer never orders anything. As we will see later that it is this restrictive nature of parameters of wholesale price contracts that prevents the supplier from achieving a perfect, 100 percent channel efficiency and from achieving an absolute zero information rent. In contrast, contracts specifying directly order quantities for the retailer as analyzed by Taylor and Xiao (2006) allow a supplier to achieve both.

Let Π_i^R be the expected profit a retailer of type $i = L, H$ receives, and let Π^S be the expected profit the supplier expects to receive before issuing the contract menus. The *information rent* can be defined as expected profit of the retailer from supplier's point of view, i.e., $p\Pi_L^R + (1-p)\Pi_H^R$.

To keep the model simple and transparent, we assume that an overproduced/unsold unit bears zero salvage value or zero disposal cost in the system, and that an unsatisfied demand does not render any shortage penalty beyond a revenue loss to both parties. In addition, we assume that the retailer incurs zero cost to sell the items to market, and his reservation profit for entering business is normalized to be zero.

Before conducting the analysis we present a property about probability distributions that we will use later in our proofs:

Lemma 1 *Suppose $F_L(\cdot)$ and $F_H(\cdot)$ are two CDF functions such that $F_L(x) \geq F_H(x)$ for all $x \geq 0$, and $F_L(0) = F_H(0)$. Let $f_L(\cdot)$ and $f_H(\cdot)$ are the corresponding PDF functions. For any $y \geq 0$, let $k(y) = F_H^{-1}[F_L(y)]$. Then, the function*

$$K(y) = \int_0^{k(y)} x f_H(x)dx - \int_0^y x f_L(x)dx$$

is non-decreasing and non-negative for all $y \geq 0$.

Proof:

First, $F_H(k(y)) = F_L(y) \geq F_H(y)$. Since $F_H(\cdot)$ is a non-decreasing function, we have $k(y) \geq y$. Further, we can show that $f_H(k(y))'(y) = f_L(y)$. Now, $K(0) = 0$ and $K'(y) = k'(y)k(y)f_H(k(y)) - yf_L(y) = [k(y) - y]f_L(y) \geq 0$, which completes the proof.

We now consider the contracting problem between the supplier and the retailer. We first consider the retailer's decision. Facing two contracts (w_L, b_L, T_L) and (w_H, b_H, T_H) offered by the supplier, the retailer chooses one of them to accept together with an order quantity Q for the supplier to deliver. The number of products to return upon observing the realized demand D_i $(i = H, L)$ will be simply $\max(Q - D_i, 0)$, realizing that the buyback prices b_L and b_H in the contracts each have to be strictly less than wholesale price w_L and w_H respectively. Thus, for a type i retailer to choose contract j, for $i, j = L$ and H, his expected profit, as a function of his commitment quantity Q to choose, can be written as

$$\Pi_i^R(w_j, b_j, T_j, Q) = (r - b_j)\int_0^Q \overline{F}_i(x)dx - (w_j - b_j)Q - T_j, \tag{2}$$

where $\overline{F}_i(x) \equiv 1 - F_i(x)$. Let Q_{ij} be the corresponding optimal commitment quantity; that is,

$$Q_{ij} \equiv \underset{Q}{\operatorname{argmax}}\, \Pi_i^R(w_j, b_j, T_j, Q).$$

We can show that Q_{ij} satisfies

$$(r - b_j)\overline{F}_i(Q_{ij}) = w_j - b_j. \tag{3}$$

Substituting Q_{ij} of (3) into (2), we calculate the expected profit achieved by a type i retailer for choosing contract j as

$$\Pi_i^R(w_j, b_j, T_j, Q_{ij}) = (r - b_j)\int_0^{Q_{ij}} x f_i(x)dx - T_j \quad \text{for } i, j = L \text{ and } H.$$

We now turn to the supplier's problem. When a type i retailer chooses contract j, the supplier's expected profit, as a function of the contract parameters, can be written as

$$\Pi_i^S(w_j, b_j, T_j) = T_j + (w_j - b_j - c)Q_{ij} + b_j \int_0^{Q_{ij}} \overline{F}_i(x)dx. \tag{4}$$

The supplier's problem of maximizing her expected profit subject to the IR and IC constraints is described as follows:

$$\begin{aligned}
\max_{w_L, b_L, T_L, w_H, b_H, T_H} \quad & \Pi^s = p\Pi_L^S(w_L, b_L, T_L) + (1-p)\Pi_H^S(w_H, b_H, T_H) & \\
\text{s.t.} \quad & \Pi_L^R(w_L, b_L, T_L, Q_{LL}) \geq 0 & \text{(IR-1)} \\
& \Pi_H^R(w_H, b_H, T_H, Q_{HH}) \geq 0 & \text{(IR-2)} \\
& \Pi_L^R(w_L, b_L, T_L, Q_{LL}) \geq \Pi_L^R(w_H, b_H, T_H, Q_{LH}) & \text{(IC-1)} \\
& \Pi_H^R(w_H, b_H, T_H, Q_{HH}) \geq \Pi_H^R(w_L, b_L, T_L, Q_{HL}) & \text{(IC-2)}
\end{aligned}$$

From the retailer's optimality condition (3) for choosing his commitment quantities Q_{ij}, we have the following identity:

$$Q_{HL} = F_H^{-1}[F_L(Q_{LL})], \tag{5}$$

from which, we further have

$$\frac{\partial Q_{HL}}{\partial Q_{LL}} = \frac{f_L(Q_{LL})}{f_H(Q_{HL})}. \tag{6}$$

We now proceed to characterize the optimal contracts for the supplier by examining the four constraints in her optimization problem. First, the following arguments demonstrate that constraints (IR-1) and (IC-2) implies constraint (IR-2), and so (IR-2) is redundant.

$$\begin{aligned}
& \Pi_H^R(w_H, b_H, T_H, Q_{HH}) \\
= \ & (r - b_H)\int_0^{Q_{HH}} \overline{F}_H(x)dx - (w_H - b_H)Q_{HH} - T_H \\
\geq \ & (r - b_L)\int_0^{Q_{HL}} \overline{F}_H(x)dx - (w_L - b_L)Q_{HL} - T_L \quad \text{(by IC-2)} \\
\geq \ & (r - b_L)\int_0^{Q_{HL}} \overline{F}_H(x)dx - (w_L - b_L)Q_{HL} - \left[(r - b_L)\int_0^{Q_{LL}} \overline{F}_L(x)dx - (w_L - b_L)Q_{LL}\right] \quad \text{(by IR-1)} \\
= \ & (r - b_L)\left[\int_0^{Q_{HL}} x f_H(x)dx - \int_0^{Q_{LL}} x f_L(x)dx\right] \quad \text{(by Eq.(3))} \\
\geq \ & 0 \quad \text{(by Lemma 1 and Eq.(5))}
\end{aligned}$$

Second, the (IR-1) and (IC-2) constraints must both be binding at the optimal solution. (Otherwise, by increasing the fixed fees T_L and T_H until the constraints become binding, one always improves supplier's profit function.) In conjunction with the retailer's optimal conditions (3) for Q_{ij}, we can show that these two binding constraints lead to the following two conditions that supplier's optimal solution must satisfy:

$$T_L = (r - b_L) \int_0^{Q_{LL}} x f_L(x) dx, \tag{7}$$

and $$T_H = (r - b_H) \int_0^{Q_{HH}} x f_H(x) dx - (r - b_L) \left[\int_0^{Q_{HL}} x f_H(x) dx - \int_0^{Q_{LL}} x f_L(x) dx \right]. \tag{8}$$

The fact that the (IR-1) constraint is binding implies that an L type retailer will always make zero profit above his reservation profit, i.e., $\Pi_L^R(w_L, b_L, T_L, Q_{LL}) = 0$. The fact that the (IR-2) constraint is redundant or always satisfied, on the other hand, implies that in general an H type retailer does make a positive profit above his reservation, that is, $\Pi_H^R(w_H, b_H, T_H, Q_{HH}) > 0$. From the supplier's point of view, $(1-p)\Pi_H^R(w_H, b_H, T_H, Q_{HH})$ represents the expected information rent that she has to pay the retailer in exchange for obtaining demand information. These conclusion are true in general for this type of gaming problem with asymmetric information. What we will see from our further analysis, however, is that for our problem here, the supplier can effectively reduce the information rent to (almost) zero.

Third, in continuing our analysis, the (IC-1) constraint is actually redundant as well. This will be verified directly in the end, by using optimal solution obtained without considering this constraint.

In summary, the supplier's problem is to maximize her expected profit function $\Pi^S = p\Pi_L^S(w_L, b_L, T_L) + (1-p)\Pi_H^S(w_H, b_H, T_H)$ subject to the set of equality constraints including (3), (7) and (8). By substituting the relevant equality constraints into the objective function, we can show that the resulting unconstrained optimization problem has an objective Π^S as a function only of b_L, b_H, Q_{LL} and Q_{HH}, namely, $\Pi^S(b_L, b_H, Q_{LL}, Q_{HH})$, which can be written as

$$\Pi^S = p\left[r \int_0^{Q_{LL}} x f_L(x)dx - cQ_{LL}\right] + (1-p)\left\{r \int_0^{Q_{HH}} x f_H(x)dx - cQ_{HH} - (r - b_L)\left[\int_0^{Q_{HL}} x f_H(x)dx - \int_0^{Q_{LL}} x f_L(x)dx\right]\right\}.$$

We present the first-order derivatives as follows:

$$\begin{aligned}
\frac{\partial \Pi^S}{\partial b_L} &= (1-p)\left[\int_0^{Q_{HL}} x f_H(x)dx - \int_0^{Q_{LL}} x f_L(x)dx\right] \geq 0 \quad \text{(by Lemma 1)}; \\
\frac{\partial \Pi^S}{\partial b_H} &= 0; \\
\frac{\partial \Pi^S}{\partial Q_{LL}} &= p[r\overline{F}_L(Q_{LL}) - c] - (1-p)(r - b_L)\left[Q_{HL} f_H(Q_{HL}) \frac{\partial Q_{HL}}{\partial Q_{LL}} - Q_{LL} f_L(Q_{LL})\right] \\
&= p[r\overline{F}_L(Q_{LL}) - c] - (1-p)(r - b_L)(Q_{HL} - Q_{LL}) f_L(Q_{LL}); \\
\frac{\partial \Pi^S}{\partial Q_{HH}} &= (1-p)(r\overline{F}_H(Q_{HH}) - c).
\end{aligned}$$

Together with Eq.(3), i.e., $Q_{HL} = F_H^{-1}[F_L(Q_{LL})]$, an optimal solution satisfying the first-order conditions is derived as follows:

$$
\begin{aligned}
& b_L^* = r - \varepsilon \quad \text{with } \varepsilon \text{ being an arbitrary small positive number;} && (9)\\
& 0 < b_H^* < r \quad \text{(that is, } b_H^* \text{ is not unique);} && (10)\\
& p[r\overline{F}_L(Q_{LL}^*) - c] = (1-p)\varepsilon(Q_{HL}^* - Q_{LL}^*) f_L(Q_{LL}^*); && (11)\\
& Q_{HL}^* = F_H^{-1}[F_L(Q_{LL}^*)]; && (12)\\
\text{and} \quad & Q_{HH}^* = \overline{F}_H^{-1}(\frac{c}{r}). && (13)
\end{aligned}
$$

The need for a small positive number ε in $b_L^* = r - \varepsilon$ is due to the strict constraints of (1) as required by any viable contract. Once $b_L^*, b_H^*, Q_{LL}^*, Q_{HL}^*$ and Q_{HH}^* are found through (9)-(13), all other parameters in the optimal solution can be determined accordingly through equations (3), (7) and (8). For example, using (3), we get $w_L^* = r - \varepsilon F_L(Q_{LL}^*)$ and $w_H^* = b_H^* + (r - b_H^*)\frac{c}{r}$. We can check that constraints (1) are always satisfied by the optimal solutions.

To conclude our analysis for the two-state model, we now verify that the (IC-1) constraint is always satisfied by the optimal solution. To this end, we derive a set of inequalities that the optimal solution has to satisfy. First, since $0 < b_H^* < r$ of (10) is not unique, without loss of optimality we can set

$$b_H^* \leq b_L^*. \tag{14}$$

Second, considering (11), since $\varepsilon > 0$, the right hand side of (11) is positive, and, hence, we have $\overline{F}_L(Q_{LL}^*) > \frac{c}{r}$. It then follows from optimality conditions (13) and (3) that $\overline{F}_H(Q_{HL}^*) = \frac{w_L^* - b_L^*}{r - b_L^*} = \overline{F}_L(Q_{LL}^*) > \frac{c}{r} = \overline{F}_H(Q_{HH}^*) = \frac{w_H^* - b_H^*}{r - b_H^*} = \overline{F}_L(Q_{LH}^*)$, which leads us to

$$Q_{LL}^* \leq Q_{LH}^* \leq Q_{HH}^*, \tag{15}$$

$$Q_{HL}^* = F_H^{-1}[F_L(Q_{LL}^*)] \text{ and } Q_{HH}^* = F_H^{-1}[F_L(Q_{LH}^*)]. \tag{16}$$

Now, using (7) and (8), the (IC-1) constraints becomes:

$$
\begin{aligned}
& \text{Left side} - \text{right side} \\
= \;& (r - b_H^*)\left[\int_0^{Q_{HH}^*} x f_H(x)dx - \int_0^{Q_{LH}^*} x f_L(x)dx\right] - (r - b_L^*)\left[\int_0^{Q_{HL}^*} x f_H(x)dx - \int_0^{Q_{LL}^*} x f_L(x)dx\right] \\
\geq \;& (r - b_L^*)\left[\int_0^{Q_{HH}^*} x f_H(x)dx - \int_0^{Q_{LH}^*} x f_L(x)dx\right] - (r - b_L^*)\left[\int_0^{Q_{HL}^*} x f_H(x)dx - \int_0^{Q_{LL}^*} x f_L(x)dx\right] \\
\geq \;& 0,
\end{aligned}
$$

where, the first inequality is due to (14) and second follows from Lemma 1 in conjunction with (15) and (16). We thus verify the (IC-1) constraint.

The next Proposition summarizes the above results and gives the optimal solution.

Proposition 1 *Consider the supplier's problem for the menu of policies* (w_L, b_L, T_L) *and* (w_H, b_H, T_H) *for the two-state (H&L) problem.*

(i) The optimal buyback prices are:

$$\begin{aligned} b_L^* &= r - \varepsilon \\ b_H^* &\leq b_L^* \end{aligned}$$

where ε is a positive, arbitrarily small number, and b_H^* is not unique and can be any value less than b_L^*.

(ii) The optimal wholesale prices are:

$$\begin{aligned} w_L^* &= (r - b_L^*)\overline{F}_L(Q_{LL}^*) + b_L^* \\ w_H^* &= (r - b_H^*)\overline{F}_H(Q_{HH}^*) + b_H^*, \end{aligned}$$

where Q_{LL}^* is the solution of Eq.(11) and (12); and $Q_{HH}^* = \overline{F}_H^{-1}(\frac{c}{r})$.

(iii) The optimal transfer fees are:

$$T_L^* = (r - b_L^*)\int_0^{Q_{LL}^*} x f_L(x)dx, \tag{17}$$

$$T_H^* = (r - b_H^*)\int_0^{Q_{HH}^*} x f_H(x)dx - (r - b_L^*)\left[\int_0^{Q_{HL}^*} x f_H(x)dx - \int_0^{Q_{LL}^*} x f_L(x)dx\right]. \tag{18}$$

The proof is straightforward and thus omitted. Note that since b_H^* is not unique, neither is w_H^* nor T_H^*. In particular, for example, we can let $b_H^* = 0$ and by substituting it into the expressions for w_H^* and T_H^* we get another optimal solution.

In any of our optimal solutions, only $Q_{HH}^* = Q_H^c$, and $Q_{LL}^* \neq Q_L^c$. So an optimal solution for the supplier is not the first-best solution of the channel. However, we will show that such a second-best solution can be arbitrarily close to the first-best solution.

It can be shown that $w_L^* \geq w_H^*$ and $T_L^* \leq T_H^*$ for any b_H^*. This means that the low type retailer pays a lower transfer fee and higher wholesale price and the high type retailer pays a higher transfer fee and lower wholesale price.

2.1 Implication of The Supplier's Optimal Contracts

We show that as the supplier chooses an arbitrarily small ε in her optimal policy, she induces the retailer to choose commitment (production) quantities that are arbitrarily close to the first-best quantities for the channel, under each demand state; and, at the same time, she squeezes the information rent that she has to pay the retailer arbitrarily close to zero. As a consequence, when using such a contract to deal with a more informed retailer, the supplier can overcome her disadvantage in information and extract nearly all the maximum channel profit for herself.

Proposition 2 *In the presence of asymmetric information, using contracts of the form (w, b, T), the supplier can (almost) coordinate the supply chain and the information rent for the supplier can be driven arbitrarily close to zero.*

Proof

First, note that the channel optimal production quantities, denoted by Q_L^* and Q_H^*, satisfy $Q_L^* = \overline{F}_L^{-1}(\frac{c}{r})$ and $Q_H^* = \overline{F}_H^{-1}(\frac{c}{r})$ respectively for the two demand states. Now, in the decentralized channel, when the supplier offers her optimal contracts, the retailer in H state is induced to choose $Q_{HH}^* = \overline{F}_H^{-1}(\frac{c}{r})$, from (13), which is exactly the quantity Q_H^* optimal for the channel. In L state, as $\varepsilon \to 0$, retailers quantity Q_{LL}^*, determined by (11), approaches arbitrarily close to $Q_{LL}^* \approx \overline{F}_L^{-1}(\frac{c}{r})$, which again is the optimal quantity Q_L^* for the channel. The information rent can be calculated as

$$(1-p)\Pi_H^R = \varepsilon(1-p)\left[\int_0^{Q_{HL}^*} xf_H(x)dx - \int_0^{Q_{LL}^*} xf_L(x)dx\right],$$

which approaches to zero as $\varepsilon \to 0$.

Discussion

When there is no information asymmetry (i.e., when the supplier knows the retailer's type i), the supplier can use simpler contracts to extract the first-best channel profit all for herself, leaving the retailer with zero (or his reservation) profit. For example, if the supplier knows the retailer is of type i, she can offer a single contract of the form (w_i, T_i). The retailer is then induced to choose the order quantity Q_i^* according to $r\overline{F}_i(Q_i^*) = w_i$, so long as he can meet his reservation profit after paying the transfer fee of T_i. Now, by simply setting $w_i = c$, the supplier induces the first-best channel quantity $Q_i^* = \overline{F}_i^{-1}(\frac{c}{r})$. Moreover, since the supplier knows the retailer's type she can compute the exact expected profit the retailer gains when ordering the channel optimal quantity at the price c, and hence by adjusting T_i, she leaves the retailer with zero profit.

Using contracts of the form (w_i, T_i) with asymmetric information, the supplier cannot extract all the first-best channel profit. Suppose that she offers the two contracts $(w_H = c, T_H)$ and $(w_L = c, T_L)$ that correspond to the two contracts under the setting with full information. Since $T_H > T_L$ (as can be easily demonstrated), retailers of both types will obviously choose the second contract. In doing so, both types of retailers will order the corresponding first-best quantity. But, a H type retailer gains an above-reservation profit of $(T_H - T_L)$, which is essentially the information rent paid by supplier. While most efficient for the channel, such contracts are not optimal for the supplier herself.

The supplier's optimal menu of contracts in this case (i.e., within the contract form of (w_i, T_i)) can be derived from the solution to the problem of our general contract form (w_i, T_i, b_i) by forcing $b_L^* = b_H^* = 0$. In particular, the resulting order quantity $Q_{HH}^* = \overline{F}_H^{-1}(\frac{c}{r})$ for H type retailer equals the first-best quantity, the order quantity Q_{LL}^* for a L type retailer, determined through $p[r\overline{F}_L(Q_{LL}^*) - c] = (1-p)r(Q_{HL}^* - Q_{LL}^*)f_L(Q_{LL}^*)$ where $Q_{HL}^* = F_H^{-1}[F_L(Q_{LL}^*)]$, however, can be shown to be *strictly* lower than the channel optimal quantity $\overline{F}_H^{-1}(\frac{c}{r})$, and furthermore, the expected profit (or information rent) gained by the H type retailer, given by $(1-p)\Pi_H^R = r(1-p)\left[\int_0^{Q_{HL}^*} xf_H(x)dx - \int_0^{Q_{LL}^*} xf_L(x)dx\right]$, is *strictly* positive or larger than his reservation profit. So, using these contracts optimally, the supplier fails to extract all the first-best channel profit.

In determining her optimal contracts, the supplier essentially has to balance the overall channel efficiency against the information rent she needs to pay. A buyback policy, b_i, when added to the above two-parameter contract, provides the supplier with additional leverage or an additional degree of freedom to adjust to a balance point so that the overall channel gets arbitrarily close to the first-best optimality and at the same time allows the information rent to become arbitrarily close to zero.

Finally, we point out that a (w_i, T_i, b_i) contract can be equivalently implemented through a call option contract, where the retailer pays the fixed fee of T_i, and is allowed to order options at the price $(w_i - b_i)$ that have a strike price b_i for exercising. Before the realization of demand, the supplier produces according to the number of options that the retailer orders. Ignoring the time value of money, the two contracts have a one-to-one correspondence and hence are equivalent to each other.

3 A Generalized Model with Continuous Demand States

In the previous section, we showed that the supplier can pay (almost) zero *information rent* in an asymmetric information scenario using the buyback contract (w, b, T). A question arises as to whether this result is a fairly general result or if it holds specifically for two states. Perhaps, for example, the optimality of these results derives from the fact that the number of free parameters relates to the number of different states. In this section, we consider extending the results to the case where the number of states forms a continuum.

In order to study this problem we need to modify our notation. Since the demand type θ is continuous, we define the support of type θ as $[\underline{\theta}, \overline{\theta}]$, where $-\infty \leq \underline{\theta} < \overline{\theta} \leq \infty$. The demand type θ is random and has CDF and PDF function $G(\cdot)$ and $g(\cdot)$ respectively. The CDF and PDF of type θ demand is $F(\cdot, \theta)$ and $f(\cdot, \theta)$ respectively. Assume both F and f is differentiable on θ in $(\underline{\theta}, \overline{\theta})$ and $\dfrac{\partial F}{\partial \theta}(x, \theta) \leq 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ and all $x \geq 0$, i.e., $F(x, \theta) \geq F(x, \phi)$ for $\theta \leq \phi$ and for all $x \geq 0$. This condition tells us that F is stochastically increasing in θ, which is consistent with our model assumption about two demand states.

The supplier's objective is to find a contract $(w(\theta), b(\theta), T(\theta))$ to maximize her expected profit conditional on all possible θ. Then, the supplier's model is changed to the following form:

$$\begin{array}{rl}
\displaystyle\max_{w(\theta), b(\theta), T(\theta)} & \displaystyle\int_{\underline{\theta}}^{\overline{\theta}} \Pi^S(\theta) g(\theta) d\theta \\
\text{s.t.} & \Pi^R(\theta, \theta) \geq 0 \quad \forall \theta \qquad \text{(IR)} \\
& \Pi^R(\theta, \theta) \geq \Pi^R(\theta, \phi) \quad \forall \theta \neq \phi \qquad \text{(IC)}
\end{array}$$

where $\Pi^R(\theta, \phi)$ is type θ retailer's optimal profit if he chooses contract type ϕ, and $\Pi^S(\theta)$ is the supplier's optimal profit when she faces a type θ retailer.

The (IR) constraint says that any type retailer must obtain at least his reservation profit (which is normalized to zero here), and the (IC) constraint says that type θ retailer will optimally choose contract type θ.

We begin our analysis of this model by considering the retailer's profit. Since

$$\Pi^R(\theta, w(\phi), b(\phi), T(\phi), Q) = [r - b(\phi)] \int_0^Q \overline{F}(x, \theta) dx - [w(\phi) - b(\phi)]Q - T(\phi), \tag{19}$$

let

$$Q(\theta, \phi) = \operatorname*{argmax}_Q \Pi^r(\theta, w(\phi), b(\phi), T(\phi), Q), \tag{20}$$

where $Q(\theta, \phi)$ satisfies

$$[r - b(\phi)]\overline{F}(Q(\theta, \phi), \theta) = w(\phi) - b(\phi). \tag{21}$$

Then

$$\frac{\partial w}{\partial \phi}(\phi) - \frac{\partial b}{\partial \phi}(\phi) = -\frac{\partial b}{\partial \phi}(\phi)\overline{F}(Q(\theta, \phi), \theta) - [r - b(\phi)]\frac{\partial Q}{\partial \phi}(\theta, \phi) f(Q(\theta, \phi), \theta). \tag{22}$$

Define

$$\overline{Q}(\theta) = Q(\theta, \theta),$$

then it is easy to show that

$$\frac{\partial \overline{Q}}{\partial \theta}(\theta) = \frac{\partial Q}{\partial \phi}(\theta, \theta). \tag{23}$$

We know that

$$\begin{aligned}
\Pi^R(\theta, \phi) &= [r - b(\phi)] \int_0^{Q(\theta,\phi)} \overline{F}(x, \theta) dx - [w(\phi) - b(\phi)]Q(\theta, \phi) - T(\phi) \\
&= [r - b(\phi)] \int_0^{Q(\theta,\phi)} \overline{F}(x, \theta) dx - [r - b(\phi)]\overline{F}(Q(\theta, \phi), \theta)Q(\theta, \phi) - T(\phi) \\
&= [r - b(\phi)] \int_0^{Q(\theta,\phi)} x f(x, \theta) dx - T(\phi).
\end{aligned}$$

The IC constraint is equivalent to

$$\frac{\partial \Pi^R}{\partial \phi}(\theta, \theta) = 0, \tag{24}$$

$$\text{and} \quad \frac{\partial^2 \Pi^R}{\partial \phi^2}(\theta, \theta) \leq 0, \tag{25}$$

where Eq.(24) is the first order condition and Eq.(25) is the second order condition. Eq.(24) can be written as:

$$\begin{aligned}
\frac{\partial \Pi^R}{\partial \phi}(\theta, \theta) &= [r - b(\theta)]\frac{\partial Q}{\partial \phi}(\theta, \theta)Q(\theta, \theta)f(Q(\theta, \theta), \theta) - \frac{\partial b}{\partial \phi}(\theta) \int_0^{Q(\theta,\theta)} x f(x, \theta) dx - \frac{\partial T}{\partial \phi}(\theta) \\
&= [r - b(\theta)]\frac{\partial Q}{\partial \phi}(\theta, \theta)\overline{Q}(\theta) f(\overline{Q}(\theta), \theta) - \frac{\partial b}{\partial \theta}(\theta) \int_0^{\overline{Q}(\theta)} x f(x, \theta) dx - \frac{\partial T}{\partial \theta}(\theta) \\
&= [r - b(\theta)]\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta) f(\overline{Q}(\theta), \theta) - \frac{\partial b}{\partial \theta}(\theta) \int_0^{\overline{Q}(\theta)} x f(x, \theta) dx - \frac{\partial T}{\partial \theta}(\theta) \\
&= 0
\end{aligned} \tag{26}$$

The retailer's optimal profit function when he tells the truth is defined as

$$V(\theta) = \Pi^R(\theta, \theta) = [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f(x,\theta) dx - T(\theta), \tag{27}$$

then

$$\begin{aligned}\frac{\partial V}{\partial \theta} &= -\frac{\partial b}{\partial \theta}(\theta) \int_0^{\overline{Q}(\theta)} x f(x,\theta) dx + [r - b(\theta)] \frac{\partial \overline{Q}}{\partial \theta}(\theta) \overline{Q}(\theta) f(\overline{Q}(\theta), \theta) \\ &\quad + [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f_\theta(x,\theta) dx - \frac{\partial T}{\partial \theta}(\theta) \\ &= [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f_\theta(x,\theta) dx,\end{aligned}$$

where

$$f_\theta(x,\theta) \equiv \frac{\partial f}{\partial \theta}(x,\theta).$$

Therefore,

$$V(\theta) = \int_{\underline{\theta}}^{\theta} \left\{ [r - b(\tau)] \int_0^{\overline{Q}(\tau)} x f_\tau(x,\tau) dx \right\} d\tau. \tag{28}$$

From Eq.(27) and Eq.(28), we can write $T(\theta)$ as

$$\begin{aligned}T(\theta) &= [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f(x,\theta) dx - V(\theta) \\ &= [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f(x,\theta) dx - \int_{\underline{\theta}}^{\theta} \left\{ [r - b(\tau)] \int_0^{\overline{Q}(\tau)} x f_\tau(x,\tau) dx \right\} d\tau.\end{aligned}$$

The supplier's profit can be written as:

$$\begin{aligned}\Pi^S(\theta) &= T(\theta) - c\overline{Q}(\theta) + [r - b(\theta)] \overline{F}(\overline{Q}(\theta,\theta) \overline{Q}(\theta) + b(\theta) \int_0^{\overline{Q}(\theta)} \overline{F}(x,\theta) dx \\ &= T(\theta) - c\overline{Q}(\theta) + r \int_0^{\overline{Q}(\theta)} \overline{F}(x,\theta) dx - [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f(x,\theta) dx \\ &= r \int_0^{\overline{Q}(\theta)} \overline{F}(x,\theta) dx - c\overline{Q}(\theta) - \int_{\underline{\theta}}^{\theta} \left\{ [r - b(\tau)] \int_0^{\overline{Q}(\tau)} x f_\tau(x,\tau) dx \right\} d\tau.\end{aligned}$$

By using integration by parts, we can obtain that

$$\begin{aligned}&\int_{\underline{\theta}}^{\overline{\theta}} g(\theta) \int_{\underline{\theta}}^{\theta} \left\{ [r - b(\tau)] \int_0^{\overline{Q}(\tau)} x f_\tau(x,\tau) dx \right\} d\tau \, d\theta \\ = &\int_{\underline{\theta}}^{\overline{\theta}} [1 - G(\theta)][r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f_\theta(x,\theta) dx \, d\theta,\end{aligned}$$

and the objective function is

$$\begin{aligned}&\int_{\underline{\theta}}^{\overline{\theta}} \Pi^S(\theta) g(\theta) \, d\theta \\ = &\int_{\underline{\theta}}^{\overline{\theta}} g(\theta) \left\{ r \int_0^{\overline{Q}(\theta)} \overline{F}(x,\theta) dx - c\overline{Q}(\theta) - \frac{1 - G(\theta)}{g(\theta)} [r - b(\theta)] \int_0^{\overline{Q}(\theta)} x f_\theta(x,\theta) dx \right\} d\theta \\ = &\int_{\underline{\theta}}^{\overline{\theta}} g(\theta) H(\theta) d\theta,\end{aligned}$$

where

$$H(\theta) = r\int_0^{\overline{Q}(\theta)} \overline{F}(x,\theta)dx - c\overline{Q}(\theta) - \frac{1-G(\theta)}{g(\theta)}[r-b(\theta)]\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx. \tag{29}$$

Now the objective is to find $\overline{Q}^*(\theta)$ and $b^*(\theta)$ that maximize $H(\theta)$ for each $\theta \in [\underline{\theta}, \overline{\theta}]$. The first order conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial b}(\theta) &= \frac{1-G(\theta)}{g(\theta)}\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx & (30)\\
\frac{\partial H}{\partial \overline{Q}}(\theta) &= r\overline{F}(\overline{Q}(\theta),\theta) - c - \frac{1-G(\theta)}{g(\theta)}[r-b(\theta)]\overline{Q}(\theta)f_\theta(\overline{Q}(\theta),\theta) & (31)
\end{aligned}$$

It seems hard to get the analytical optimal solution from the first order conditions. However, Lemma 2 tells us more about $\displaystyle\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx$.

Lemma 2 *Suppose all notation are defined above. And $F_\theta(x,\theta) \leq 0$. Then*

$$\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx \geq 0$$

Proof:

The integral $\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx$, can be written as

$$\lim_{\phi\to\theta} \frac{\int_0^{Q(\phi,\theta)} xf(x,\phi)dx - \int_0^{Q(\theta,\theta)} xf(x,\theta)dx}{\phi-\theta}.$$

Without loss of generality, let's assume $\phi > \theta$. We need only to show the numerator is positive. By using Lemma 1, the result is immediate.

By Lemma 2 , we see that $\dfrac{\partial H}{\partial b}(\theta) \geq 0$, which means $b^*(\theta)$ can be as big as possible. Recall that $b(\theta) < r$, thus, $b^*(\theta) = r - \varepsilon(\theta)$, where $\varepsilon(\theta)$ is an arbitrarily small positive number depending on type θ.

By Lemma 3 in Appendix, b^* is non-increasing in θ. This guarantees that $\overline{Q}(\theta)$ is the maxima of $\Pi^R(\theta,\cdot)$. Since $r - b^*(\theta) = \varepsilon(\theta) \to 0$, from Eq.(31), we have:

$$r\overline{F}(\overline{Q}^*(\theta),\theta) \approx c, \tag{32}$$

or, $\overline{Q}^*(\theta) \to \overline{F}^{-1}(\frac{c}{r},\theta)$ as $\varepsilon(\theta)\to 0$. And from Eq.(21), $w^*(\theta) = r - \varepsilon(\theta)F(\overline{Q}^*(\theta),\theta)$.

We are now ready to state our main proposition.

Proposition 3 *The optimal solution for the buyback contract $(w(\theta), b(\theta), T(\theta))$ with continuous demand type θ is:*

$$\begin{aligned}
b^*(\theta) &= r - \varepsilon(\theta),\\
w^*(\theta) &= (r - b^*(\theta))\overline{F}(\overline{Q}^*(\theta),\theta) + b^*(\theta),
\end{aligned}$$

where $\overline{Q}^*(\theta)$ satisfies

$$r\overline{F}(\overline{Q}^*(\theta),\theta) - c = \varepsilon(\theta)\overline{Q}^*(\theta) f_\theta(\overline{Q}^*(\theta),\theta)\frac{1-G(\theta)}{g(\theta)},$$

and $\varepsilon(\theta)$ is very small, positive and nondecreasing in θ. Finally, the transfer price is:

$$T^*(\theta) = [r - b^*(\theta)]\int_0^{\overline{Q}^*(\theta)} x f(x,\theta)dx - \int_{\underline{\theta}}^{\theta}\left\{[r-b^*(\tau)]\int_0^{\overline{Q}^*(\tau)} x f_\tau(x,\tau)dx\right\} d\tau. \tag{33}$$

Proof :

The proof follows directly from the above analysis and is straightforward, so omitted.

In continuous states, the *information rent* is defined as the retailer's expected profit from supplier's point of view, which is:

$$\int_{\underline{\theta}}^{\overline{\theta}} V(\theta)g(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} [1-G(\theta)][r-b^*(\theta)]\int_0^{\overline{Q}^*(\theta)} x f_\theta(x,\theta)dx\, d\theta. \tag{34}$$

We can show that *information rent* goes to zero as $\varepsilon(\theta)$ goes to zero.

From the discussion above, we see that as $\varepsilon(\theta) \to 0$, $\overline{Q}^*(\theta) \to \overline{F}^{-1}(\frac{c}{r},\theta)$, which is the system optimal order quantity. Then we have the following proposition.

Proposition 4 *Using a buyback contract of the type (w,b,T) in continuous demand type scenario, the supplier's optimal contract $(w^*(\theta), b^*(\theta), T^*(\theta))$ can (almost) coordinate the channel and the* information rent *for the supplier is (almost) zero.*

Proof :

The proof is similar to the proof of Proposition 1 and thus omitted.

From Proposition 1 and Proposition 4 we see that the results of continuous demand type model is consistent with two demand type model. For example, state H (L) for the two state model corresponds to state $\overline{\theta}$ $(\underline{\theta})$. From (28), $V(\underline{\theta}) = 0$, which corresponds to $\Pi_L^R = 0$; Also, $b^*(\theta)$ is non-increasing in θ corresponds to $b_L^* \geq b_H^*$; Further, both $b^*(\theta)$ and b_L^* converge to r as $\varepsilon(\theta)$ or ε is an arbitrarily small positive number; Finally, $w^*(\theta)$ and w_i^* have the same structure.

From Eq.(17), (18) and Eq.(33), T_i is discrete form of $T^*(\theta)$. These comparisons show that the two-state model is just a special case of the continuous-state model. This results suggests that similar results may hold for any n-state model.

4 Conclusion

There is an array of contracts that exist between suppliers and retailers. The terms of contracts vary with the topology of the problem. The cost structure in preparing goods for delivery, the nature of the demand distribution, the flexibility the retailer has in altering retail prices, the degree of substitutability, and other factors all impact terms of trade. This paper has been focused on the case where there is a powerful supplier who is less informed about the nature of demand for a seasonal good that is sold by the more informed retailer. In the literature much attention has been placed on contracting relationships that coordinate the supply chain. Typically these contracts have involved features such as fixed wholesale prices plus a transfer fee, or fixed wholesale prices plus an option feature allowing the retailer to buy more at predetermined prices, or to return unsold goods. Combining these option features, gives the retailer more flexibility to manage uncertain demands, but this in turn allows the Stackelberg leader, namely the supplier, to extract more rent from the retailer. Without asymmetric information, simple policies exist for the supplier to extract all surplus from the retailer and obtain the first-best solution. However, the general opinion has been that with asymmetric information, the supplier must give up some profit in return for being less informed than the retailer. The informational rent indeed is the expected value of information costs viewed through the lens of the supplier. Our paper contributes to the literature by showing that even with large asymmetries of information, the supplier can design contracts that essentially extract all the surplus and attain almost the first-best solution for the supply chain.

Appendix

Lemma 3 *$b^*(\theta)$ is non-increasing in θ.*

Proof

From Eq.(26), we have

$$\frac{\partial T}{\partial \theta}(\theta) \quad = \quad [r-b(\theta)]\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) - \frac{\partial b}{\partial \theta}(\theta)\int_0^{\overline{Q}(\theta)} xf(x,\theta)dx,$$

then we get

$$\begin{aligned}
\frac{\partial^2 T}{\partial \theta^2}(\theta) \quad = \quad & -\frac{\partial b}{\partial \theta}(\theta)\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) + [r-b(\theta)]\frac{\partial^2 \overline{Q}}{\partial \theta^2}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) \\
& +[r-b(\theta)]\left[\frac{\partial \overline{Q}}{\partial \theta}(\theta)\right]^2 f(\overline{Q}(\theta),\theta) + [r-b(\theta)]\left[\frac{\partial \overline{Q}}{\partial \theta}(\theta)\right]^2 \overline{Q}(\theta)f'(\overline{Q}(\theta),\theta) \\
& -\frac{\partial^2 b}{\partial \theta^2}(\theta)\int_0^{\overline{Q}(\theta)} xf(x,\theta)dx - \frac{\partial b}{\partial \theta}(\theta)\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) \\
& -\frac{\partial b}{\partial \theta}(\theta)\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx.
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial^2 \Pi^R}{\partial \phi^2}(\theta,\theta) \quad = \quad & -\frac{\partial b}{\partial \theta}(\theta)\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) + [r-b(\theta)]\frac{\partial^2 \overline{Q}}{\partial \theta^2}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) \\
& +[r-b(\theta)]\left[\frac{\partial \overline{Q}}{\partial \theta}(\theta)\right]^2 f(\overline{Q}(\theta),\theta) + [r-b(\theta)]\left[\frac{\partial \overline{Q}}{\partial \theta}(\theta)\right]^2 \overline{Q}(\theta)f'(\overline{Q}(\theta),\theta) \\
& -\frac{\partial^2 b}{\partial \theta^2}(\theta)\int_0^{\overline{Q}(\theta)} xf(x,\theta)dx - \frac{\partial b}{\partial \theta}(\theta)\frac{\partial \overline{Q}}{\partial \theta}(\theta)\overline{Q}(\theta)f(\overline{Q}(\theta),\theta) \\
& -\frac{\partial^2 T}{\partial \theta^2}(\theta) \\
= \quad & \frac{\partial b}{\partial \theta}(\theta)\int_0^{\overline{Q}(\theta)} xf_\theta(x,\theta)dx.
\end{aligned}$$

From Eq.(25) on Page 12 and Lemma 2 on Page 14, we obtain that $\frac{\partial b}{\partial \theta}(\theta) \leq 0$, which proves that $b^*(\theta)$ is non-increasing in θ.

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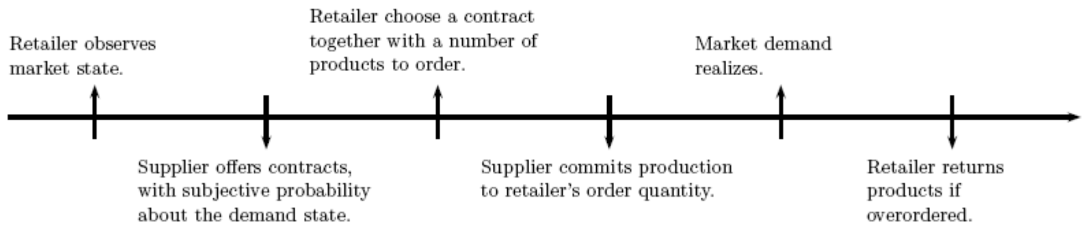

Figure 1: The Sequence of Events